

**Project PAJ3 - Combined Cyclic Loading
and Hostile Environments 1996-1999**

Report No 16

Statistical Analysis of Durability Data

T J Twine (Beta Technology) and M Hall (Xyratex, Havant)

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ABSTRACT

The performance of adhesively bonded joints is often considered to be too variable for all but the least demanding of applications. The variation often observed is due to a lack of knowledge and control of critical variables in the joint manufacturing and testing processes. Such variation can be magnified by durability testing to a point where no statistically valid conclusions can be made even though significant time, effort and cost have usually been expended. Design of experiments (DOE) is a rational scientific approach which enables quantification of input variables and their interactions on the output (e.g. joint strength). The technique allows the user to refine a process in order to optimise the output according to specific requirements. This approach avoids subjectivity and when correctly applied can improve performance characteristics, reduce costs and decrease product development and production time.

This report presents a compilation of case studies, demonstrating the effectiveness of DOE and other statistical analysis techniques in assessing durability data, quantifying critical factors and their interactions which give rise to environmental failure, and determining the statistical validity of any conclusions drawn by the case study authors. The case studies presented in this report are based on durability data supplied by the Defence Evaluation and Research Agency (Farnborough, United Kingdom) and generated within the Department of Trade and Industry funded Measurement Technology and Standards (MTS) projects "Environmental Durability of Adhesive Bonds, ADH3 (1993-1996)" and "Performance of Adhesive Joints - Combined Loading and Hostile Environments PAJ3 (1996-1999)".

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National Physical Laboratory
Teddington, Middlesex, UK, TW11 0LW

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Approved on behalf of Managing Director, NPL, by Dr C Lea,
Head of Centre for Materials Measurement and Technology.

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FOREWORD

The performance of adhesively bonded joints is often considered to be too variable for all but the least demanding of applications. The variation often observed is due to a lack of knowledge and control of critical variables in the joint manufacturing and testing processes. Such variation can be magnified by durability testing to a point where no statistically valid conclusions can be made even though significant time, effort and cost have usually been expended. Design of experiments (DOE) is a rational scientific approach which enables quantification of input variables and their interactions on the output (e.g. joint strength). The technique allows the user to refine a process in order to optimise the output according to specific requirements. This approach avoids subjectivity and when correctly applied can improve performance characteristics, reduce costs and decrease product development and production time. Experimental design provides:

- An effective method for identification of the key input factors.
- An efficient method of understanding the relationship between input factors and the response.
- A means of constructing a mathematical model relating response to input factors.
- A means of adjusting input parameters to optimise the response according to the requirements.
- A scientific method for controlling process/experiment tolerances.

This report presents a compilation of case studies, demonstrating the effectiveness of statistical analysis techniques in assessing durability data, quantifying critical factors and their interactions which give rise to environmental failure, and determining the statistical validity of any conclusions drawn by the case study authors. The case studies presented in this report are based on durability data supplied by the Defence Evaluation and Research Agency (Farnborough, United Kingdom) and generated within the Engineering Industries Directorate of the United Kingdom Department of Trade and Industry (DTI) funded Measurement Technology and Standards (MTS) projects “Environmental Durability of Adhesive Bonds, ADH3 (1993-1996)” and “Performance of Adhesive Joints - Combined Loading and Hostile Environments PAJ3 (1996-1999)”.

Four case studies are considered:

Case Study 1: Data from DTI MTS Project 3 Report No 8.

Case Study 2: Data from Report DRA / SMC / CR951184.

Case Study 3: NPL Durability Data Analysis and Modelling.

Case Study 4: Statistical Analysis of NPL Experiment on Single-Lap Joints.

Case studies 3 and 4 relate to experimental work carried out at the National Physical Laboratory (NPL) on perforated single-lap joints immersed in distilled/deionised water at three temperatures for up to 42 days. The two studies consider different approaches to data interpretation.

The research discussed in this report forms part of the DTI funded project on “Performance of Adhesive Joints - Combined Cyclic Loading and Hostile Environments”, which aims to develop and validate test methods and environmental conditioning

procedures that can be used to measure parameters required for long-term performance predictions. This project is one of three technical projects forming the programme on “Performance of Adhesive Joints - A Programme in Support of Test Methods”.

Case Study 1
Data from DTI MTS Project 3 Report Number 8

1. INTRODUCTION

The first case study relates to work on the single-lap shear test that formed part of an experimental assessment of durability test methods carried out within the DTI funded Adhesives Programme ADH Project 3 “Environmental Durability of Adhesive Joints” [1]. Although, testing with this configuration is relatively straightforward and cost effective, there is concern about the variability of results within groups of tests and, hence, the reliability of conclusions when comparing the performance of different systems. The case study considers the results of an initial round-robin exercise involving five organisations (Test Houses), which showed that there was considerable variation in test results. The large uncertainties observed were attributed to inconsistencies in specimen preparation rather than strength testing procedures. As part of the durability programme, considerable effort was employed in refining specimen preparation procedures in order to reduce variation in strength data (see Table 1 in Appendix 1).

The original objective of the work within Project 3 was to determine the sensitivity of the single-lap shear test to discriminate between two surface treatments for each substrate/adhesive combination exposed to one specific ageing environment. The surface treatments were selected to reflect a “Normal” general engineering requirement and a “High” performance demanding environmental requirement. A general conclusion drawn at the time of the experimental programme was that nearly all tests showed improved performance with the “High” performance surface treatment (Table 1.1 [1]). However, there are many caveats relating to the sensitivity of the lap shear test method to unknown variables, which could mask changes attributable to controlled factors being investigated.

This report covers a subsequent statistically based analysis of the test results from the case study to determine whether there is a significant difference between:

- “Normal” and “High” treatments; and
- Test Houses nominally testing the same substrate/adhesive/surface treatment combinations.

2. ANALYSIS

The results of the case study analysed are shown in Appendix 1 [1]. It should be noted that the test for significant difference between the two surface treatments has been performed on the Control (time zero, no exposure) data sets and 12 weeks environmental exposure data sets. The test for significance between Test Houses nominally testing the same specimens has been performed on the Control data sets only.

2.1 INITIAL DATA ANALYSIS

An initial simple visual assessment on the integrity of the data shown in Table 1.1 (Appendix 1) can be made using a whisker plot where the mean and ± 2 standard deviation lines for each cell are chartered together. Appendix 2 shows these charts for both the Control and 12 weeks exposure data sets for the “Normal” and “High” surface

treatments. These charts compare data cells from the individual substrate/adhesive/Test House combinations.

For each substrate/adhesive/surface treatment group of whisker plots, the visual assessment showed a number of cases where there appeared to be significant differences in both means and distributions, which indicated that a more detailed analysis was required.

2.2 TEST FOR SIGNIFICANCE BETWEEN “NORMAL” AND “HIGH” TREATMENTS

The results were examined for significant differences between the “Normal” and “High” treatments for specimens tested at each Test House. Analysis was conducted on the control and 12 Weeks data sets, which are shown in Appendix 3. For the Control data sets, it can be seen from Columns 12 and 13 in Table 2 (Appendix 3) that in 8 out of 19 cases there was a significant difference in the mean bond strength between “Normal” and “High” surface treatments. For the 12 Weeks data sets, it can be seen from Columns 12 and 13 of Table 3 (Appendix 3) that in 12 out of 19 there was also a significant difference in mean bond strength, thus showing that the “high” treatments improved environmental resistance.

2.3 TEST FOR SIGNIFICANCE BETWEEN TEST HOUSES

The results were also examined to determine any significant differences between the Test Houses. Each Test House was nominally testing the same specimens. The results, Columns 11 and 12 in Table 4 (Appendix 4), show that for the Control data sets with “Normal” treatment, that in 6 out of 9 cases there was a significant difference in mean bond strength.

3. CONCLUSIONS

- The statistical data analysis used were successful in identifying the cases where there was significant difference due to surface treatment and differences between Test Houses.
- The large number of cases where there was significant difference between Test Houses (nominally testing the same adhesive/substrate system) indicates problems with the sample sizes (i.e. number of tests per condition) used for the single-lap shear tests.
- The original report for this case study [1] discussed general patterns of response whereas the analysis performed in this report provides a much clearer differentiation between significant and non-significant effects. This clarity can aid researchers to focus their attention on causes of particular significant effects. The following are suitable areas for this attention:

The variation observed could be investigated by understanding the differences in techniques between the Test Houses that produced tight distributions versus those

that produced broad distributions and then running a Design of Experiments exercise to identify key variables.

Further statistical techniques are available to design experiments for a Test House where the variability is known so that:

Where a sample size is fixed, the delta between means for a significant difference can be established. NB. As a demonstration this has been included in some of the analysis performed - see Appendix 3 (Column 14 in Tables 2 and 3).

Where a particular delta needs exploring, the sample size to establish significance can be defined.

REFERENCES

1. Report No.8 "Experimental Assessment of Durability Test Methods", MTS Adhesive Programme, Project 3: Environmental Durability of Adhesive Bonds, 1995.
2. Duncan, "Quality Control and Industrial Statistics", Irwin.
3. Snedecor and Cochran, "Statistical Methods", Iowa State University Press.

APPENDIX 1**CASE STUDY DATA SETS USED FOR ANALYSIS BY STATISTICAL METHODS**

Table 1 lists the data sets analysed [1] in this report. It should be noted that each data point (mean and deviation) represents results obtained from six single-lap shear tests.

The control samples were tested in an unaged condition (at least 7 days from the end of the normal cure cycle) the other samples were conditioned in deionised water at 60 °C and withdrawn after selected periods of time. Testing of unaged and conditioned specimens was conducted under ambient conditions. For the comparison of performance between the “Normal” and “High” treatments, the data sets from the Control and 12 Weeks conditioning were used. The Control/”Normal” treatment data sets were used for comparing results between Test Houses.

Table 1.1 Results of main test series

Substrate	Adhesive	Test House	NORMAL TREATMENT						HIGH TREATMENT					
			Control kN	Failure Load (kN)					Control kN	Failure Load (kN)				
				3 Weeks	6 Weeks	12 Weeks	36 Weeks	60 Weeks		3 Weeks	6 Weeks	12 Weeks	36 Weeks	60 Weeks
Mild steel CR1	2001	1	4.74 ±0.13	2.34 ±0.08	1.95 ±0.10	2.14 ±0.05	-	2.10 ±0.16	4.80 ±0.61	3.85 ±0.25	3.06 ±0.24	3.17 ±0.38	-	3.13 ±0.56
		2	3.58 ±0.10	2.07 ±0.32	1.83 ±0.25	2.50 ±0.31	2.09 ±0.08	-	5.49 ±0.53	4.38 ±0.57	3.74 ±0.47	3.05 ±0.62	2.19 ±0.35	-
		5	4.08 ±0.60	2.136 ±0.68	1.57 ±0.32	1.42 ±0.26	1.28 ±0.31	1.27 ±0.28	4.23 ±0.61	2.47 ±0.45	2.36 ±0.32	2.14 ±0.18	2.32 ±0.42	-
	2007	1	5.24 ±0.35	4.06 ±0.15	5.10 ±0.31	5.41 ±0.12	-	4.71 ±0.12	5.16 ±0.26	5.12 ±0.22	5.85 ±0.48	5.61 ±0.27	-	5.169 ±0.2
		4	4.85 ±0.14	4.19 ±0.49	4.99 ±0.21	4.97 ±0.20	4.08 ±0.29	-	5.23 ±0.19	5.13 ±0.42	5.85 ±0.08	5.48 ±0.08	5.10 ±0.14	-
	F241	1	3.68 ±0.37	3.05 ±0.38	2.71 ±0.32	2.27 ±0.33	-	1.66 ±0.12	4.40 ±0.24	4.35 ±0.28	3.65 ±0.20	3.47 ±0.54	-	2.681 ±0.22
		3	4.70 ±0.42	3.81 ±0.15	3.69 ±0.11	2.37 ±0.29	1.70 ±0.14	-	4.98 ±0.32	4.31 ±0.20	3.78 ±0.24	2.90 ±0.26	1.18 ±0.23	-
	PU3532	1	3.28 ±0.16	1.82 ±0.19	2.04 ±0.31	2.00 ±0.22	-	0.21 ±0.07	3.15 ±0.09	2.36 ±0.11	2.15 ±0.33	2.08 ±0.17	-	0.61 ±0.33
		5	2.09 ±0.83	1.10 ±0.31	0.90 ±0.44	0.73 ±0.19	0.02 ±0.04	-	1.73 ±0.43	2.03 ±0.42	1.68 ±0.21	1.58 ±0.46	0.80 ±0.23	-
	2001	1	4.90 ±0.26	3.05 ±0.26	2.97 ±0.30	2.78 ±0.48	-	2.62 ±0.24	5.19 ±0.08	1.13 ±0.90	1.48 ±1.60	1.64 ±1.23	-	2.19 ±1.70
		4	4.38 ±0.41	3.06 ±0.27	2.95 ±0.23	2.79 ±0.21	1.42 0.73±	-	4.20 ±0.83	3.72 ±0.29	3.19 ±0.56	3.03 ±0.33	3.27 ±0.22	-
Polypropylene GWM 213	2001	1	1.65 ±0.05	1.13 ±0.21	1.00 ±0.21	1.01 ±0.11	-	0.46 ±0.07	1.32 ±0.16	1.37 ±0.21	1.51 ±0.20	1.36 ±0.21	-	1.23 ±0.15
		2	1.64 ±0.14	1.02 ±0.13	1.13 ±0.21	0.75 ±0.063	0.58 ±0.10	-	1.45 ±0.17	1.36 ±0.12	1.42 ±0.11	1.19 ±0.17	0.85 ±0.08	-
	PU3532	1	0.99 ±0.14	0.84 ±0.11	0.58 ±0.16	0.51 ±0.21	-	0.06 ±0.02	0.36 ±0.07	0.97 ±0.17	0.78 ±0.15	0.52 ±0.20	-	0.11 ±0.04
		2	0.89 ±0.25	0.57 ±0.08	0.56 ±0.06	0.46 ±0.06	0.0 ±0.0	-	0.46 ±0.13	0.97 ±0.16	0.80 ±0.10	0.86 ±0.24	0.51 ±0.06	-
GRP	2001	1	2.25 ±0.36	2.25 ±0.42	1.17 ±0.23	1.28 ±0.27	-	0.98 ±0.28	3.39 ±0.17	2.27 ±0.14	2.04 ±0.41	1.829 ±0.18	-	1.78 ±0.28
		3	2.96 ±0.28	1.66 ±0.12	1.87 ±0.26	1.81 ±0.08	0.84 ±0.09	-	3.12 ±0.57	2.16 ±0.27	2.01 ±0.36	1.72 ±0.32	0.86 ±0.13	-
	F241	1	2.99 ±0.63	2.99 ±0.63	1.25 ±0.23	1.10 ±0.06	-	0.80 ±0.14	3.10 ±0.41	2.10 ±0.52	1.91 ±0.31	1.64 ±0.29	-	1.29 ±0.21
		3	3.31 ±0.97	2.31 ±0.44	2.13 ±0.35	1.72 ±0.29	0.66 ±0.12	-	2.30 ±0.30	1.67 ±0.37	1.62 ±0.22	1.54 ±0.34	0.65 ±0.08	-
Mild steel pin/collar	638	1	12.83 ±1.23	9.90 ±1.15	10.89 ±1.61	12.74 ±0.83	-	12.41 ±1.48	12.6 ±0.70	12.24 ±1.29	13.18 ±1.26	13.27 ±1.71	-	13.22 ±0.27
		5	16.69 ±1.03	11.02 ±2.0	10.87 ±0.97	10.65 ±1.30	12.17 ±1.55	-	-	-	-	-	-	-

APPENDIX 2

WHISKER PLOTS OF DATA SETS

Charts 1 and 2 show whisker plots for the Control and 12 Weeks data sets from Table 1 (Appendix 1) for the “Normal” and “High” treatments. Each data cell is represented by a diamond for the mean value of the lap shear strength and by crosses for the ± 2 standard deviation points.

The key for the chart headings and footings is given below:

S	Adherend / substrate material
A	Adhesive type
TH	Test House designation
TR	Surface treatment

Surface Treatments [1]

“Normal”

GB	Grit blast
CAE	Chromic acid etch
SA	“Scotchbrite” abrade

“High”

GBS	Grit blast + silane
PA	Phosphoric acid anodise
SA	Chlorinated primer
FG	Fine grit blast

CHART 1: LAP SHEAR STRENGTHS FOR THE CONTROL SAMPLES

S	MILD STEEL				AL	POLY.		GRP		MILD STEEL				AL	POLY		GRP	
A	2001	2007	F241	P 3532	2001	2001	P 3532	2001	F241	2001	2007	F241	P 3532	2001	2001	P 3532	2001	F241

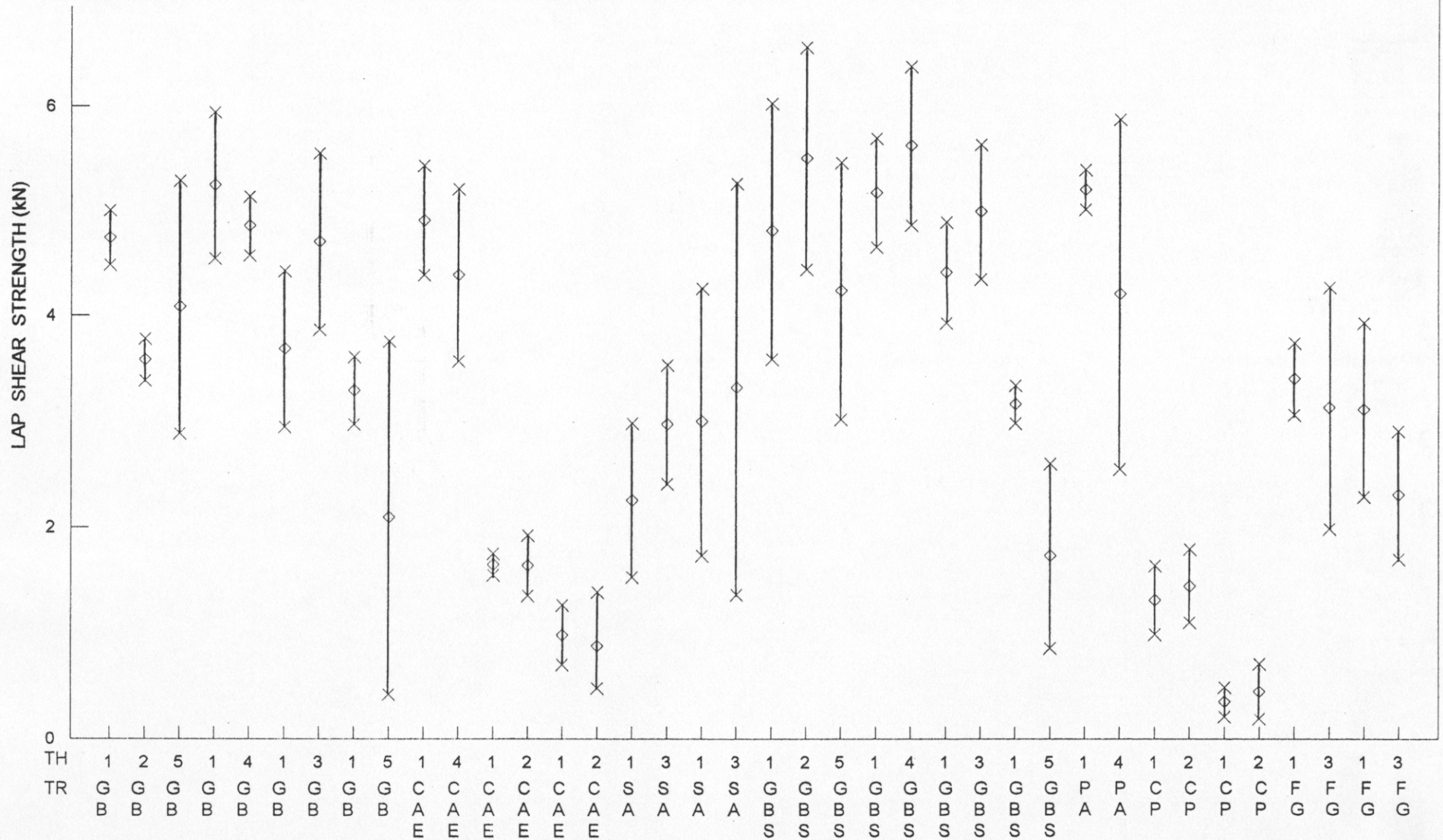
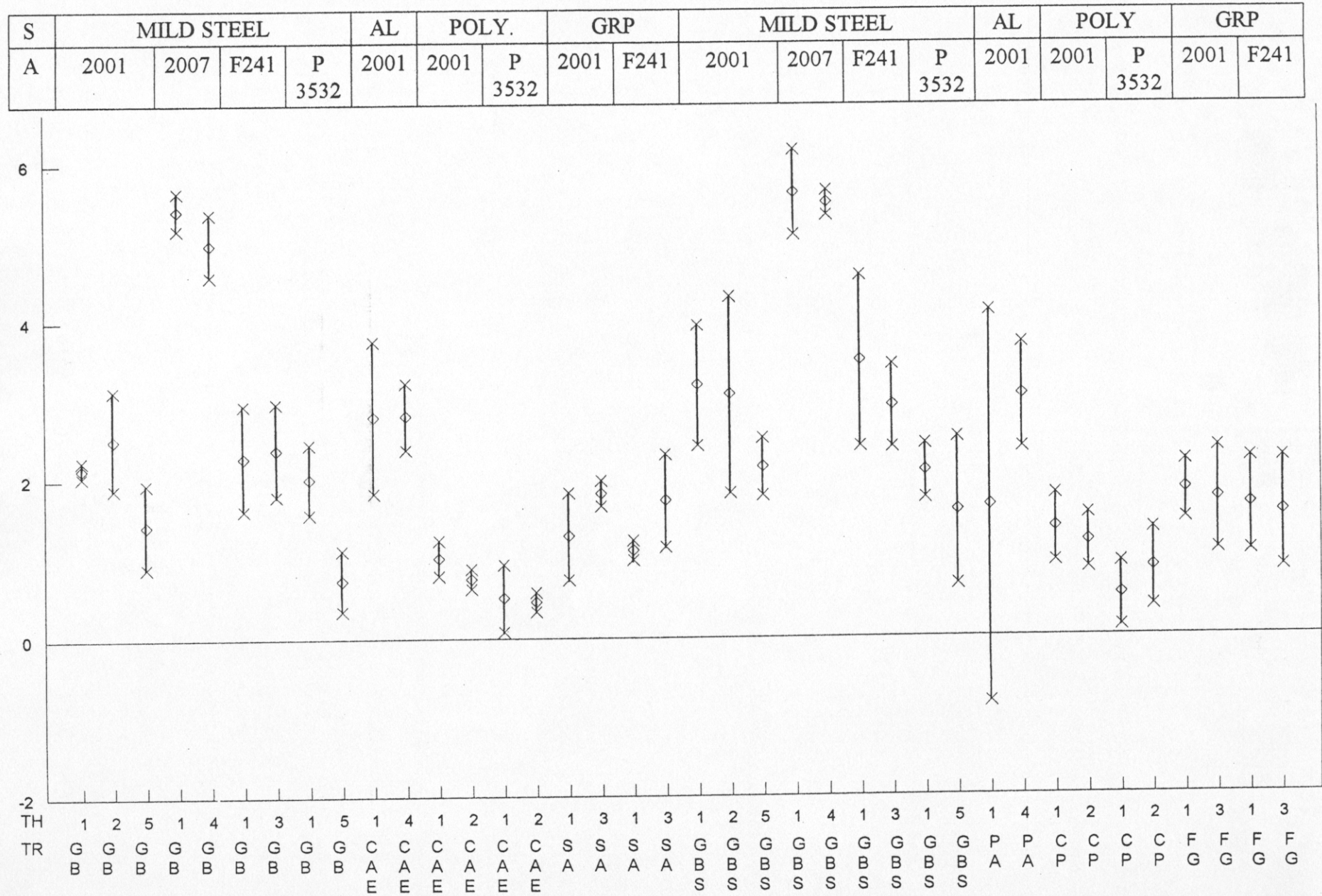


CHART 2: LAP SHEAR STRENGTHS FOR THE 12 WEEKS SAMPLES



APPENDIX 3

TEST FOR SIGNIFICANCE BETWEEN “NORMAL” AND “HIGH” TREATMENTS

This analysis was performed on the Control data sets and the 12 Week data sets and is shown in Tables 2 and 3, respectively.

The key describing the columns in Tables 2 and 3 is as follows:

- Column 1:** Adherend substrate material.
- Column 2:** Adhesive type.
- Column 3:** Test House designation.
- Column 4:** Mean failure load kN ($\bar{x}_N$) and standard deviation (s_N) for the “Normal” treatment.
- Column 5:** Mean failure load kN ($\bar{x}_H$) and standard deviation (s_H) for the “High” treatment.
- Column 6:** $s^2_{\max} \div s^2_{\min}$ from **Columns 4** and **5** quantifies the variability between the “Normal” and “High” treatment populations [2].
- Column 7:** Decision if the variability is significant (value from **Column 6** > 7.15 [2]).
- Column 8:** $\sqrt{(s_N^2 + s_H^2) \div 2}$ which is the pooled standard deviation value (s) from the “Normal” and “High” populations.
- Column 9:** $\bar{x}_H - \bar{x}_N$ which is the difference in mean failure loads ($\bar{x}$) from **Columns 5** and **4**, respectively.
- Column 10:** $\sqrt{2s^2 \div 6}$ which is the standard error of the difference between the mean failure loads for the “Normal” and “High” treatments.
- Column 11:** $\bar{x} \div \sqrt{2s^2 \div 6}$ which is the **t** statistic [3].
- Column 12:** Decision if there is a significant difference in mean failure load between the “Normal” and “High” treatments using the **t** statistic (from **Column 11**) where there is no significant difference between s_N and s_H (see **Column 7**) then the value from **Column 11** for 5% significance is > 2.23.

NB. In this case where there was no significant difference between the standard deviations and the sample numbers in each population (6) were the same then the t_{10} value is used (i.e. **2(n - 1)** degrees of freedom [3]).

Column 13: Decision if there is a significant difference in mean failure load between the “Normal” and “High” treatments using the **t** statistic (from **Column 11**) where there is no significant difference between ***s_N*** and ***s_H*** (see **Column 7**) then the value from **Column 11** for 5% significance is 2.571.

NB. In this case where there is a significant difference between standard deviations it is still valid to use a pooled standard deviation because the sample numbers in each population were the same, but the ***t₅*** value is used (i.e. ***n - 1*** degrees of freedom [3]).

Column 14: ***t₁₀*** x ***Xbar*** which is the minimum delta means (i.e. ***xbar_H*** - ***xbar_N***) required for a significant difference where there is no significant difference between ***s_N*** and ***s_H***.

t₅ x ***Xbar*** which is the minimum delta means (i.e. ***xbar_H*** - ***xbar_N***) required for a significant difference where there is a significant difference between ***s_N*** and ***s_H***.

**TABLE 2: ANALYSIS FOR SIGNIFICANCE BETWEEN “NORMAL” AND “HIGH” TREATMENTS
FROM THE CONTROL DATA SETS**

1	2	3	4		5		6	7	8	9	10	11	12	13	14	
			$\bar{x}_N$	s_N	$\bar{x}_H$	S_H							t_{10}	t_5	$t_{10} \times 10$	$t_5 \times 10$
Mild Steel CR1	2001	1	4.74	0.13	4.80	0.61	22.0	yes	0.44	0.06	0.25	0.24		no		0.64
		2	3.58	0.10	5.49	0.53	28.1	yes	0.38	1.91	0.22	8.68		yes		0.56
		5	4.08	0.60	4.23	0.61	1.0	no	0.61	0.15	0.35	0.42	no		0.78	
	2007	1	5.24	0.35	5.16	0.26	1.8	no	0.31	-0.08	0.18	-0.44	no		0.40	
		4	4.85	0.14	5.23	0.19	1.8	no	0.17	0.38	0.10	3.80	yes		0.22	
	F241	1	3.68	0.37	4.40	0.24	2.4	no	0.31	0.72	0.18	4.00	yes		0.40	
		3	4.70	0.42	4.98	0.32	1.7	no	0.37	0.28	0.21	1.30	no		0.47	
	PU3532	1	3.28	0.16	3.15	0.09	3.2	no	0.13	-0.13	0.08	-1.63	no		0.18	
		5	2.09	0.83	1.73	0.43	3.7	no	0.66	-0.36	0.38	-0.95	no		0.85	
Al Alloy 5251	2001	1	4.90	0.26	5.19	0.08	10.6	yes	0.19	0.29	0.11	2.64		yes		0.28
		4	4.38	0.41	4.20	0.83	4.1	no	0.65	-0.18	0.38	-0.47	no		0.85	
Polypropylene GWM213	2001	1	1.65	0.05	1.32	0.16	10.2	yes	0.12	-0.33	0.07	-4.71		yes		0.18
		2	1.64	0.14	1.45	0.17	1.5	no	0.16	-0.19	0.09	-2.11	no		0.20	
	PU3532	1	0.99	0.14	0.36	0.07	4.0	no	0.11	-0.63	0.06	10.50	yes		0.13	
		2	0.89	0.25	0.46	0.13	3.7	no	0.20	-0.43	0.11	-3.91	yes		0.25	
GRP	2001	1	2.25	0.36	3.39	0.17	4.5	no	0.28	1.14	0.16	7.13	yes		0.36	
		3	2.96	0.28	3.12	0.57	4.1	no	0.45	0.16	0.26	0.62	no		0.58	
	F241	1	2.99	0.63	3.10	0.41	2.4	no	0.53	0.11	0.31	0.35	no		0.69	
		3	3.31	0.97	2.30	0.30	10.5	yes	0.71	-1.01	0.41	-2.46		no		1.05

**TABLE 3: ANALYSIS FOR SIGNIFICANCE BETWEEN "NORMAL" AND "HIGH" TREATMENTS
FROM THE 12 WEEKS DATA SETS**

1	2	3	4		5		6	7	8	9	10	11	12	13	14	
			xbar _N	s _N	xbar _H	S _H									t ₁₀ x 10	t ₅ x 10
Mild Steel CR1	2001	1	2.14	0.05	3.17	0.38	57.8	yes	0.27	1.03	0.16	6.44		yes		0.41
		2	2.50	0.31	3.05	0.62	4.0	no	0.49	0.55	0.28	1.96	no		0.62	
		5	1.42	0.26	2.14	0.18	2.1	no	0.22	0.72	0.13	5.54	yes		0.29	
	2007	1	5.41	0.12	5.61	0.27	5.1	no	0.21	0.20	0.12	1.67	no		0.27	
		4	4.97	0.20	5.48	0.08	6.3	no	0.16	0.51	0.09	5.67	yes		0.20	
	F241	1	2.27	0.33	3.47	0.54	2.7	no	0.45	1.20	0.26	4.62	yes		0.58	
		3	2.37	0.29	2.90	0.26	1.2	no	0.28	0.53	0.16	3.31	yes		0.36	
	PU3532	1	2.00	0.22	2.08	0.17	1.7	no	0.20	0.08	0.12	0.67	no		0.27	
		5	0.73	0.19	1.58	0.46	5.9	no	0.35	0.85	0.20	4.25	yes		0.45	
Al Alloy 5251	2001	1	2.78	0.48	1.64	1.23	6.6	no	0.93	-1.14	0.54	-2.11	no		1.20	
		4	2.79	0.21	3.03	0.33	2.5	no	0.28	0.24	0.16	1.50	no		0.36	
Polypropylene GWM213	2001	1	1.01	0.11	1.36	0.21	3.6	no	0.17	0.35	0.10	3.50	yes		0.22	
		2	0.75	0.06	1.19	0.17	8.0	yes	0.13	0.44	0.08	5.50		yes		0.21
	PU3532	1	0.51	0.21	0.52	0.20	1.1	no	0.21	0.01	0.12	0.08	no		0.27	
		2	0.46	0.06	0.86	0.24	16.0	yes	0.17	0.40	0.09	4.44		yes		0.23
GRP	2001	1	1.28	0.27	1.83	0.18	2.3	no	0.23	0.55	0.13	4.23	yes		0.29	
		3	1.81	0.08	1.72	0.32	16.0	yes	0.23	-0.09	0.13	-0.69		no		0.33
	F241	1	1.10	0.06	1.64	0.29	23.4	yes	0.21	0.54	0.12	4.50		yes		0.27
		3	1.72	0.29	1.54	0.34	1.4	no	0.32	-0.18	0.18	-1.00	no		0.40	

APPENDIX 4

TEST FOR SIGNIFICANCE BETWEEN TEST HOUSES NOMINALLY TESTING THE SAME SUBSTRATE/ADHESIVE SURFACE TREATMENT COMBINATIONS

This analysis was performed on the Control data sets and the 12 Week data sets and is shown in Table 4, respectively.

The key describing the columns in Table 4 is as follows:

Column 1: Adherend substrate material.

Column 2: Adhesive type.

Column 3: Test House designation.

Column 4: Mean failure load kN ($\bar{x}$) and standard deviation (s) at each Test house.

Column 5: $s^2_{\max} \div s^2_{\min}$ from Column 4 quantifies the variability between the result populations from the different Test Houses [2].

Column 6: Decision if the variability is significant (value from **Column 5** > 7.15 for a comparison of two Test Houses and > 10.8 for a comparison of three Test Houses [2]).

Column 7: $\sqrt{(s_A^2 + s_B^2) \div 2}$ which is the pooled standard deviation (S) from Test House “A” and Test House “B” populations.
 $\sqrt{(s_A^2 + s_B^2 + s_C^2) \div 3}$ in the case where there are three Test Houses

Column 8: $\bar{x}_{\max} - \bar{x}_{\min}$ which is the difference between the highest and lowest mean failure loads ($\bar{X}$) from **Column 4**

Column 9: $\sqrt{2s^2 \div 6}$ which is the standard error of the difference between the mean failure loads from each Test.

Column 10: $\bar{X} \div \sqrt{2s^2 \div 6}$ which is **t** statistic for comparing two data groups [3]. For the mild steel CR1 substrate / Araldite® 2001 epoxy adhesive combination where three Test Houses were used then the Studentized range **q** statistic of $\bar{X} \div (S/\sqrt{n})$ is used [2] where n is the number of samples tested in each data set (6).

Column 11: Decision if there is a significant difference in mean failure load between the Test Houses using the **t** statistic (from **Column 10** where there is no significant difference in the variability between the Test House populations (see **Column 6**) then the value for 5% significance from **Column 10** is 2.23

NB. In this case where there was no significant difference between the standard deviations and the sample numbers in each population (6) were the same then the t_{10} value is used (i.e. **2(n - 1)** degrees of freedom [3]).

Column 12: Decision if there is a significant difference in mean failure load between two Test Houses using the **t** statistic (from **Column 10** where there is a significant difference in the variability between the Test House populations (see **Column 6** then the value of 5% significance from **Column 10** is > 2.571).

NB. In this case where there was a significant difference in the variabilities it is still valid to use a pooled standard deviation because the sample numbers in each population (6) were the same, but the **t_{15}** value is used (i.e. **n - 1** degrees of freedom [3]). For the mild steel CR1 substrate/ Araldite® 2001 epoxy adhesive combination where three Test Houses were used, it is again valid to use the pooled standard deviation, but **$q_{0.05}(3, 15)$** value is used (i.e. **3(n - 1)** degrees of freedom [2], which is 3.67 for 5% significance.

TABLE 4: ANALYSIS FOR SIGNIFICANCE BETWEEN TEST HOUSES

1	2	3	4		5	6	7	8	9	10	11	12
			xbar	s								
Mild Steel CR1	2001	1	4.74	0.13	36	yes	0.36	1.16		7.89		yes
		2	3.58	0.10								
		5	4.08	0.60								
	2007	1	5.24	0.35	6.3	no	0.27	0.39	0.16	2.44	yes	
		4	4.85	0.14								
	F241	1	3.68	0.37	1.3	no	0.40	1.02	0.23	4.43	yes	
		3	4.70	0.42								
	PU3532	1	3.28	0.16	26.9	yes	0.60	1.19	0.35	3.40		yes
		5	2.09	0.83								
Al Alloy 5251	2001	1	4.90	0.26	2.5	no	0.34	0.52	0.20	2.60	yes	
		4	4.38	0.41								
Polypropylene	2001	1	1.65	0.05	7.8	yes	0.11	0.01	0.06	0.17		no
		2	1.64	0.14								
	PU3532	1	0.99	0.14	3.2	no	0.20	0.10	0.12	0.83	no	
		2	0.89	0.25								
GRP	2001	1	2.25	0.36	1.7	no	0.32	0.71	0.18	3.94	yes	
		3	2.96	0.28								
	2001	1	2.99	0.63	2.4	no	0.82	0.32	0.47	0.68	no	
		3	3.31	0.97								

Case Study 2
Data from Report DRA/SMC/CR951184

1. INTRODUCTION

The overall strategy for Project 3 is to establish a methodology for designing adhesively bonded joints for cyclic loading and hostile environments. Two of the main objectives are “to assess available durability test methods and data” and “to develop and validate test methods and environmental conditioning procedures that can be used to measure the parameters required for long-term performance predictions”. A key component in the approach adopted has been to employ statistical methods for evaluating the criticality of various factors and their interactions.

In the activities that have been undertaken to satisfy these objectives it has been obvious that relationships between test regimes (e.g. short-term accelerated tests versus long-term life condition tests) and physical parameters (e.g. adhesive moisture content versus joint strength) need to be established. The quality of such relationships is an essential ingredient to developing a successful predictive model and can be quantified by the use of regression equations and correlation coefficients.

The case study being investigated in this report is work performed by the Defence Evaluation and Research Agency (DERA formerly DRA) at Farnborough on developing an adhesive joint durability predictive capability [1]. Part of this work involved studies of two types of correlation, namely:

Association Correlation: relating joint strengths after long-term weathering to those after accelerated tests (i.e. where the magnitude of results from one timescale seem to be associated with results from another timescale).

Causation Correlation: relating joint moisture content and joint strength (i.e. where joint strength is expected to be a function of joint moisture content).

In both cases the DERA Report (DRA/SMC/CR951184 [1]) concluded that correlation existed, but the extent of the correlation was stated as being from “rough” to “reasonable” and that it was “indicating a promising approach to long-term prediction”. This report details analysis of data extracted from the DRA report to determine quantifiable correlation coefficients and the regression equations for both association correlation and causation correlation, together with a means of evaluating their significance.

2. ANALYSIS

In this case study, for both association correlation and causation correlation, the hypothesis is that there is a linear relationship between the two factors being compared.

To test this hypothesis and to determine the quality of the correlation the approach is to use the tools of regression analysis, namely, regression equations, analysis of variance and correlation coefficient determination.

The linear relationship can be expressed by the following equation:

$$Y = B_0 + B_1 X \quad (1)$$

This equation is known as the regression equation of $\mathbf{Y}$ and $\mathbf{X}$, where $\mathbf{B}_0$ is the intercept and $\mathbf{B}_1$ is the slope of the line. The data points $(\mathbf{X}, \mathbf{Y})$ are used to determine the constants $\mathbf{B}_0$ and $\mathbf{B}_1$ for the equation.

Constants $\mathbf{B}_0$ and $\mathbf{B}_1$ in Equation (1) can be defined by the following expressions:

$$B_0 = \frac{\sum Y}{n} - B_1 \frac{\sum X}{n} \quad (2)$$

and

$$B_1 = \frac{\sum XY - \frac{\sum X \sum Y}{n}}{\sum X^2 - \frac{(\sum X)^2}{n}} \quad (3)$$

Analysis of Variance (ANOVA) is a method of apportioning the variation which can be explained by the regression equation and that which is left unexplained. The latter is effectively the error, or deviation, of the Y values from the best line fit. The “F” test is the statistical test which assesses from the sources of variation whether the regression line is a good representation of the data. ANOVA (utilising the AGSS statistical software package used at Xyratex, Havant) of the data from Appendices 1 and 2 is given in Appendices 3 and 4 for association correlation and causation correlation, respectively.

The correlation coefficient r can be defined by the following equation:

$$r = \frac{\sum XY - \frac{\sum X \sum Y}{n}}{\left(\sum X^2 - \frac{(\sum X)^2}{n} \right) \left(\sum Y^2 - \frac{(\sum Y)^2}{n} \right)} \quad (4)$$

In a similar approach “F” test (which checks the quality of the regression line), it is possible to assess the correlation coefficients for their significance (see [4], which specifies critical values for different sample sizes against which the significance of particular experimental correlation coefficients can be assessed). The critical values pertinent to each of the case study examples are given in Appendices 3 and 4. It should be noted that the 5% and 1% values represent thresholds that relate to the percentage risk of concluding incorrectly that a correlation exists.

3. CONCLUSIONS

The following conclusions can be made in regard to the analysis and durability data.

- The statistical data analysis techniques used were successful in quantifying the extent of the relationships between the factors of interest “to the experimenters”.

- The regression equations and correlation coefficients enable the user to differentiate the significance, or quality, of the correlation between factors being compared.
- In the case study examples that were studied there was a statistically significant relationship and correlation between joint strengths from long-term weathering and joint strengths obtained from accelerated laboratory tests. This relationship can therefore be used as a predictor of long-term weathering performance provided information on the accelerated laboratory test is known. It should be noted that this is only true over the range of data included in the experiment/field trials.
- In the other two case study examples, the data analysis showed that there was not a statistically significant relationship between long-term weathering joint strength and joint moisture content. Therefore, this relationship cannot be used as a predictor of joint strength for long-term weathering based on joint moisture content. The original hypothesis of a linear causal relationship between the two factors is not supported by the data.

However, there could be reasons for this situation, as follows:

- a) The factors are not related.
- b) The relationship is not linear, but some other function.
- c) In the presence of experimental variation, the sample sizes (i.e. number of pairs of X and Y) were insufficient to uncover the true relationship.
- d) There are other factors which also contribute to the joint strength (i.e. the model is incomplete). Methods of multiple regression which seek to establish relationships involving more than one independent variable may be appropriate.

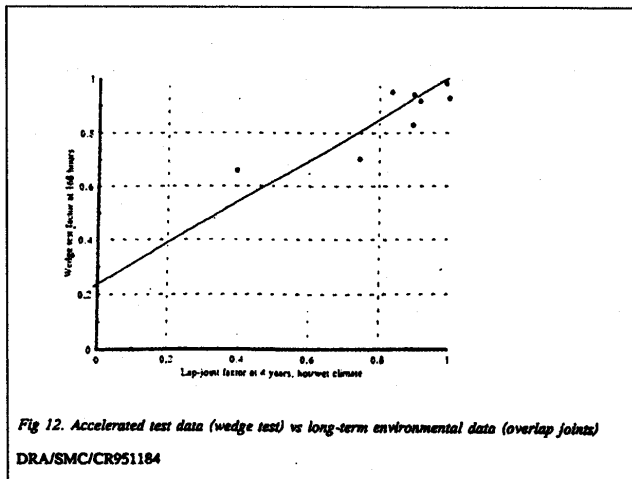
REFERENCES

1. B M Parker and S J Shaw, "The Development of An Adhesive Joint Durability Predictive Capability", Defence Evaluation and Research Agency (Farnborough, United Kingdom) Report DRA/SMC/CR951184, 1995 (restricted commercial).
2. Kleinbaum, Kupper and Muller, "Applied Regression Analysis and Other Multivariable Methods" PWS Kent.
3. Box, Hunter and Hunter, "Statistics for Experimenters, Wiley.
4. Snedecor and Cochran, "Statistical Methods", Iowa State University Press.

APPENDIX 1

DATA FROM REPORT DRA/SMC/CR951184 SELECTED FOR ASSOCIATION
CORRELATION ANALYSIS

DATA SET 1: Ref. Page 29; Fig 12; Accelerated Test Data (Wedge Test) vs
Long-Term Environmental Data (Overlap Joints)



NB. The data shown in this table are derived from the graph and are estimated values.

Lap Joint Factor	Wedge Test Factor
0.4	0.65
0.75	0.7
0.85	0.94
0.9	0.83
0.9	0.92
0.91	0.91
0.99	1
1	0.94

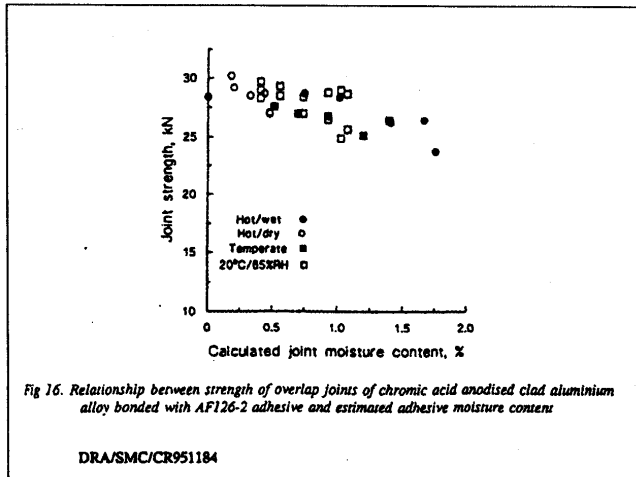
APPENDIX 2

DATA FROM REPORT DRA/SMC/CR951184 SELECTED FOR CAUSATION

CORRELATION ANALYSIS

DATA SET 2: Ref. Page 31; Fig 16; Relationship Between Strength of Overlap

Joints and Estimated Adhesive Moisture Content



NB. The data shown in this table are derived from the graph and are estimated values.

Hot/Dry Environment		20C/65%RH Environment	
Joint Moisture (%)	Joint Strength (kN)	Joint Moisture (%)	Joint Strength (kN)
0.2	30.5	0.4	30
0.22	29	0.4	29
0.35	28.5	0.4	28.5
0.45	29	0.6	29.5
0.5	27	0.6	28.5
		0.75	28.5
		0.75	27
		0.92	26.5
		0.92	29
		1.05	25
		1.05	29.5
		1.1	25.7
		1.1	29

APPENDIX 3**AGSS ANALYSIS FOR ASSOCIATION CORRELATION ON THE DATA GIVEN IN****APPENDIX 1****REGRESSION EQUATIONS**

The values for B_0 and B_1 were calculated to be 0.392 and 0.560, respectively, which gives the following regression equation:

$$Y = 0.392 + 0.56X$$

ANALYSIS OF VARIANCE TABLE

SOURCE	SS	DF	MS	F
GRAND MEAN	5.934	1		
REGRESSION	0.082	1	0.082	18.329
RESIDUAL	0.027	6	0.004	
TOTAL	6.043	8		

THE SIGNIFICANCE LEVEL FOR REGRESSION = 0.0052 (1)

COEFFICIENT OF DETERMINATION (RSQUARE) = 0.753

CORRELATION COEFFICIENT $\left(\sqrt{RSQUARE}\right) = 0.868 (2)$

NOTE: (1) A small value of significance level, such as less than 0.01, suggests a significant effect and a relatively large value, such as greater than 0.2, gives no evidence of a significant effect.

(2) In this case with a residual 6 degrees of freedom the correlation coefficient needs to be greater than 0.707 @ 5% and 0.834 @ 1% for statistical significance.

APPENDIX 4**AGSS ANALYSIS FOR CAUSATION CORRELATION ON THE DATA GIVEN IN APPENDIX 2****EXAMPLE 1: DATA FROM THE HOT/DRY ENVIRONMENT****REGRESSION EQUATIONS**

The values for B_0 and B_1 were calculated to be 31.347 and -7.404, respectively, which gives the following regression equation:

$$Y = 31.347 - 7.404X$$

ANALYSIS OF VARIANCE TABLE

SOURCE	SS	DF	MS	F
GRAND MEAN	4147.2	1		
REGRESSION	3.931	1	3.931	4.979
RESIDUAL	2.369	3	0.790	
TOTAL	4153.5	5		

THE SIGNIFICANCE LEVEL FOR REGRESSION = 0.1118 (1)

COEFFICIENT OF DETERMINATION (RSQUARE) = 0.624

CORRELATION COEFFICIENT $\left(\sqrt{RSQUARE}\right) = 0.790 (2)$

NOTE: (1) A small value of significance level, such as less than 0.01, suggests a significant effect and a relatively large value, such as greater than 0.2, gives no evidence of a significant effect.

(2) In this case with a residual 3 degrees of freedom the correlation coefficient needs to be greater than 0.878 @ 5% and 0.959 @ 1% for statistical significance.

APPENDIX 4 CONT.**AGSS ANALYSIS FOR CAUSATION CORRELATION ON THE DATA GIVEN IN APPENDIX 2****EXAMPLE 2: DATA FROM THE 20C/65%RH ENVIRONMENT****REGRESSION EQUATIONS**

The values for B_0 and B_1 were calculated to be 30.349 and -2.872, respectively, which gives the following regression equation:

$$Y = 30.349 - 2.872X$$

ANALYSIS OF VARIANCE TABLE

SOURCE	SS	DF	MS	F
GRAND MEAN	10287.422	1		
REGRESSION	7.333	1	7.333	3.628
RESIDUAL	22.235	11	2.021	
TOTAL	10316.99	13		

THE SIGNIFICANCE LEVEL FOR REGRESSION = 0.0833 (1)

R SQUARE = 0.248

CORRELATION COEFFICIENT $\left(\sqrt{RSQUARE}\right)$ = 0.498 (2)

NOTE: (1) A small value of significance level, such as less than 0.01, suggests a significant effect and a relatively large value, such as greater than 0.2, gives no evidence of a significant effect.

(2) In this case with a residual 11 degrees of freedom the correlation coefficient needs to be greater than 0.553 @ 5% and 0.684 @ 1% for statistical significance.

Case Study 3

NPL Durability Data Analysis and Modelling

1. INTRODUCTION

Work already undertaken for PAJ3 has identified statistical methods that successfully augmented the data analysis from case studies. The statistical methods enabled quantifiable conclusions to be drawn about the significance of variables such as different surface finishes and about correlation between both testing regimes (short-term accelerated laboratory tests versus long-term field conditions) and physical parameters (joint moisture content versus joint strength).

The work detailed in this case study is further development of these statistical analysis techniques to investigate data from current experimental work at NPL and to develop a analytical relationships relating exposure time and temperature to joint strength.

2. ANALYSIS

The durability data from tensile tests on single-lap joints with and without perforations that have been immersed in distilled/deionised water at three different temperatures for varying lengths of time [1-2]. The construction of the test specimens was as follows:

Geometry:	Single-lap (unperforated, single hole, double hole and triple hole)
Adherend:	CR1 mild rolled steel supplied by British Steel
Adhesive:	AV119 (Araldite® 2007) supplied by Ciba Speciality Polymers
Surface treatment:	Grit blast + 1,1,1 trichloroethane wipe
Spacer/filler:	1% 250 µm diameter ballontini glass beads
Cure:	140 °C for 75 minutes

Further details on specimen geometry, specimen preparation and test procedure are obtainable in references [1-2].

The specimens were divided into three groups which were immersed in distilled/deionised water at 25 °C, 40 °C and 60 °C. After exposure times of 0, 3, 7, 14, 21 and 42 days a number of specimens were removed and tested to determine failure load (Appendices 1, 2 and 3 show these results together with means and standard deviations for the tests at 25 °C, 40 °C and 60 °C, respectively).

2.1 INITIAL DATA REVIEW

An initial simple visual assessment on the integrity and form of the data presented in Appendices 1, 2 and 3 can be made using box/whisker plots. The box/whisker plot depicts the data from each test cell with the maximum, minimum and mean data points shown along the whisker line and the 25th, 50th, and 75th percentile values shown by the horizontal box lines. Appendix 4 shows box/whisker plots failure load against exposure time for each of the test temperatures and each of the specimen configurations (i.e. non-perforated and the three levels of perforation). These graphic displays of how failure load changes against exposure time gives an indication of the type of relationship which connects the two factors (i.e. curvilinear, polynomial, etc.). From the charts in Appendix 4, it can be seen that the perforation condition appears to have little affect on the failure

load. However, there is an unexpected effect with the 60 °C/42 days results where the trend for reducing failure load with increasing exposure time reverts very noticeably.

The box/whisker plots can be rearranged to give a graphical display of the data in another dimension. In Appendix 5, there are a series of charts showing failure load against temperature for each of the specimen configurations at each exposure time. From these charts, it can be seen in most cases a common relationship exists related to temperature (i.e. generally failure load reduces as temperature increases). This relationship is true over all exposure times until 42 days where the data appears to be incongruous. This requires further investigation to establish whether this is a real effect that can be associated to other data or some discontinuity that should be treated separately. For the purpose of this analysis, it will be assumed that the 60 °C/42 days data is valid and associated with other data.

Note: The increase in “apparent” strength, also observed by AEA Technology [3], is attributed to plasticisation of the adhesive. This results in a reduction in peel and shear stresses at the joint ends to a level lower than experienced by drier specimens, thereby masking interfacial effects. This effect may be also occurring at lower immersion temperatures, albeit to a much lesser extent.

2.2 FUNCTIONAL (EMPIRICAL) RELATIONSHIP DETERMINATION

The objective now is to try to understand from the analyses already conducted, whether there exist any clues to assist in constructing the functional empirical relationship between failure load and temperature (**TP**), exposure time (**TM**) and perforations (**H**).

The charts in Appendix 5, show that there exists essentially a linear relationship between failure load and temperature (**TP**). This holds true for all perforation conditions and exposure times with the exception of the results for 60 °C/42 days.

The charts in Appendix 4, show that a simple linear relationship exists between failure load and exposure time (**TM**). Again, this is exacerbated by what appears to be the “strange” results at 60 °C/42 days.

Therefore, a polynomial/quadratic function was found to be necessary to get anywhere near to fitting all the data. Appendix 6 shows the curves obtained from the mean failure loads at given temperatures against exposure times for each of the specimen configurations (i.e. unperforated, single hole, double hole and triple hole).

Although a higher order polynomial would give a better fit to the data, stability prediction of the model would be lost, together with simplicity. In addition, some rationale behind the physics/chemistry of the system, which would result in more than one slope change in the function would be needed.

Generally, stability and simplicity are called for and hence the choice of a quadratic function connecting failure load with exposure time (**TM**). The charts in Appendix 4 do not indicate a very strong relationship between failure load and perforation (**H**).

2.3 LINEAR REGRESSION MODELLING

2.3.1 Full Model

Using the AGSS statistical software package a full linear regression was produced using all the terms as given in the following relationship:

$$\text{Failure Load (N/mm)} = f [\text{Temperature } (\mathbf{TP}), \text{ Time } (\mathbf{TM}), \text{ Holes } (\mathbf{H}), \text{ Temperature x Temperature } (\mathbf{TP^*2}), \text{ Time x Time } (\mathbf{TM^*2}), \text{ Temperature x Time } (\mathbf{TP \times TM}), \text{ Temperature x Holes } (\mathbf{TP \times H}), \text{ Holes x Holes } (\mathbf{H^*2}), \text{ Time x Holes } (\mathbf{TM \times H})]$$

Appendix 7 shows the features of the model and from the Significance Level Column in the Table of Coefficients it can be seen that certain effects are not significant and can be excluded from that model (I.e. **H**, **H*2** and **TM x H**). The R-SQUARED value in the table shows that this model accounts for 65% of the total variation observed, which is considered to be reasonable in view of the unexpected effect at 60 °C/42 days. Additionally, the Analysis of Variance table shows that **lack of fit** is significant, which indicates that there is something missing in the overall model.

It would be possible to reduce the **lack of fit** but at the expense of model simplicity and physical understanding of the results. A more profitable route would be to establish whether:

- Other factors were at work during the experiments (i.e. data accuracy/experimental conditions).
- Some new physical change is taking place during the experiment (i.e. mode of failure).

Note: There are two competing processes: (i) interfacial degradation; and (ii) plasticisation of the adhesive.

2.3.2 Reduced Model

Again AGSS was used to produce a reduced linear regression model, which excluded the non-significant terms **H**, **H*2** and **TM x H**. Appendix 8 shows the features of the reduced model, which still explains almost 65% of the variation seen together with a significant **lack of fit**. The reduced model is as follows:

$$\begin{aligned} \text{Failure Load(N/mm)} = & 417.44 - 4.87 (\mathbf{TP}) - 9.03(\mathbf{TM}) + 0.04(\mathbf{T^*2}) + 0.16(\mathbf{TM^*2}) \\ & + 0.03(\mathbf{TP \times TM}) - 0.1(\mathbf{TP \times H}) \end{aligned}$$

This model/function can be represented in a number of graphical forms to aid interpretation and use in predicting expected failure load performance. Appendix 9 shows two types of graphical output for the condition of Single Hole, namely, a surface plot and a contour plot. These plots show graphically how the failure load changes as a function of temperature (**TP**) and exposure time (**TM**). The contour plot is just the plan

view of the surface plot. The contours represent conditions that produce constant failure load. Projection to the temperature and exposure time scales produces combinations of the two factors which result in the same failure load.

Both of these plots can be used to predict failure load under different conditions of temperature and time. In addition, the plots provide a very useful method of visual interpretation of the empirical model.

3. CONCLUSIONS

The following conclusions can be drawn from the analysis:

- The analysis approach adopted has been successful in being able to cope with the complexity of the experiment on single-lap shear tests involving 3 factors (temperature, time and number of holes), together with the multiple levels of each of these factors.
- The analysis has been able to identify the relative importance of the factors included in the experiment. The relative importance can be ranked as follows:
 - a) Time.
 - b) Temperature.
 - c) Some smaller effect due to the number of holes in combination with temperature.
- The various intermediate stages of analysis through box plots are very useful in attempting to understand the type of relationship for each factor which should be incorporated in the final model.
- The analysis tools and approach used proved successful in constructing an empirical model which showed the link between failure load and the 3 factors.
- The empirical model linking the above factors can be displayed as both a response graph and a contour plot. These methods are very useful in conveying the essence of the relationships of the model, and assists significantly in communicating the results.
- Although the empirical model was a reasonable representation of the data, the lack of fit means that some are factors are affecting the results. These need to be investigated in order that “wrong trails” are avoided. It is tempting to add more terms into the model, however, a more profitable route is to understand the physical mechanisms behind the data, and then proceed with the modelling based on this new insight.

REFERENCES

1. W R Broughton, R D Mera and G Hinopoulos, "Environmental Degradation of Adhesive Joints. Single-Lap Joint Geometry", NPL Report CMMT(A) 196, 1999.
2. A Olusanya, K Tully, R Mera and W R Broughton, "A Comparison of Commercial Design of Experiment Software Programs for the Analysis of Durability Data", NPL Report CMMT(A) 98, 1998.
3. Report No.8 "Experimental Assessment of Durability Test Methods", MTS Adhesive Programme, Project 3: Environmental Durability of Adhesive Bonds, 1995.

ADDITIONAL READING

1. Box, Hunter and Hunter, "Statistics for Experimenters", Wiley.
2. Montgomery, "Design and Analysis of Experiments", Wiley.

APPENDIX 1**TABLE 1: JOINT STRENGTH RESULTS FROM SINGHAP SPECIMENS AFTER IMMERSION IN DISTILLED/DEIONISED WATER AT 23C (FAILURE LOAD PER UNIT WIDTHN/mm)**

Exposure Time (days)	Unperforated	Single Hole	Double Hole	Triple Hole
0	340	313	288	321
	320	294	291	320
	346	279	288	271
	339	320	297	312
	325	298	338	298
3	298	244	263	279
	292	264	266	300
	287	270	260	297
	301	273	265	292
	278	265	264	309
7	279	242	257	261
	265	265	248	262
	271	253	268	247
	269	271	263	251
	273	259	259	278
14	275			
	240	219	220	228
	243	234	212	220
	229	235	219	219
	237	236	218	
21	231	233	215	
	230	232		
	242	260	273	251
	257	253	264	230
	244	279	270	249
42	239	262	252	249
	256	247	268	256
	252	235	235	258
	236	223	227	280
	237	237	252	274
	249	238	245	251
	246	240	268	268

APPENDIX 1 (CONT.)**EMPIRICAL COMPARISON OF MARGINAL DISTRIBUTIONS
FOR 25°C JOINT STRENGTH TEST RESULTS GIVEN IN TABLE 1****TABLE 1a: UNPERFORATED**

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	334	10.98
3	5	291	9.15
7	6	272	4.86
14	6	235	5.83
21	5	248	8.32
42	5	244	7.18
Pooled Standard Deviation Estimate			7.82

TABLE 1b: SINGLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	301	16.18
3	5	263	11.35
7	5	258	11.18
14	6	232	6.28
21	5	260	12.07
42	5	235	6.73
Pooled Standard Deviation Estimate			10.99

TABLE 1c: DOUBLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	300	21.34
3	5	264	2.30
7	5	259	7.45
14	5	217	3.27
21	5	265	8.17
42	5	245	15.82
Pooled Standard Deviation Estimate			11.86

TABLE 1d: TRIPLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	304	20.82
3	5	295	11.06
7	6	260	12.03

14	3	222	4.93
21	5	247	9.92
42	5	266	11.76
Pooled Standard Deviation Estimate			13.14

APPENDIX 2**TABLE 2: JOINT STRENGTH RESULTS FROM SINGHAP SPECIMENS AFTER IMMERSION IN DISTILLED/DEIONISED WATER AT 40C (FAILURE LOAD PER UNIT WIDTHN/mm)**

Exposure Time (days)	Unperforated	Single Hole	Double Hole	Triple Hole
0	340	313	288	321
	320	294	291	320
	346	279	288	271
	339	320	297	312
	325	298	338	298
3	242	244	263	279
	227	264	266	300
	219	270	260	297
	231	273	265	292
	223	265	264	309
7	235	228	229	230
	235	237	229	240
	219	231	299	226
	239	227	203	235
	236	224	212	219
	227			
14	228	191	242	213
	206	203	250	205
	212	193	231	209
	222	205	236	199
	221	200	258	195
	225			
21	197	213	218	205
	193	210	203	202
	197	204	185	207
	209	211	208	201
	189	203		
42	220	237	211	184
	209	207	219	189
	210	220	219	187
	204	208	194	200
	221	207	207	294

APPENDIX 2 (CONT.)**EMPIRICAL COMPARISON OF MARGINAL DISTRIBUTIONS
FOR 40°C JOINT STRENGTH TEST RESULTS GIVEN IN TABLE 2****TABLE 2a: UNPERFORATED**

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	334	10.98
3	5	228	8.82
7	6	232	7.44
14	6	219	8.34
21	5	197	7.48
42	5	213	7.39
Pooled Standard Deviation Estimate			8.46

TABLE 2b: SINGLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	301	16.18
3	5	272	6.58
7	5	229	4.93
14	6	198	6.15
21	5	208	4.44
42	5	216	13.07
Pooled Standard Deviation Estimate			9.64

TABLE 2c: DOUBLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	300	21.34
3	5	260	19.27
7	5	223	7.50
14	5	243	10.81
21	5	204	13.82
42	5	210	10.34
Pooled Standard Deviation Estimate			14.74

TABLE 2d: TRIPLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	304	20.82
3	5	243	8.79

7	5	230	8.09
14	5	204	7.29
21	4	204	2.75
42	5	191	6.30
Pooled Standard Deviation Estimate			10.83

APPENDIX 3

TABLE 3: JOINT STRENGTH RESULTS FROM SINGHAP SPECIMENS AFTER IMMERSION IN DISTILLED/DEIONISED WATER AT 60C (FAILURE LOAD PER UNIT WIDTHN/mm)

Exposure Time (days)	Unperforated	Single Hole	Double Hole	Triple Hole
0	340	313	288	321
	320	294	291	320
	346	279	288	271
	339	320	297	312
	325	298	338	298
3	212	219	223	210
	202	205	206	212
	219	226	204	201
		209	195	207
		222	211	207
7	226	223	212	214
	223	217	220	223
	226	216	219	213
	222	216	204	208
	209	206	212	204
	227			
14	208	198	198	184
	213	173	197	178
	209	181	190	168
	210	166	196	180
	218	196	164	194
	205			
21	174	148	159	175
	176	149	187	171
	188	156	166	165
	181	143	193	163
	166	159	163	162
42	312	270	244	238
	308	296	266	236
	308	288	262	250
	300	290	258	244
	294	272	252	246

APPENDIX 3 (CONT.)**EMPIRICAL COMPARISON OF MARGINAL DISTRIBUTIONS
FOR 60°C JOINT STRENGTH TEST RESULTS GIVEN IN TABLE 3****TABLE 3a: UNPERFORATED**

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	334	10.98
3	3	211	8.54
7	6	222	6.74
14	6	211	4.51
21	5	177	8.19
42	5	305	7.23
Pooled Standard Deviation Estimate			7.73

TABLE 3b: SINGLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	301	16.18
3	5	216	8.87
7	5	216	6.11
14	6	183	14.03
21	5	151	6.44
42	5	283	11.83
Pooled Standard Deviation Estimate			11.22

TABLE 3c: DOUBLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	300	21.34
3	5	208	10.28
7	5	213	6.46
14	5	189	14.32
21	5	174	15.32
42	5	256	8.65
Pooled Standard Deviation Estimate			13.65

TABLE 3d: TRIPLE HOLE

Exposure Time	No. of Data Points	Mean Failure Load	Standard Deviation
0	5	304	20.82
3	5	207	4.16
7	5	212	7.16

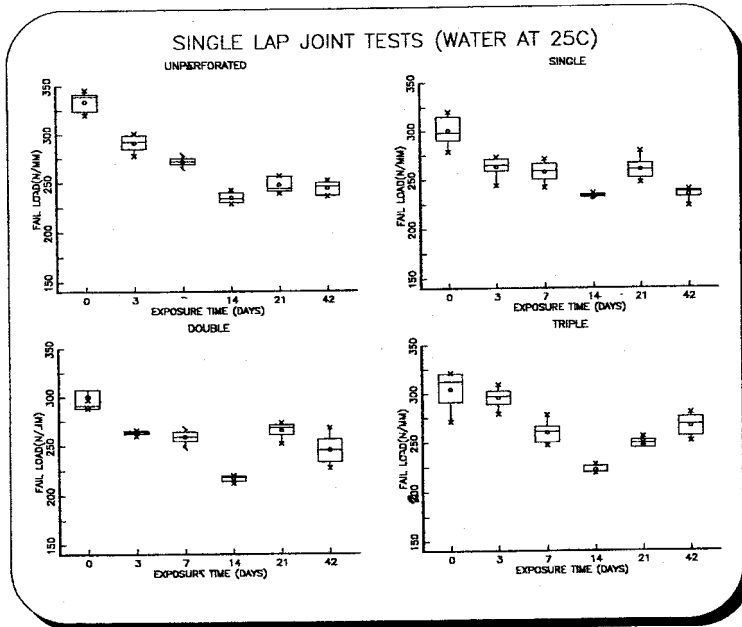
14	5	181	9.44
21	4	167	5.59
42	5	243	5.76
Pooled Standard Deviation Estimate			10.45

APPENDIX 4

BOX/WHISKER PLOTS FOR FAILURE LOAD VERSUS EXPOSURE AT 25

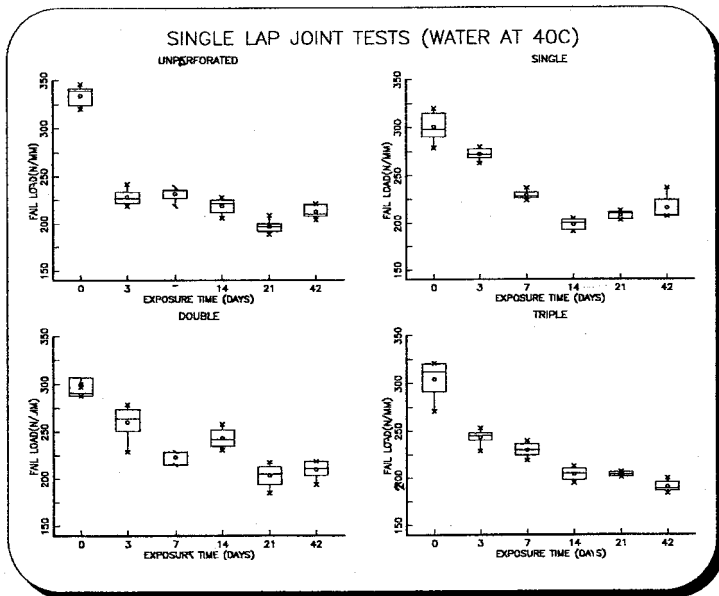
APPENDIX 4

BOX/WHISKER PLOTS FOR FAILURE LOAD VERSUS EXPOSURE AT 25 °C



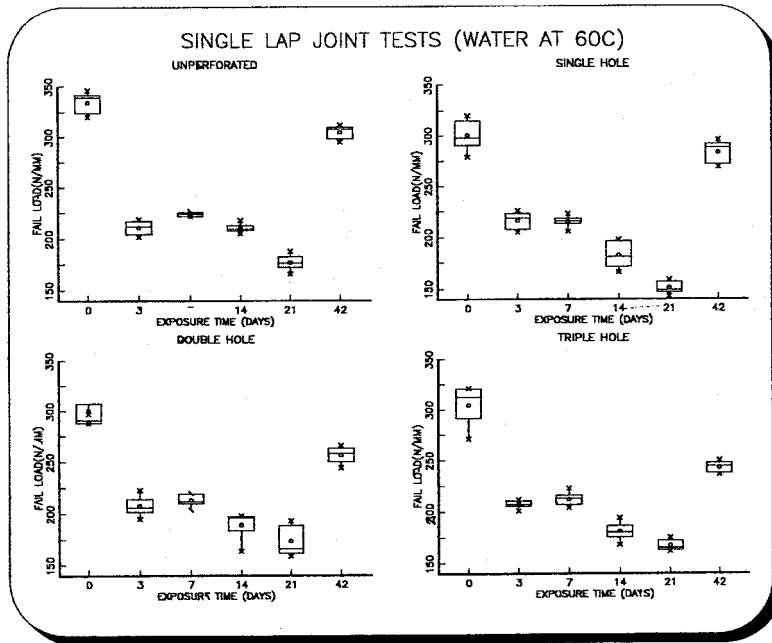
APPENDIX 4 (CONT.)

BOX/WHISKER PLOTS FOR FAILURE LOAD VERSUS EXPOSURE AT 40 °C



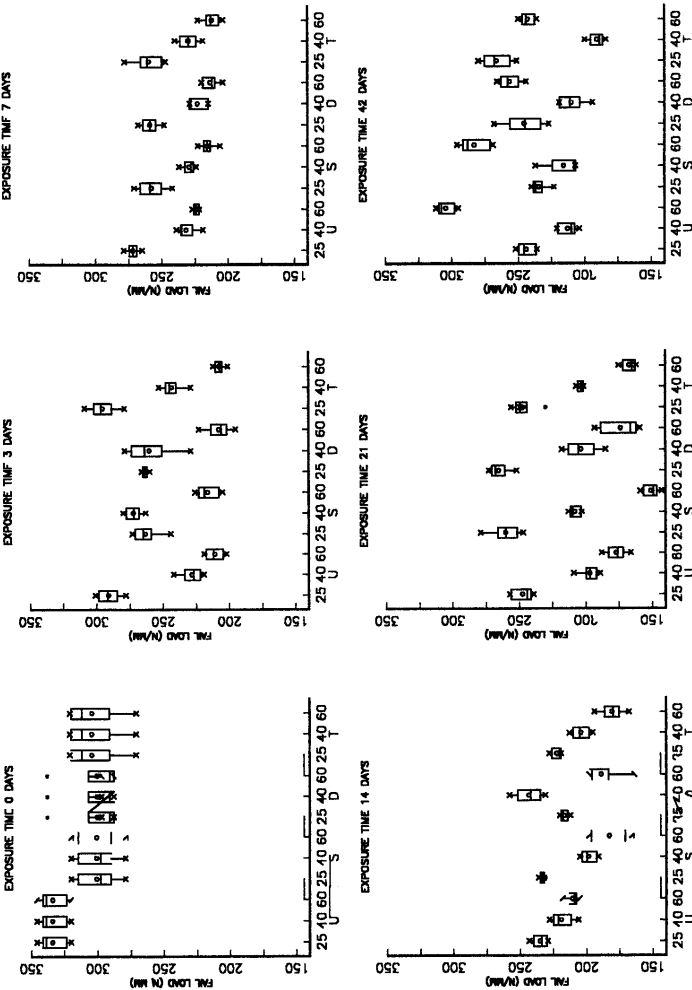
APPENDIX 4 (CONT.)

BOX/WHISKER PLOTS FOR FAILURE LOAD VERSUS EXPOSURE AT 60 °C



APPENDIX 5

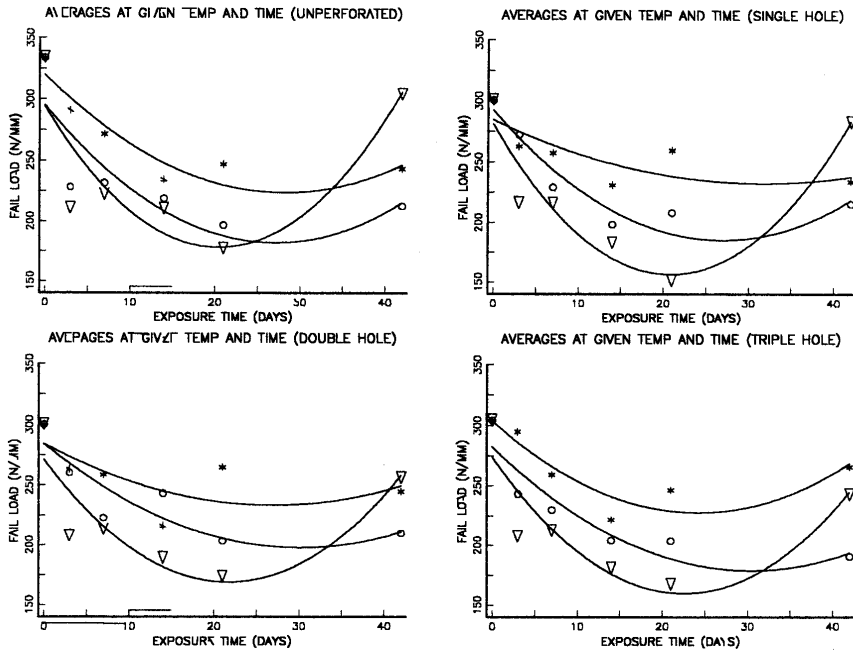
BOX/WHISKER PLOTS FOR FAILURE LOAD AGAINST TEMPERATURE
AT EACH EXPOSURE TIME AND EACH HOLE CONDITION
(U = Unperforated, S = single hole, D = double hole, T = triple hole)



APPENDIX 6

POLYNOMIAL/QUADRATIC FUNCTION FITTING FOR FAILURE LOAD VERSUS EXPOSURE TIME FOR EACH TEMPERATURE/SPECIMEN GEOMETRY COMBINATION

(* = 25 °C, ° = 40 °C, ▽ = 60 °C)



APPENDIX 7**FULL MODEL****TABLE OF COEFFICIENTS**

361 OBSERVATIONS R-SQUARED = 0.65368 STANDARD ERROR =
 26.155
 10 VARIABLES ADJ R-SQUARED = 0.6448

COEFFICIENT	ESTIMATE	STD ERROR	T STATISTIC	SIG LEVEL
INTERCEPT	412.58	18.12	22.77	0.0000E0
TP	4.75	0.86	5.54	4.4240E7
TM	8.99	0.47	19.19	0.0000E0
H	1.45	5.68	0.26	7.8484E1
TP*2	0.04	0.01	4.33	4.0600E5
TM*2	0.16	0.01	19.99	0.0000E0
TP-TM	0.03	0.01	4.62	1.4492E5
TP-H	0.21	0.09	2.40	1.6178E2
H*2	2.20	1.37	1.60	1.0713E1
TM-H	0.03	0.09	0.29	7.6608E1

ANALYSIS OF VARIANCE

SOURCE	DF	SS	MS	F	SIG LEVEL
SECURE	9	4.5322E5	50,358	73.61	0.0000E0
RESIDUAL	351	2.4012E5	684.09		
LACK OF FIT	62	2.0499E5	3,306.4	27.21	1.1102E6
PURE ERROR	289	3.5121E4	121.53		
TOTAL	360	6.9334E5			

APPENDIX 8**REDUCED MODEL****TABLE OF COEFFICIENTS**

361 OBSERVATIONS R-SQUARED = 0.65368 STANDARD ERROR = 26.155
 10 VARIABLES ADJ R-SQUARED = 0.6448

COEFFICIENT	ESTIMATE	STD ERROR	T STATISTIC	SIG LEVEL
INTERCEPT	417.44	17.18	24.30	0.0000E0
TP	-4.87	0.85	-5.72	2.1231E-7
TM	-9.03	0.45	-20.05	0.0000E0
TP*2	0.04	0.01	4.29	4.5943E-5
TM*2	0.16	0.01	19.98	0.0000E0
TP-TM	0.03	0.01	4.62	1.4455E-5
TP-H	0.10	0.03	-3.75	2.9155E-4

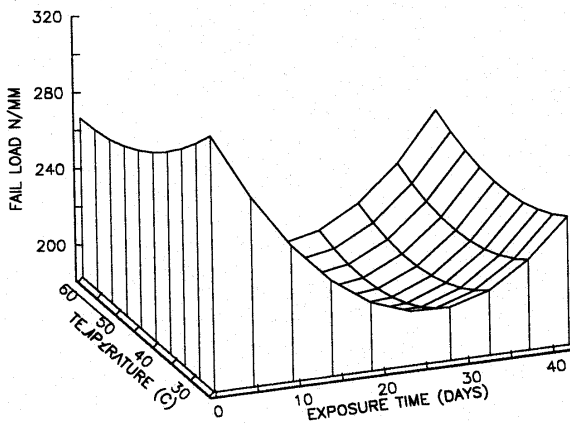
ANALYSIS OF VARIANCE

SOURCE	DF	SS	MS	F	SIG LEVEL
SECURE	6	4.5040E5	75,067	109.39	0.0000E0
RESIDUAL	354	2.4294E5	686.26		
LACK OF FIT	65	2.0781E5	3,197.1	26.31	0.0000E0
PURE ERROR	289	3.5121E4	121.53		
TOTAL	360	6.9334E5			

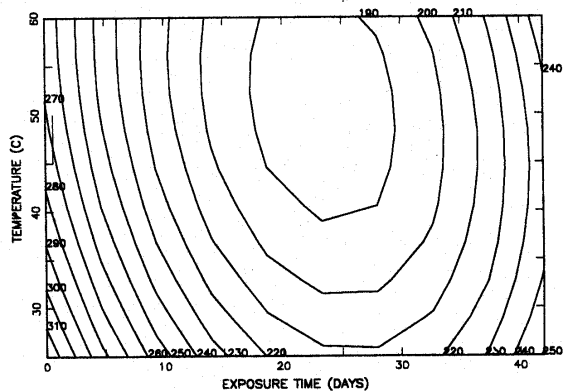
APPENDIX 9

GRAPHICAL OUTPUT OF THE REDUCED MODEL FOR THE SINGLE HOLE CONDITION

**RESPONSE PLOT FOR FITTED MODEL
FAILURE LOAD VERSUS TEMPERATURE AND TIME**



**CONTOUR PLOT FOR FITTED MODEL
FAILURE LOAD VERSUS TEMPERATURE AND TIME**



Case Study 4

Statistical Analysis of NPL Single-Lap Joint Tests

1. INTRODUCTION

In the previous case study the following factors were investigated by Xyratex:

Geometry:	Single-lap (unperforated, single hole, double hole and triple hole)
Adherend:	CR1 mild rolled steel supplied by British Steel
Adhesive:	AV119 (Araldite® 2007) supplied by Ciba Speciality Polymers
Surface treatment:	Grit blast + 1,1,1 trichloroethane wipe
Spacer/filler:	1% 250 µm diameter ballontini glass beads
Cure:	140 °C for 75 minutes

Note: Further details on specimen geometry, specimen preparation and test procedure are obtainable in references [1-2].

The analysis lead the authors to make a number of recommendations for a follow-up activity on the analysis. These were:

- To perform another analysis which specifically excludes the 60 °C/42 days data from the analysis. This analysis would examine the extent of improvement in the model fit. This recommendation was based on the fact that the 60 °C/42 days data seemed untypical and could account for the “lack of fit” found in the analysis.
- The experiment performed was a full factorial for the above factors. It would be useful to understand whether only a fraction of the whole test matrix would have provided nearly the same information as a full factorial. This could be useful for future programmes by reducing the amount of testing required.
- There were some indication that the variation in the results was affected by the geometry of the single-lap joint (unperforated, single hole, double hole and triple holes). If this proved to be the case it could have significance in terms of developing a better method for test repeatability and the knock on effect of test quantities.

The objective of the analysis in this case study is to explore the above. The data used in the analysis is documented in Case Study 3.

2. FULL FACTORIAL ANALYSIS

2.1 LINEAR REGRESSION / ANOVA COMPARISONS

The exclusion of the data for 60 °C/42 days reduced the total data set from 361 to 341 values. Linear Regression data was performed on the new data set, and the coefficients of the model established. The model structure and the factor labels used were:

Failure Load (N/mm) = f [Temperature (**TP**), Time (**TM**), Holes (**H**), Temperature x Temperature (**TP*2**), Time x Time (**TM*2**), Temperature x Time (**TP x TM**), Temperature x Holes (**TP x H**), Holes x Holes (**H*2**), Time x Holes (**TM x H**)]

This construction is referred to as a FULL model. On the basis of the significance of the Coefficients, a smaller model was also constructed which included only the significant factors (i.e. REDUCED model). The analyses for both models are shown in Appendix 2.1 with the ANOVA for the two models included in Appendix 2.2.

The essential features of the two models are as follows:

Full Model		
All data included	Residual sigma = 26.15	R2 = 0.65
60 °C / 42days excluded	Residual sigma = 22.78	R2 = 0.74
	Residual sigma reduction = 3.37	
Reduced Model		
All data included	Residual sigma = 26.20	R2 = 0.65
60 °C / 42days excluded	Residual sigma = 23.20	R2 = 0.73
	Residual sigma reduction = 3.00	

The exclusion of the 60 °C / 42 days data has had the effect of reducing the residual sigma by over 3 for both the FULL and REDUCED models. This is a reduction of more than 11%. The significance of the results on the model fit can be seen with the change in R2 values (R-square statistics). The R2 values have increased from 65% variation to 74%, a significant improvement in model fit.

This implies that the exclusion of the 60C/42 days data has improved the ability to model environmental degradation. Conversely, it points to the fact, that it is very difficult to include in a model which seeks to represent an effect over a smooth response domain, factors which may cause a discontinuity over that response domain. The 60 °C/42 days data is not part of the model appropriate for the bulk of the data. It exhibits a discontinuity and needs separate assessment.

2.2 RESIDUAL VERSUS PARAMETER ANALYSIS

The analysis performed above resulted in the evaluation of the coefficients of the model equation. The experimental configuration for each recorded failure load (i.e. load / width) was inserted into the equation. Failure load is given in N / mm. This resulted in a set off “predicted values”. The difference between the “actual” and “predicted” values is called the residual. The chart in Appendix 2.3 shows a plot of the Residual for each of the 341 values, against some of the parameters used in the model. These plots can be very useful in determining whether any more terms should be included in the model. It can be seen that the Residual/Temperature (TP) plot shows no relationship and therefore the TP term included in the model is sufficient. However, there is definitely a relationship between time (TM) and Residual, possibly a 3rd order effect. The other parameters included in the analysis also show some relationships. **It is important that the experimenter does not get carried away by developing esoteric terms for inclusion in the model, but instead to concentrate on explaining the results in terms of physical phenomena. If physical factors are at play, then by all means enhance the model, but use caution.**

2.3 RESIDUAL SIGNIFICANCE ANALYSIS

A further development of the data on Residuals from the above analysis was used to develop the charts shown in Appendices 2.4 and 2.5. The basic idea was to investigate the residuals to determine any “extreme” data points (i.e. the difference between “actual” and “predicted” value is larger than would be expected). If large differences are present then either the model is insufficient to explain durability behaviour or that data itself is suspect. The experimenter needs to identify which is applicable. The table in Appendix 2.4 shows the residual analysis for each of the 341 data values. It is supplemented with two further columns “STUD T” which normalises the residual to enable significance tests to be performed, and the “SIG” which shows whether the residual is indeed significant. It can be seen from the chart that there are indeed some residuals that are “extreme”. If other members of the same group are not significant, it points to some peculiarity of the data point. Several points fall in this category and are denoted with a “*” in the “SIG” column.

Appendix 2.5 shows the graph of “STUD T” plotted against observation number. This graph is very useful in not only showing “extreme” values, but also in providing a useful pictorial for more detailed examination of the residuals. A residual analysis from a “well behaved” set of data would show residuals randomly spread around a value of zero. It can be seen that there are several instances where unusual patterns are present. Particular zones of interest are over observations 80 to 100, and 260 to 280. It can be seen from the table in Appendix 2.4 that these zones correspond to 25/U21 to 25/T/21 and 60/U/3 to 60/T/3.

As with previous comments, this points to either the inability of the model to predict accurately in these cases, or the data is suspect. If the latter is the case, it points to experimental preparation. Again, these are points for the experimenter to investigate.

Both the residual studies have been useful in augmenting the main analysis. Residual analysis has been shown to provide very useful insights into the data/model combination, which would remain hidden by a Linear Regression Analysis and ANOVA alone.

3. FRACTION ANALYSIS

3.1 DATA SELECTION

There were 3 main factors involved in the original experiment:

Geometry:	Unperforated, single hole, double hole and triple holes (4 levels)
Temperature:	25 °C, 40 °C and 60 °C (3 levels)
Time:	0, 3, 7, 14, 21 and 42 days (6 levels)

There are many possible fractions which could be taken from the above set, but for purposes of evaluating the benefits of a fraction analysis the following rationale was used:

- All levels of Geometry to be included. In a real situation these would require accurate evaluation and therefore all of the U/S/D/T data has been included.

- From the original work, it was noted that Temperature had a strong linear effect on joint strength. It is therefore reasonable to only include the end points of the temperature range (i.e. 25 °C and 60 °C) and to exclude the 40 °C data.
- As 6 levels of Time were included in the previous analysis it would be most profitable to reduce the number of levels. For purposes of this work, various combinations of 3 levels, 4 levels and 5 levels were selected from the full range of 0, 3, 7, 14, 21 and 42 days. Again from the original work, it was noted that Time had a strong curvilinear effect, so the interest was to see which levels of time would best produce the curvilinear effect. In all cases selected, the end points of the range 0 and 42 days were taken as a base. These will always provide significant information and must be selected in any new application. The final choices of data selected are shown in Appendix 3.1. Nine combinations of Time were selected in all.

3.2 INVESTIGATION PROCESS

The investigation process adopted is shown in Appendix 3.1, but in essence it was as follows:

1. Take a fraction of the data, perform linear regression on this data to develop a regression model from the coefficients analysis.
2. Project the model developed in (1), on all the 25 °C and 60 °C data (241 values in total) and predict the failure loads (N/mm) for each of the data points> This is labelled REDUCED DATA in the analysis.
3. Establish the Residual as Actual value- Predicted value.
4. Determine the Residual Sum of Squares as $\Sigma (\text{Actual} - \text{Predicted})^2$.
5. Compare the Residual Sum of Squares against that shown from an ANOVA performed on all the 25 °C and 60 °C data, using this as a benchmark for comparison.

The process was repeated from stage 2 above, for all 25 °C, 40 °C and 60 °C data. This is labelled FULL DATA in the analysis.

3.3 MODEL COEFFICIENTS

The linear regression coefficients for each of the nine selected data sets are shown in Appendix 3.2. This table also includes the R-square and Residual Sigma values computed as part of the ANOVA analysis. It is noted that the best fit model was for selection 3 (0/14/42 days), which gave an R-square value of 0.86 and a Residual Sigma value of 18.39. Conversely, the worst fit model was for selection 7 (0/3/21/42 days), which gave an R-square value of 0.61 and a Residual Sigma of 31.54. This points to the data for selection 3 being “Better behaved” than for selection 7, making it easier to model with confidence.

3.4 FRACTION MODEL RESIDUAL ANALYSIS

The model coefficients for each of the selected 9 data sets were then applied to both the FULL DATA (25 °C/40 °C/60 °C) and the REDUCED DATA (25 °C/60 °C) to establish

predicted values, residuals and the Residual Sum of Squares. The base for comparison purposes was that derived from ANOVA on the FULL DATA and the REDUCED DATA. The ANOVA analyses are shown in Appendix 3.3. In each case, the significant result of Residual Sum of Squares required for further analysis is highlighted. The measure used for comparison purposes was to establish the extent to which the computed Residual Sum of Squares for a particular fraction selection was different from that shown from the FULL or REDUCED DATA analysis. The derived measure of goodness of fit was evaluated as:

$$\text{Residual Sum of Squares (selection) / Residual Sum of Squares (total data)}$$

The measure, which is a multiple of the total data Residual Sum of Squares, is used as the base for comparison. A selection that produces a multiple close to 1 is almost as good at modelling the experiment environment as the full experiment using all 6 Time levels. The measure was evaluated for both FULL and REDUCED data sets. The fraction model residual analysis is shown in Appendix 3.3.

3.5 RESULTS

A number of observations, listed below, are worth noting:

- As expected, no selection exactly produces the information which was obtained by examining all 6 Time levels. Selection 9 with the 5 levels (0/3/7/21/42 days) came very close with multiples of 1.09 (FULL) and 1.0 (REDUCED).
- It is very significant that even selections with 4 levels (i.e. selection 6 with multipliers of 1.21 (FULL) and 1.20 (REDUCED), and selection 7 with multipliers of 1.13 (FULL) and 1.03 (REDUCED)), gave very good results.
- The degree of fitness decreases with a reduction in levels. Two selections from the 3 level selections (selection 3 with multipliers of 1.43 (FULL) and 1.48 (REDUCED) and selection 4 with multipliers of 1.48 (FULL) and 1.41 (REDUCED)) gave almost identical results.
- The worst selection by a considerable margin was selection 1 with levels of 0/3/42 days. This had multipliers of 10.16 (FULL) and 11.06 (REDUCED).
- The only difference between FULL and REDUCED was that FULL included the 40 °C data. The fact that the FULL and REDUCED multipliers are almost identical for each selection combination implies that the inclusion of the 40 °C data has provided no additional information.

4. HOLE GEOMETRY VARIATION ANALYSIS

A key feature of a measurement system is it's ability to produce consistent results when measuring the same feature. In the present study, the Geometry of the test specimens could provide insights into whether different geometries exhibit different abilities to be measured with consistency. The four configurations evaluated were Unperforated (**U**), Single Hole (**S**), Double Hole (**D**) and Triple Hole (**T**).

The full data set of 361 values was used for this analysis. Each factor combination in the original experiment was replicated 5 times, hence the standard deviation (**s**) was calculated for each combination of Perforation/Time/Temperature. The values for **U**, **S**, **D** and **T** were compared for significance using BARTLETT'S test for homogeneity of variances. This analysis was performed for each Temperature and Time combination. The results are included in Appendix 4.1. In addition to the table in Appendix 4.1, Box plots are also included to give a visual impression of the results. The results show that there is no difference in the variation exhibited the different levels of hole. Hence, U/S/D/T effectively behave the same and this is true regardless of temperature and time.

5. CONCLUSIONS

The following conclusions can be drawn from the analysis carried out within the study:

- The exclusion of the 60 °C./42 days data from the analysis had a significant beneficial effect on the ability to model the experiment environment.
- Rather than complicate the modelling by including additional terms, it is advisable to first identify the physical mechanisms responsible for any discontinuities in response, and instead of using a single model to cover the complete data set, try to establish whether another model can better explain that data responsible for a discontinuity in the response.
- The residual analysis was seen to provide very useful insights into the data and model being used. In some cases, this could indicate the need for a complex model and in other cases it could point to deficiencies in the experimental approach.
- The fraction analysis showed that it is possible to perform just a fraction of a planned experiment and gain almost as much information as a full factorial experiment.
- The best fraction analysis results were obtained with five levels of Time (0/3/7/21/42 days), but there were cases where even 3 levels provided satisfactory results. Notably 0/14/42 days and 0/21/42 days.
- The reduction of temperature levels in the analysis (i.e. 25 °C and 60 °C) was justified. The analysis showed that no more information was provided with the inclusion of the 40 °C data.
- The best opportunities for level reduction are on quantitative factors, which are normally performed over several levels. The experimenter could reduce those quantitative factors which are considered essentially linear. The opportunity for level reduction on qualitative factors is not clear, since in most cases accurate information is sought at each distinct level (as in the Geometry case included in this case study).

- The hole variation analysis showed that there is no difference in variation exhibited by the different hole configurations. This means that there is no “best” approach that could prove beneficial as the best for measurement consistency.

REFERENCES

1. W R Broughton, R D Mera and G Hinopoulos, “Environmental Degradation of Adhesive Joints. Single-Lap Joint Geometry”, NPL Report CMMT(A) 196, 1999.
2. A Olusanya, K Tully, R Mera and W R Broughton, “A Comparison of Commercial Design of Experiment Software Programs for the Analysis of Durability Data”, NPL Report CMMT(A) 98, 1998.

ADDITIONAL READING

1. Snedecor and Cochran, “Statistical Methods”, Iowa State University Press.
2. Draper and Smith, “Applied Regression Analysis”, Wiley.
3. Box, Hunter and Hunter, “Statistics for Experimenters”, Wiley.
4. Montgomery, “Design and Analysis of Experiments”, Wiley.

APPENDIX 1.1

FULL MODEL - COEFFICIENT ANALYSIS

COEF.	ESTIMATE	STD.ERROR	T STAT	SIG. LEVEL
INTERCEPT	347.19	17.286	20.085	0
TP	-1.874	0.797	-2.35	0.02
TM	-5.016	0.566	-8.857	2E-13
H	-6.236	5.227	-1.193	0.23
TPxTM	-0.05	0.01	-5.19	2E-6
TP*2	0.014	0.009	1.609	0.1
TM*2	0.117	0.008	14.965	0
TPxH	-0.123	0.08	-1.534	0.12
H*2	2.28	1.234	1.847	0.06
TMxH	0.128	0.088	1.46	0.14

$$N/mm = 347.19 - 1.874TP - 5.016 TM - 6.236 H - 0.05 TPxTM + 0.014 TP^2 + 0.117 TM^2 - 0.123 TPxH + 2.28 H^2 + 0.128TMxH.$$

REDUCED MODEL - COEFFICIENT ANALYSIS

COEF.	ESTIMATE	STD.ERROR	T STAT	SIG. LEVEL
INTERCEPT	321.52	6.046	53.177	0
TP	-0.813	0.134	-6.085	5E-8
TM	-4.635	0.549	-8.444	2E-12
TP-TM	-0.053	0.009	-5.544	4E-7
TM*2	0.115	0.008	14.571	0

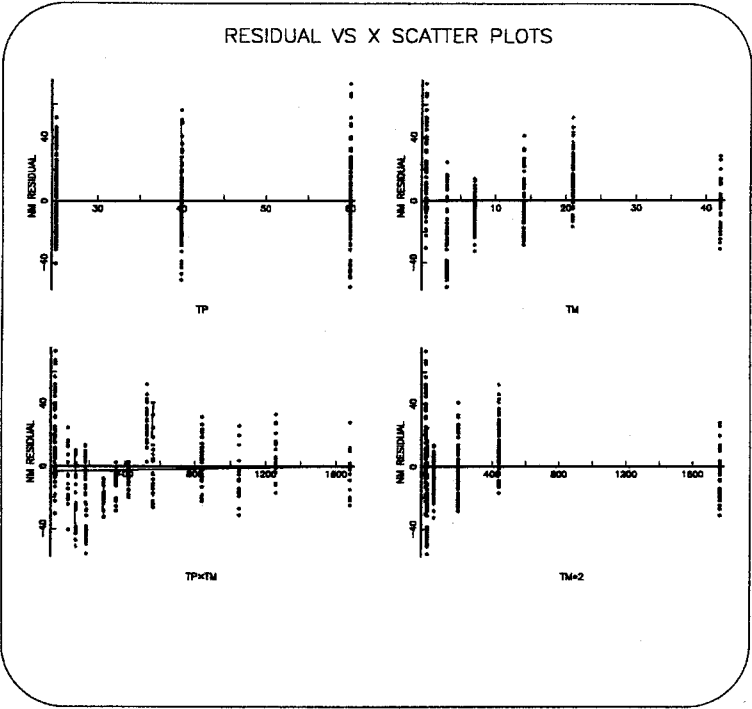
$$N/mm = 321.52 - 0.813 TP - 4.635 TM - 0.053 TPxTM + 0.115 TM^2$$

ANALYSIS OF VARIANCE

DATA/ MODEL	SOURCE	DF	SS	MS	F	SIG.	R-SQUARED	RESIDUAL SIGMA
INCL 60C/42 (FULL)	SECURE	9	4.5322E5	50358	73.61	0		26.15
	RESIDUAL	351	2.4012E5	684.09				
	LACK OF FIT	62	2.0499E5	3306.4	27.21	1.11E-16		
	PURE ERROR	289	3.5121E4	121.53				
	TOTAL	360	6.9334E5				0.65	
INCL 60C/42 (REDUCED)	SECURE	6	4.504E5	75067	109.39	0		26.20
	RESIDUAL	354	2.4294E5	686.26				
	LACK OF FIT	65	2.0781E5	3197.1	26.31	0		
	PURE ERROR	289	3.5121E4	121.53				
	TOTAL	360	6.9334E5				0.65	
EXCL 60C/42 (FULL)	SECURE	9	4.9108E5	54565	105.16	0		22.78
	RESIDUAL	331	1.7175E5	518.89				
	LACK OF FIT	58	1.3783E5	2376.4	19.126	1.11E-16		
	PURE ERROR	273	3.392E4	124.25				
	TOTAL	340	6.6283E5				0.74	
EXCL 60C/42 (REDUCED)	SECURE	4	4.8203E5	1.2051E5	223.96	1.11E-16		23.2
	RESIDUAL	336	1.808E5	538.09				
	LACK OF FIT	12	1.0505E5	8754.3	37.445	3.33E-16		
	PURE ERROR	324	7.5748E4	233.79				
	TOTAL	340	6.6283E5				0.73	

APPENDIX 1.3

RESIDUAL v. MODEL PARAMETERS



APPENDIX 1.4

RESIDUAL ANALYSIS									
OBS	TEMP	PERF	TIME	ACT	PRED	RESID	STUD.T	SIG.	
1	25	U	0	340	301	39	.71		
2				320	301	19	.83		
3				346	301	45	1.97		
4				339	301	38	1.66		
5				325	301	24	1.05		
6	25	S	0	313	301	12	.52		
7				294	301	-7	-.32		
8				279	301	-22	-.98		
9				320	301	19	.83		
10				298	301	-3	-.14		
11	25	D	0	288	301	-13	-.58		
12				291	301	-10	-.45		
13				288	301	-13	-.58		
14				297	301	-4	-.18		
15				338	301	37	1.62		
16	25	T	0	321	301	20	.87		
17				320	301	19	.83		
18				271	301	-30	-1.33		
19				312	301	11	.47		
20				298	301	-3	-.14		
21	25	U	3	298	284	14	.59		
22				292	284	8	.33		
23				287	284	3	.11		
24				301	284	17	.73		
25				278	284	-6	-.28		
26	25	S	3	244	284	-40	-1.76		
27				264	284	-20	-.89		
28				270	284	-14	-.63		
29				273	284	-11	-.50		
30				265	284	-19	-.85		
31	25	D	3	263	284	-21	-.93		
32				266	284	-18	-.80		
33				260	284	-24	-1.06		
34				265	284	-19	-.85		
35				264	284	-20	-.89		
36	25	T	3	279	284	-5	-.23		
37				300	284	16	.68		
38				297	284	13	.55		
39				292	284	8	.33		
40				309	284	25	1.07		
41	25	U	7	279	265	14	.60		
42				265	265	0	-.01		
43				271	265	6	.25		
44				269	265	4	.17		
45				273	265	8	.34		
46				275	265	10	.43		
47	25	S	7	242	265	-23	-1.01		
48				265	265	0	-.01		
49				253	265	-12	-.53		
50				271	265	6	.25		
51				259	265	-6	-.27		
52	25	D	7	257	265	-8	-.36		
53				248	265	-17	-.75		
54				268	265	3	.12		
55				263	265	-2	-.09		
56				259	265	-6	-.27		
57	25	T	7	261	265	-4	-.18		
58				262	265	-3	-.14		
59				247	265	-18	-.79		
60				251	265	-14	-.62		
61				278	265	13	.56		

ACT = OBSERVED VALUE
 PRED = PREDICTED FROM MODEL
 RES = ACT - PRED
 STUD.T = RESIDUAL NORMALISED
 SIG. = RESIDUAL SIGNIFICANCE (YES = *)

RESIDUAL ANALYSIS									
OBS	TEMP	PERF	TIME	ACT	PRED	RESID	STUD.T	SIG.	
62	25	U	14	240	240	0	-.02		
63				243	240	3	.11		
64				229	240	-11	-.50		
65				237	240	-3	-.15		
66				231	240	-9	-.41		
67				230	240	-10	-.45		
68	25	S	14	219	240	-21	-.93		
69				234	240	-6	-.28		
70				235	240	-5	-.24		
71				236	240	-4	-.19		
72				233	240	-7	-.32		
73				232	240	-8	-.37		
74	25	D	14	220	240	-20	-.89		
75				212	240	-28	-1.24		
76				219	240	-21	-.93		
77				218	240	-22	-.98		
78				215	240	-25	-1.11		
79	25	T	14	228	240	-12	-.54		
80				220	240	-20	-.89		
81				219	240	-21	-.93		
82	25	U	21	242	227	15	.66		
83				257	227	30	1.32		
84				244	227	17	.75		
85				239	227	12	.53		
86				256	227	29	1.27		
87	25	S	21	260	227	33	1.45		
88				253	227	26	1.14		
89				279	227	52	2.28	*	
90				262	227	35	1.54		
91				247	227	20	.88		
92	25	D	21	273	227	46	2.02	*	
93				264	227	37	1.63		
94				270	227	43	1.89		
95				252	227	25	1.10		
96				268	227	41	1.80		
97	25	T	21	251	227	24	1.06		
98				230	227	3	.13		
99				249	227	22	.97		
100				249	227	22	.97		
101				256	227	29	1.27		
102	25	U	42	252	254	-2	-.09		
103				236	254	-18	-.81		
104				237	254	-17	-.76		
105				249	254	-5	-.23		
106				246	254	-8	-.36		
107	25	S	42	235	254	-19	-.85		
108				223	254	-31	-1.39		
109				237	254	-17	-.76		
110				238	254	-16	-.72		
111				240	254	-14	-.63		
112	25	D	42	235	254	-19	-.85		
113				227	254	-27	-1.21		
114				252	254	-2	-.09		
115				245	254	-9	-.41		
116				268	254	14	.62		
117	25	T	42	258	254	4	.17		
118				280	254	26	1.16		
119				274	254	20	.89		
120				251	254	-3	-.14		
121				268	254	14	.62		

APPENDIX 1.4 (Cont.)

RESIDUAL ANALYSIS									
OBS	TEMP	PERF	TIME	ACT	PRED	RESID	STUD	T	SIG
122	40	U	0	340	289	51	2.22	*	
123				320	289	31	1.35		
124				346	289	57	2.48	*	
125				339	289	50	2.18	*	
126				325	289	36	1.57		
127	40	S	0	313	289	24	1.04		
128				294	289	5	.22		
129				279	289	-10	-.44		
130				320	289	31	1.35		
131				298	289	9	.39		
132	40	D	0	288	289	-1	-.04		
133				291	289	2	.09		
134				288	289	-1	-.04		
135				297	289	8	.35		
136				338	289	49	2.13	*	
137	40	T	0	321	289	32	1.39		
138				320	289	31	1.35		
139				271	289	-18	-.78		
140				312	289	23	1.00		
141				298	289	9	.39		
142	40	U	3	242	270	-28	-1.21		
143				227	270	-43	-1.86		
144				219	270	-51	-2.20	*	
145				231	270	-39	-1.68		
146				223	270	-47	-2.03	*	
147	40	S	3	270	270	0	.01		
148				277	270	7	.31		
149				263	270	-7	-.30		
150				272	270	2	.09		
151				280	270	10	.44		
152	40	D	3	229	270	-41	-1.77		
153				279	270	9	.40		
154				264	270	-6	-.25		
155				272	270	2	.09		
156				258	270	-12	-.51		
157	40	T	3	253	270	-17	-.73		
158				245	270	-25	-1.08		
159				246	270	-24	-1.03		
160				229	270	-41	-1.77		
161				244	270	-26	-1.12		
162	40	U	7	235	247	-12	-.54		
163				235	247	-12	-.54		
164				219	247	-28	-1.23		
165				239	247	-8	-.37		
166				236	247	-11	-.50		
167				227	247	-20	-.88		
168	40	S	7	228	247	-19	-.84		
169				237	247	-10	-.45		
170				231	247	-16	-.71		
171				227	247	-20	-.88		
172				224	247	-23	-1.01		
173	40	D	7	229	247	-18	-.80		
174				229	247	-18	-.80		
175				228	247	-19	-.84		
176				215	247	-32	-1.40		
177				215	247	-32	-1.40		
178	40	T	7	230	247	-17	-.76		
179				240	247	-7	-.32		
180				226	247	-21	-.93		
181				235	247	-12	-.54		
182				219	247	-28	-1.23		

ACT = OBSERVED VALUE
 PRED = PREDICTED FROM MODEL
 RES = ACT - PRED
 STUD T = RESIDUAL NORMALISED
 SIG. = RESIDUAL SIGNIFICANCE (YES = *)

RESIDUAL ANALYSIS									
OBS	TEMP	PERF	TIME	ACT	PRED	RESID	STUD	T	SIG
183	40	U	14	228	217	11	.47		
184				206	217	-11	-.48		
185				212	217	-5	-.22		
186				222	217	5	.21		
187				221	217	4	.17		
188				225	217	8	.34		
189	40	S	14	191	217	-26	-1.14		
190				203	217	-14	-.61		
191				193	217	-24	-1.05		
192				205	217	-12	-.53		
193				200	217	-17	-.74		
194	40	D	14	242	217	25	1.08		
195				250	217	33	1.42		
196				231	217	14	.60		
197				236	217	19	.82		
198				258	217	41	1.77		
199	40	T	14	213	217	-4	-.18		
200				205	217	-12	-.53		
201				209	217	-8	-.35		
202				199	217	-18	-.79		
203				195	217	-22	-.96		
204	40	U	21	197	198	-1	-.05		
205				193	198	-5	-.22		
206				197	198	-1	-.05		
207				209	198	11	.47		
208				189	198	-9	-.40		
209	40	S	21	213	198	15	.65		
210				210	198	12	.51		
211				204	198	6	.25		
212				211	198	13	.56		
213				203	198	5	.21		
214	40	D	21	218	198	20	.86		
215				203	198	5	.21		
216				185	198	-13	-.57		
217				208	198	10	.43		
218	40	T	21	205	198	7	.30		
219				202	198	4	.17		
220				207	198	9	.38		
221				201	198	3	.12		
222	40	U	42	220	209	11	.50		
223				209	209	0	.01		
224				210	209	1	.06		
225				204	209	-5	-.21		
226				221	209	12	.55		
227	40	S	42	237	209	28	1.26		
228				207	209	-2	-.08		
229				220	209	11	.50		
230				208	209	-1	-.03		
231				207	209	-2	-.08		
232	40	D	42	211	209	2	.10		
233				219	209	10	.46		
234				219	209	10	.46		
235				194	209	-15	-.66		
236				207	209	-2	-.08		
237	40	T	42	184	209	-25	-1.11		
238				189	209	-20	-.88		
239				187	209	-22	-.97		
240				200	209	-9	-.39		
241				194	209	-15	-.66		

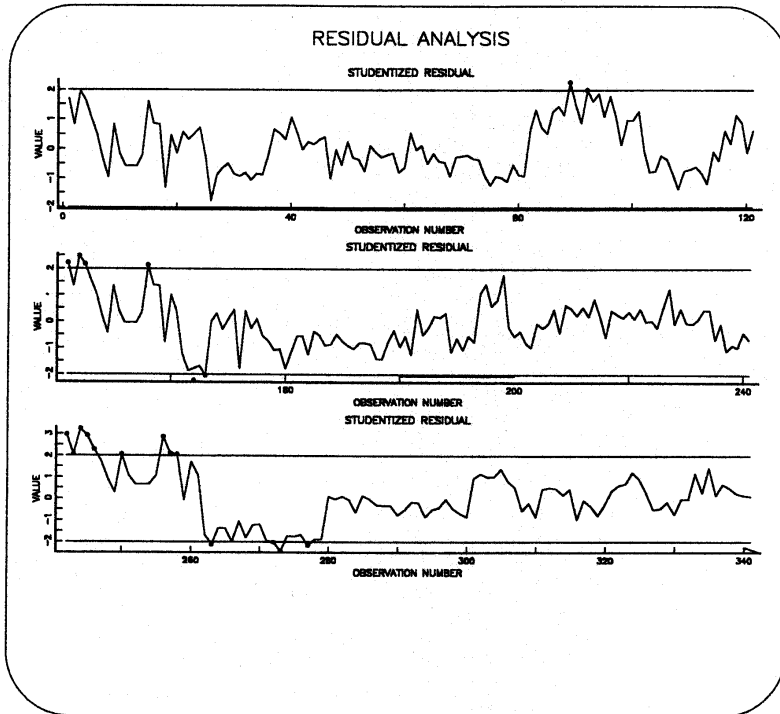
APPENDIX 1.4 (Cont.)

RESIDUAL ANALYSIS								
OBS	TEMP	PERF	TIME	ACT	PRED	RESID	STUD.T	SIG
242	60	U	0	340	273	67	2.96	*
243				320	273	47	2.08	*
244				346	273	73	3.22	*
245				339	273	66	2.92	*
246				325	273	52	2.30	*
247	60	S	0	313	273	40	1.77	
248				294	273	21	.94	
249				279	273	6	.28	
250				320	273	47	2.08	*
251				298	273	25	1.11	
252	60	D	0	288	273	15	.67	
253				291	273	18	.80	
254				288	273	15	.67	
255				297	273	24	1.07	
256				338	273	65	2.87	*
257	60	T	0	321	273	48	2.12	*
258				320	273	47	2.08	*
259				271	273	-2	-.08	
260				312	273	39	1.73	
261				298	273	25	1.11	
262	60	U	3	212	250	-38	-1.68	
263				202	250	-48	-2.12	*
264				219	250	-31	-1.37	
265	60	S	3	219	250	-31	-1.37	
266				205	250	-45	-1.98	
267				226	250	-24	-1.07	
268				209	250	-41	-1.81	
269				222	250	-28	-1.24	
270	60	D	3	223	250	-27	-1.20	
271				206	250	-44	-1.94	
272				204	250	-46	-2.03	*
273				195	250	-55	-2.42	*
274				211	250	-39	-1.72	
275	60	T	3	210	250	-40	-1.77	
276				212	250	-38	-1.68	
277				201	250	-49	-2.16	
278				207	250	-43	-1.90	
279				207	250	-43	-1.90	
280	60	U	7	226	224	2	.09	
281				223	224	-1	-.04	
282				226	224	2	.09	
283				222	224	-2	-.08	
284				209	224	-15	-.65	
285				227	224	3	.14	
286	60	S	7	223	224	-1	-.04	
287				217	224	-7	-.30	
288				216	224	-8	-.34	
289				216	224	-8	-.34	
290				206	224	-18	-.78	
291	60	D	7	212	224	-12	-.51	
292				220	224	-4	-.17	
293				219	224	-5	-.21	
294				204	224	-20	-.86	
295				212	224	-12	-.51	
296	60	T	7	214	224	-10	-.43	
297				223	224	-1	-.04	
298				213	224	-11	-.47	
299				208	224	-16	-.69	
300				204	224	-20	-.86	

ACT = OBSERVED VALUE
 PRED = PREDICTED FROM MODEL
 RES = ACT - PRED
 STUD.T = RESIDUAL NORMALISED
 SIG. = RESIDUAL SIGNIFICANCE (YES = *)

RESIDUAL ANALYSIS								
OBS	TEMP	PERF	TIME	ACT	PRED	RESID	STUD.T	SIG
301	60	U	14	208	186	22	.95	
302				213	186	27	1.17	
303				209	186	23	1.00	
304				210	186	24	1.04	
305				218	186	32	1.39	
306				205	186	19	.82	
307	60	S	14	198	186	12	.52	
308				173	186	-13	-.57	
309				181	186	-5	-.23	
310				166	186	-20	-.88	
311				196	186	10	.43	
312	60	D	14	198	186	12	.52	
313				197	186	11	.47	
314				190	186	4	.17	
315				196	186	10	.43	
316				164	186	-22	-.97	
317	60	T	14	184	186	-2	-.09	
318				178	186	-8	-.36	
319				168	186	-18	-.79	
320				180	186	-6	-.27	
321				194	186	8	.34	
322	60	U	21	174	160	14	.63	
323				176	160	16	.71	
324				188	160	28	1.24	
325				181	160	21	.93	
326				166	160	6	.27	
327	60	S	21	148	160	-12	-.52	
328				149	160	-11	-.48	
329				156	160	-4	-.17	
330				143	160	-17	-.74	
331				159	160	-1	-.03	
332	60	D	21	159	160	-1	-.03	
333				187	160	27	1.20	
334				166	160	6	.27	
335				193	160	33	1.46	
336				163	160	3	.14	
337	60	T	21	175	160	15	.67	
338				171	160	11	.49	
339				165	160	5	.23	
340				163	160	3	.14	
341				162	160	2	.10	

APPENDIX 1.5

RESIDUAL ANALYSIS**NOTE**

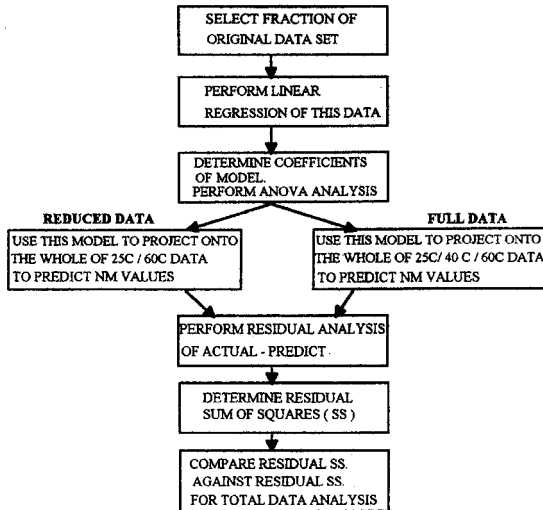
OBSERVATION	1 to 121	25 degrees C data
OBSERVATION	122 to 241	40 degrees C data
OBSERVATION	242 to 341	60 degrees C data

APPENDIX 2.1

DATA USED TO DETERMINE MODEL

TIME DAYS	TEMPERATURE																												
	25 deg C									40 deg C									60 deg C										
	SELECTION									SELECTION									SELECTION										
	1	2	3	4	5	6	7	8	9											1	2	3	4	5	6	7	8	9	
0	Y	Y	Y	Y	Y	Y	Y	Y	Y											Y	Y	Y	Y	Y	Y	Y	Y	Y	
3	Y				Y	Y	Y	Y	Y											Y				Y	Y	Y	Y	Y	
7		Y				Y			Y	Y											Y				Y			Y	Y
14			Y				Y		Y													Y				Y		Y	
21				Y				Y		Y													Y				Y		Y
42	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y										Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

Y = DATA SELECTED

FRACTION ANALYSIS (MODEL STRUCTURE)

FRACTION MODEL COEFFICIENTS

COEF.	SELECTION								
	1	2	3	4	5	6	7	8	9
INTERCEPT	342.88	339.86	334.53	345.93	345.92	338.91	343.72	347.11	347.26
TP	-0.376	-0.358	-0.148	-0.524	-0.737	-0.635	-0.909	-0.895	-1.081
TM	-24.071	-13.249	-11.311	-8.986	-13.44	-10.995	-8.691	-11.079	-8.869
H	-14.058	-12.058	-13.455	-6.892	-14.204	-14.756	-9.852	-13.838	-9.76
TPxTM	0.037	0.033	0.023	0.016	0.04	0.037	0.027	0.04	0.028
TM*2	0.508	0.256	0.216	0.167	0.258	0.205	0.162	0.205	0.166
TPxH	-0.361	-0.246	-0.272	-0.285	-0.264	-0.29	-0.287	-0.218	-0.216
H*2	7.754	5.434	5.491	4.217	6.623	6.731	5.586	5.801	4.821
TMxH	-0.009	0.001	0.048	0.052	-0.017	0.003	0.023	-0.022	0.004
R-SQUARED	0.72	0.76	0.86	0.69	0.66	0.73	0.61	0.74	0.63
RES. SIGMA	21.37	19.07	18.39	29.7	23.33	24.1	31.54	22.18	28.42

NOTES:

1. THE SELECTION NUMBERS DEFINE THE DATA FRACTION (SEE APPENDIX 3.1)
2. RES. SIGMA IS THE RESIDUAL SIGMA DERIVED FROM THE ABILITY OF THE MODEL TO PREDICT THE OBSERVED VALUES (I.E. ACTUAL - PREDICT FROM MODEL)

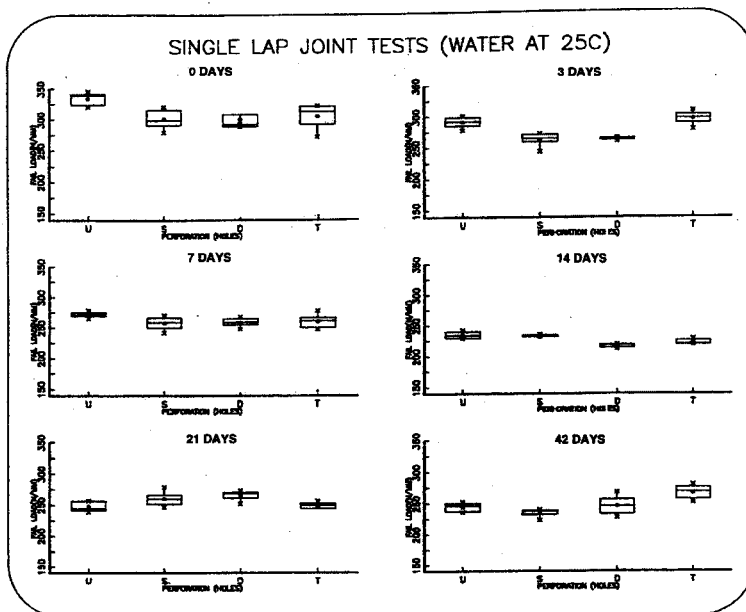
APPENDIX 2.3

ANALYSIS OF VARIANCE

DATA	SOURCE	DF	SS	MS	F	SIG.
25C/60C TOTAL DATA	SECURE	8	3.3865E5	42,332	61.74	2.22E-16
	RESIDUAL	232	1.5907E5	685.65		
	LACK OF FIT	39	1.3574E5	3,480.5	28.79	0
	PURE ERROR	193	2.3332E4	120.89		
	TOTAL	240	4.9772E5			
25C/40C/60C TOTAL DATA	SECURE	8	4.4041E5	55,051	76.61	2.22E-16
	RESIDUAL	352	2.5293E5	718.55		
	LACK OF FIT	63	2.1781E5	3,457.3	28.449	0
	PURE ERROR	289	3.5121E4	121.53		
	TOTAL	360	6.9334E5			

FRACTION MODEL RESIDUAL ANALYSIS

SELECTION	DATA INCLUDED	RESIDUAL SS. ANALYSIS			
		FULL		REDUCED	
		RESIDUAL SS	MULTIPLES OF TOTAL DATA SS.	RESIDUAL SS.	MULTIPLES OF TOTAL DATA SS.
1	0 / 3 / 42	2570459	10.16	1759984	11.06
2	0 / 7 / 42	398959	1.58	264310	1.66
3	0 / 14 / 42	361683	1.43	235342	1.48
4	0 / 21 / 42	375653	1.48	224372	1.41
5	0 / 3 / 7 / 42	405177	1.60	264606	1.66
6	0 / 3 / 14 / 42	306924	1.21	191023	1.20
7	0 / 3 / 21 / 42	284699	1.13	163276	1.03
8	0 / 3 / 7 / 14 / 42	296365	1.17	183260	1.15
9	0 / 3 / 7 / 21 / 42	276943	1.09	159643	1.00
TOTAL DATA	0 / 3 / 7 / 14 / 21 / 42	252930		159070	

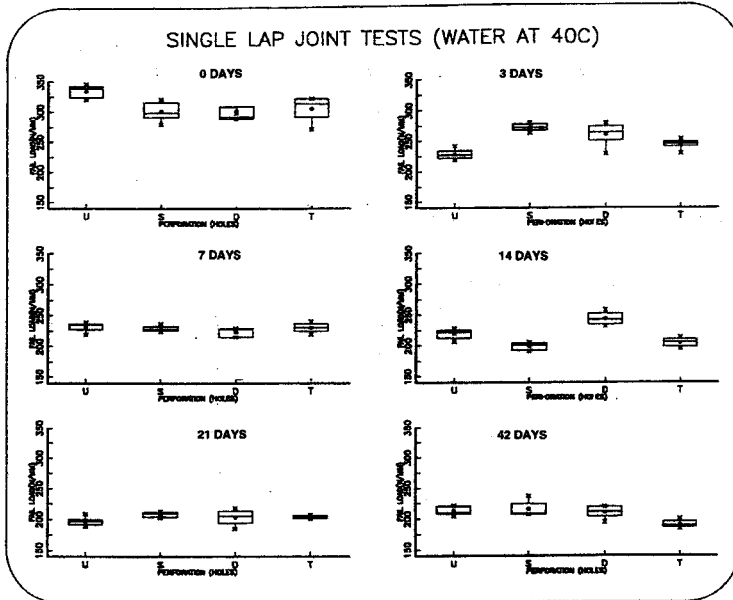
HOLE GEOMETRY VARIATION ANALYSIS - 25C**BARTLETT'S TEST FOR HOMOGENEITY OF VARIANCE**

CONDITIONING TIME (DAYS)	SIGNIFICANCE OF COMPARISON OF VARIANCES ACROSS PERFORATION U/S/D/T	ARE VARIANCES DIFFERENT ACROSS PERFORATION U/S/D/T?
0	0.62	NO
3	0.06	NO
7	0.29	NO
14	0.66	NO
21	0.86	NO
42	0.31	NO

NOTE

THE CONVENTIONAL SIGNIFICANCE LEVEL CHOSEN IS 0.05.
 VALUES LARGER THAN THIS ARE NOT CONSIDERED SIGNIFICANT
 I.E. THE VARIANCES ACROSS THE COMPARISON CAN BE CONSIDERED TO BE THE SAME

APPENDIX 3.1 (Cont.)

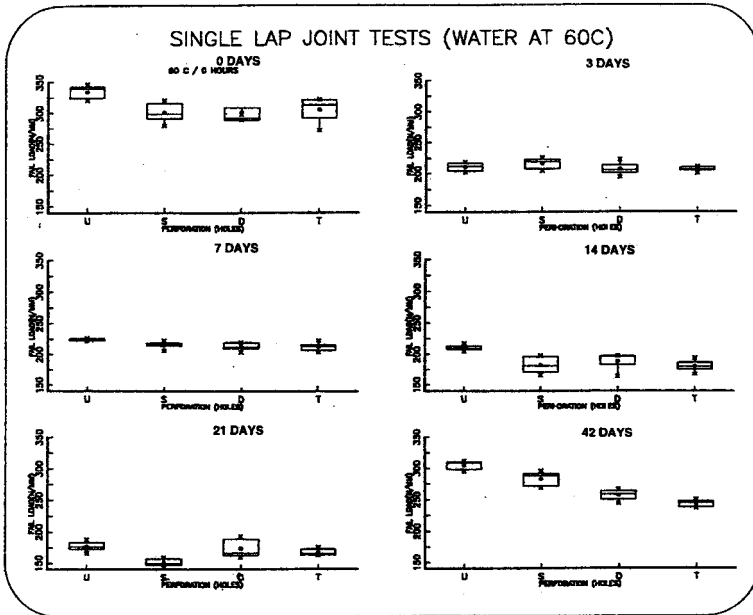
HOLE GEOMETRY VARIATION ANALYSIS - 40C**BARTLETT'S TEST FOR HOMOGENEITY OF VARIANCE**

CONDITIONING TIME (DAYS)	SIGNIFICANCE OF COMPARISON OF VARIANCES ACROSS PERFORATION U/S/D/T	ARE VARIANCES DIFFERENT ACROSS PERFORATION U/S/D/T?
0	0.62	NO
3	0.15	NO
7	0.82	NO
14	0.74	NO
21	0.06	NO
42	0.52	NO

NOTE

THE CONVENTIONAL SIGNIFICANCE LEVEL CHOSEN IS 0.05.
 VALUES LARGER THAN THIS ARE NOT CONSIDERED SIGNIFICANT
 I.E., THE VARIANCES ACROSS THE COMPARISON CAN BE CONSIDERED TO BE THE SAME

APPENDIX 3.1 (Cont.)

HOLE GEOMETRY VARIATION ANALYSIS - 60C**BARTLETT'S TEST FOR HOMOGENEITY OF VARIANCE**

CONDITIONING TIME (DAYS)	SIGNIFICANCE OF COMPARISON OF VARIANCES ACROSS PERFORATION U/S/D/T	ARE VARIANCES DIFFERENT ACROSS PERFORATION U/S/D/T?
0	0.62	NO
3	0.45	NO
7	0.98	NO
14	0.13	NO
21	0.18	NO
42	0.57	NO

NOTE

THE CONVENTIONAL SIGNIFICANCE LEVEL CHOSEN IS 0.05.
 VALUES LARGER THAN THIS ARE NOT CONSIDERED SIGNIFICANT
 I.E., THE VARIANCES ACROSS THE COMPARISON CAN BE CONSIDERED TO BE THE SAME