

REPORT

Ensemble Decimation Factors for Reverberation Chamber Stirrer Data

L R ARNAUT

AUGUST 1999

NPL Report CETM 12

Ensemble Decimation Factors for Reverberation Chamber Stirrer Data

L R Arnaut

Center for Electromagnetic and Time Metrology
National Physical Laboratory
Queens Road
Teddington
Middlesex TW11 0LW
United Kingdom

August 1999

Abstract

A spectral domain technique is presented for the accurate determination of decimation factors of stirrer data. The technique is based on using the *ensemble* autocorrelation function, which is obtained using low-pass filtering of the power spectral density function and application of the Wiener-Khinchine theorem. The technique yields decimation factors which are independent of the reference position of the stirrer. The technique is considerably faster than computation of decimation factors in the stirring domain. For sufficiently large sample sizes, it is found that the ensemble decimation factor are slightly, but consistently lower than the values obtained from the autocorrelation function in the stirring domain for a single realization of the process.

Keywords: reverberation chamber, mode-stirred cavity, EMC test environments, EMC immunity and vulnerability testing, data correlation, decimation factor, spectral analysis.

© Crown Copyright 1999
Reproduced by permission of the Controller of HMSO

ISSN 1467-3932

National Physical Laboratory
Teddington, Middlesex TW11 0LW, United Kingdom

Extracts of this report may be reproduced provided that the source is acknowledged and the extract is not taken out of context

Approved on behalf of the Managing Director of NPL by
Stuart Pollitt, Head of the Center for Electromagnetic Metrology

Contents

1	Introduction	4
2	Ensemble autocorrelation function	5
3	Numerical results	6
4	Conclusion	8
A	Figures	9

List of Figures

1	Stirrer data @ 8.2GHz	9
2	Power spectral density functions @ 8.2GHz: unfiltered data (single sample) vs. filtered data (ensemble)	10
3	Power density autocorrelation functions @ 8.2GHz: single sample vs. ensemble functions	11
4	Dependence of decimation factors m_α on sample size M @ 8.2GHz: single sample vs. ensemble values	12

1 Introduction

The analysis of correlation of stirrer data forms an essential part in the characterization of a reverberation chamber. The degree of correlation shown by the statistical power or the statistical electric field, as measured for different stirrer positions (steps), depends on the electrical size of the stirrer, incremental stirrer step, frequency of operation and room size (volume). The use of an appropriate decimation (subsampling) factor, as defined below, enables one to select a reduced set of approximately uncorrelated, i.e. linearly independent data [3]. This then better justifies the use of $\chi_{2p}^{(2)}$ distributions functions for constituent sums of squares of *independent* Gauss normal variables ($p = 1, 2, 3$), from which various statistics of interest can be calculated.

Decimation factors are in general computed from the power density autocorrelation function (acf) of the stirrer data based on its definition:

$$\rho(\tau_0, k\Delta\tau) = \frac{\sigma_{|E|^2|E|^2}}{\sigma_{|E|^2}^2} \frac{\sum_{i=0}^{M-1} \{ [|E(\tau = \tau_0 + i\Delta\tau)|^2 - \langle |E(\tau)|^2 \rangle] [|E(\tau = \tau_0 + (k+i)\Delta\tau)|^2 - \langle |E(\tau)|^2 \rangle] \}}{\sum_{i=0}^{M-1} [|E(\tau = \tau_0 + i\Delta\tau)|^2 - \langle |E(\tau)|^2 \rangle]^2} \quad (1)$$

for stirrer step size $\Delta\tau$, stirrer reference position τ_0 and where the data vector of length M has been periodically continued, i.e. $E(\tau_0 + (M+i)\Delta\tau) \equiv E(\tau_0 + i\Delta\tau)$ with $k, i = 0, 1, \dots, (M-1)$. The smallest integer value of k for which $|\rho(\tau_0 + k\Delta\tau)| \leq \rho_0$ with $\rho_0 \triangleq 1/e \approx 0.368$ is then defined as the decimation factor, denoted as m . A set of $N = \lfloor M/m \rfloor$ approximately uncorrelated data points can then be obtained as the subsampled set $|E(\tau)| = |E(\tau_0 + m(i\Delta\tau))|$ with $i = 0, 1, 2, \dots, \lfloor (M-1)/m \rfloor$.

A few comments are in order. Firstly, in the statistical analysis it may be advantageous to use noninteger values for m , i.e. for which the strict equality $\rho(\tau_0 + m\Delta\tau) = \rho_0$ holds. This enables field statistics such as mean, standard deviation and confidence intervals to be determined more accurately by using a more accurate value of $N = \lfloor M/m \rfloor$. Secondly, the choice of the value $1/e$ for ρ_0 does not have a rigorous justification and requires further study. Nevertheless, it is widely used and appears to yield statistics that are in good agreement with measured values, i.e. yielding a reasonably accurate value for the equivalent N . [4]. Other values for ρ_0 (e.g. 0.1, 0, ...) have been suggested. In any case, the correction proposed in the following analysis applies equally well to other choices, with the exception¹ of $\rho_0 = 0$ which would yield $m = \infty$.

¹Essentially this is because the method makes use of the Wiener–Khinchine relation between the acf and power spectral density (psdf), resulting in an acf that is always strictly positive after inverse Fourier transformation,

Generally, the smaller the chosen value of ρ_0 the larger the correction will be, because of the asymptotic behaviour of the ensemble acf.

The acf for the field magnitude can be defined in a similar way to (1) as:

$$\begin{aligned}\rho'(\tau_0, k\Delta\tau) &= \frac{\sigma_{|E||E|}}{\sigma_{|E|}^2} \\ &= \frac{\sum_{i=0}^{M-1} \{[|E(\tau = \tau_0 + i\Delta\tau)| - \langle |E(\tau)| \rangle][|E(\tau = \tau_0 + (k+i)\Delta\tau)| - \langle |E(\tau)| \rangle]\}}{\sum_{i=0}^{M-1} [|E(\tau = \tau_0 + i\Delta\tau)| - \langle |E(\tau)| \rangle]^2}\end{aligned}\quad (2)$$

Generally, ρ' yields decimation factors very similar to those for ρ . The acf for the (complex) field E follows from its generalized definition:

$$\begin{aligned}\rho''(\tau_0, k\Delta\tau) &= \frac{\sigma_{EE^*}}{\sigma_E \sigma_{E^*}} \\ &= \frac{\sum_{i=0}^{M-1} \{[E(\tau = \tau_0 + i\Delta\tau) - \langle E(\tau) \rangle][E^*(\tau = \tau_0 + (k+i)\Delta\tau) - \langle E^*(\tau) \rangle]\}}{\sqrt{\sum_{i=0}^{M-1} [E(\tau = \tau_0 + i\Delta\tau) - \langle E(\tau) \rangle]^2} \sqrt{\sum_{i=0}^{M-1} [E^*(\tau = \tau_0 + (k+i)\Delta\tau) - \langle E^*(\tau) \rangle]^2}}\end{aligned}\quad (3)$$

which is real and, unlike the previous definitions for the acf, positive for all τ

2 Ensemble autocorrelation function

The acf $\rho(\tau)$ is itself a random process with its own distribution function and, therefore, subject to statistical variations upon realization of the process. To eliminate these variations, one may collect a large number of samples of stirrer data, through multiple realizations of the experiment, after which the mean and standard deviation of the acf follow. A more economical way to obtain the ensemble acf makes use of a Fourier transform pair, which can be constructed on the basis of the Wiener-Khinchine theorem (see e.g. [2]):

$$\rho(\tau) = \int_0^\infty g(\omega) \exp(j\omega\tau) d\omega \quad (4)$$

where τ and ω constitute a pair of Fourier variables in the stirrer and spectral stirrer domain, respectively. We shall further denote the latter by the ‘spectral domain’ for short. The one-sided power spectral density function (psdf) for the stirrer data, $g(\omega)$, is itself obtained after a Fourier transformation (or equivalently, since $g(\omega)$ is real and positive, a Fourier cosine transformation) of the stirrer data. Typically, $g(\omega)$ shows random (noise-like) fluctuations around an, on average,

and which may be considered as an envelope to the absolute value of the true acf for a particular realization, i.e. as obtained by its definition Eqn (1).

exponential decay. These fluctuations are a result of the realization itself and cannot be reduced by e.g. increasing the sample size [1, Sec. 6.1]. By low-pass filtering $g(\omega)$ prior to applying Eqn (4) one uses the ensemble psdf to obtain the ensemble acf. Other smoothing procedures for the sample spectrum to estimate the ensemble spectrum exist (Bartlet smoothing procedure for subseries, Blackman–Tukey spectral filtering, Bartlett window, Parzen window, ... [1, Sec. 6.3]), but all show a typical compromise between variance and bias

This spectral technique not only avoids the need for multiple realizations for the experiment, but also obviates the need for calculation of the acf from its definition, Eqn (1), which becomes very time consuming for large sample sizes.

The specification of the filter for the power transfer function is a compromise between retaining sufficient spectral information (requiring high order of the filter) and eliminating statistical fluctuations (requiring low order of the filter). If one restricts the filter to be of 1st order, which is justifiable given the measured spectral psdf and characterization as a 1st order spectral autoregressive process [4], i.e.:

$$g(\omega) = \exp(\beta\omega) \quad (5)$$

then inverse Fourier transformation results in a simple acf, viz.:

$$\rho(\tau) = [1 + (\tau/\beta)^2]^{-1} \quad (6)$$

from which the decimation factor m , defined by the correlation length $\tau_m = m\Delta\tau$ for which $\rho(\tau_m) = \rho_0 = 1/e$, follows as

$$m = \frac{|\beta| \sqrt{e-1}}{\Delta\tau} \quad (7)$$

In general, m and thus τ_m can be estimated using interpolation for the discretized acf. If more complex (higher-order) models for the filtered $g(\omega)$ are permitted, then the inversion can be performed numerically.

3 Numerical results

We used stirrer data collected with a 3-axis electric field probe (Schmidt & Partner ER3DV4R (SN:1228)) in the *NPL* untuned stadium reverberation chamber. The chamber was operated at 8.2GHz using mechanical wall stirring, previously shown to yield a properly stirred statistical

field at this frequency [4]. The chamber was excited by a CW signal with nominal input power P_{in} . For each field component $|E_\alpha|$ ($\alpha = x, y, z$ and $t(otal)$), a maximum of 1200 sample points is taken.

Figure 1 shows the stirrer data $|E_\alpha|$ as recorded at an arbitrary position inside the cavity. The measured data was then normalized as $|E_\alpha|^2/P_{in}$. We applied a spectral filter of 5th order to the psdfs, i.e. after filtering $g_\alpha(\omega) \sim \exp[\sum_{i=0}^5 \beta_{\alpha i} \omega^i]$. The psdfs, before and after filtering, are shown in Fig 2. The corresponding acfs for $M = 1200$ are shown in Fig 3, yielding the decimation factors $m_\alpha = \rho_\alpha^{-1}(1/e)$.

The solid lines in Fig 4 show the decimation factors m_α for the energy densities $|E_\alpha|^2$ as a function of the sample size M . These apply to subvectors obtained by taking the first M data points from the complete 1200-point data vectors. Hence for each M all correlations were computed with respect to the same reference stirrer position τ_0 . It was found that $M > 50$ is required to yield 'stable' values for m_α . With the exception of m_x , all m_α show variations within about 0.25 from the ensemble averaged value, i.e. $m_\alpha(M) = \langle m_\alpha \rangle \pm 0.25$. The values for m_x show a marked increase around $M = 550$, indicating an increased correlation for the stirrer positions for which $M > 550$ for $|E_x|$. The dependencies $m_\alpha(M)$ are generally relatively smooth, because they result from a single realization, i.e. they are deterministic.

The dotted lines show the m_α obtained after spectral filtering of the psdf. The dependencies $m_\alpha(M)$ now show larger uncertainties because they apply to the ensemble of correlation functions rather than a single realization, i.e. they are statistics, and because of the gradually increasing size of the truncating spectral window. They are a result of jitter of the psdf, caused by decreasing the uncertainty of the psdf (through interpolation) at the expense of bias in its estimation [1]. Overall, however, the *averaged* ensemble decimation factors show less variation across the range for M than m_α for a single realization (compare e.g. a moving local average of $m_x(M)$ or $m_t(M)$ for these functions with the dotted lines). No differences were found between linear and cubic spline interpolation used to estimate m_α .

In summary, for arbitrary M the m_α obtained for the single realization are good estimates for the ensemble averaged values. However, the former yield a small but consistent overestimation of typically 0.5. The ensemble method should be avoided for relatively small sample sizes ($M < 200$), because the limited number of points in the FFT-IFFT then yields lower accuracies.

4 Conclusion

A spectral method has been proposed for determination of the decimation factor of the ensemble of autocorrelation functions and applied to stirrer data obtained in the *NPL* untuned stadium reverberation chamber using a 3-axis electric field probe. This FFT-IFFT technique yields accurate values for the decimation factor which are independent of the starting position of the stirrer, and is also considerably faster than computation of decimation factors in the stirring domain. For large sample sizes, it is found that the thus obtained decimation factor is slightly, but consistently lower than the values obtained from the autocorrelation function in the stirring domain for a single realization of the process.

The FFT-IFFT technique can also be used to advantage for obtaining estimates for other ensemble averages in cases where the spectral, i.e. Fourier transformed random process decreases exponentially with spatial frequency, so that low-pass spectral filtering minimally affects the ensemble stirrer data characteristics.

References

- [1] Jenkins, G M and Watts, D G: *Spectral Analysis and its Applications*, Holden-Day, San Francisco, CA (1968), Ch 6.1.
- [2] Vanmarcke, E: *Random Fields: Analysis and Synthesis*; MIT Press, Cambridge, MA (1983), Ch 3.
- [3] Corona, P, Ferrara, G and Migliaccio, M: Reverberation chambers as sources of stochastic electromagnetic fields; *IEEE Trans EMC*, vol 38 no 3 (1996), pp 348-356.
- [4] Arnaut, L R and West, P D: Evaluation of the *NPL* untuned stadium reverberation chamber using mechanical and electronic stirring techniques; *NPL Technical Report CEM 11* (August 1998).

A Figures

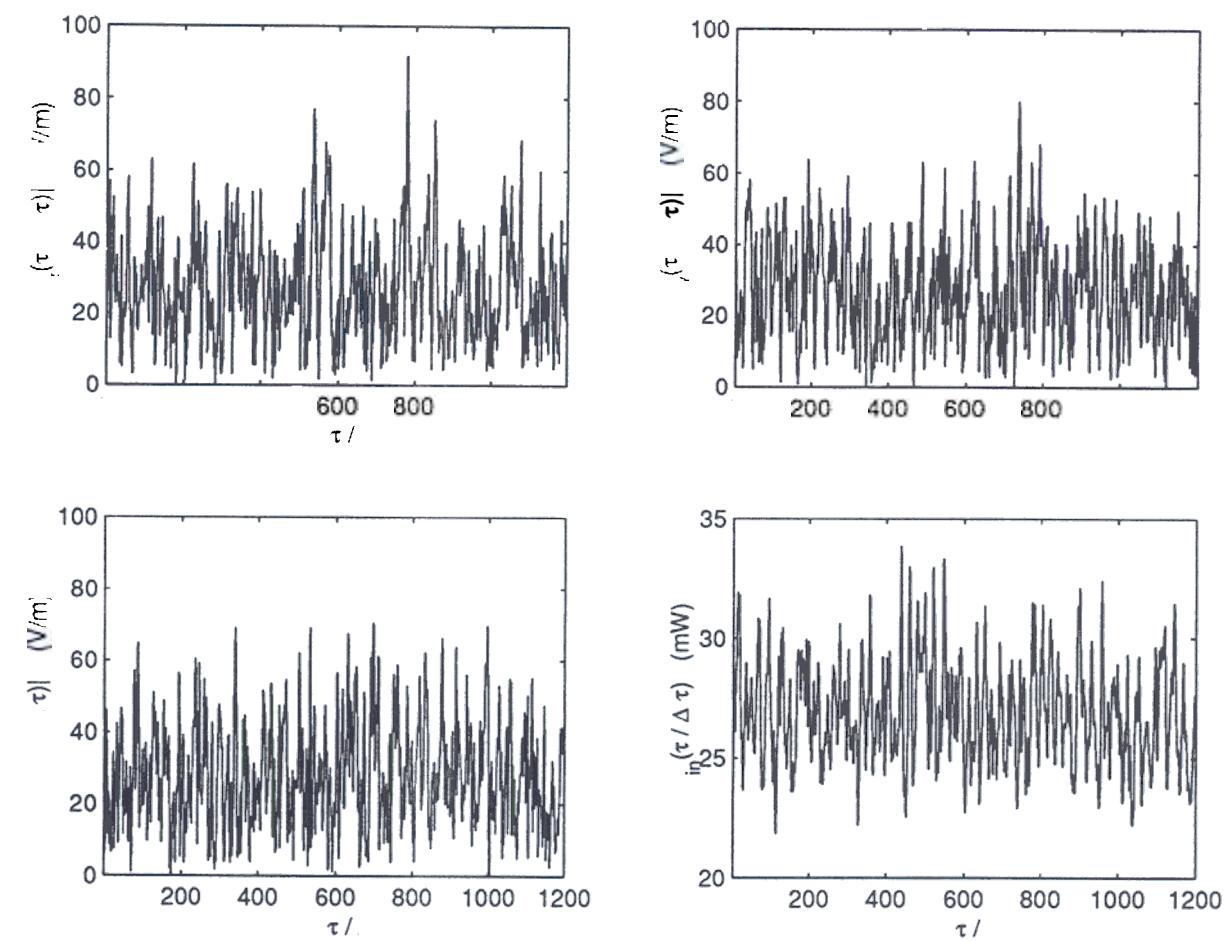
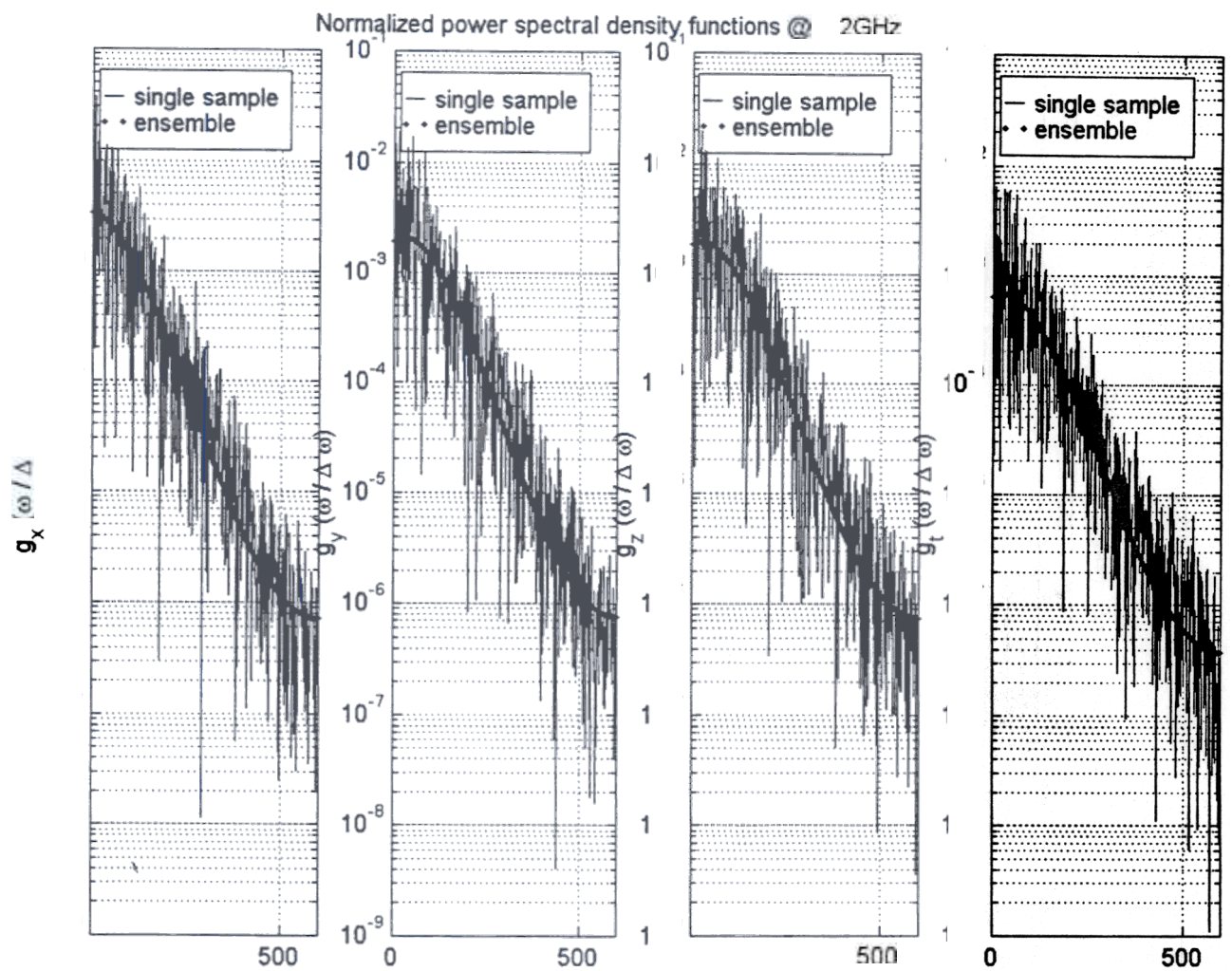


Figure 1 Stirrer data @ 2GHz



(un
 en
 len
 ion: @ 2GHz
 fi red
 gl sample)
 ered

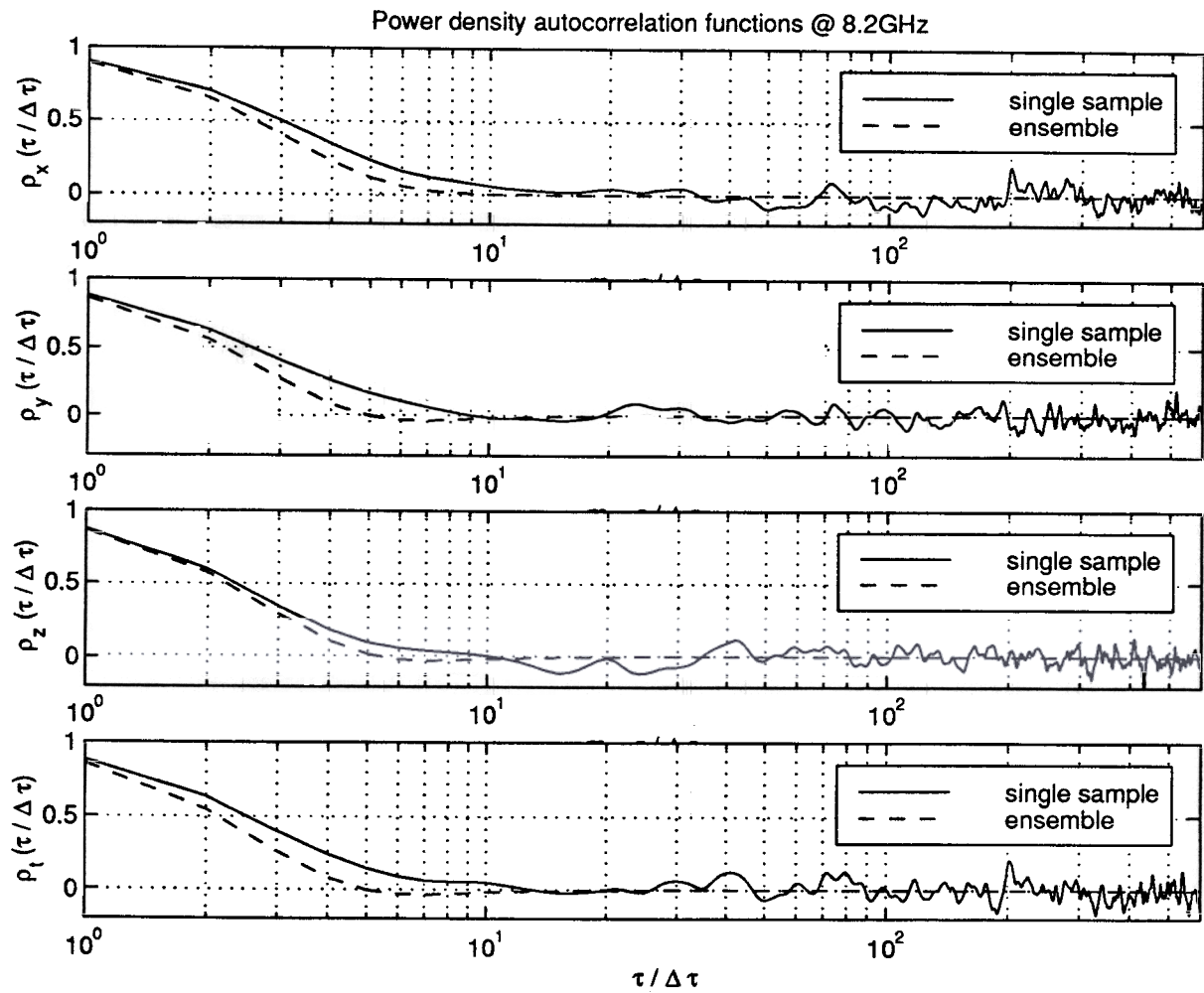


Figure 3: Power density autocorrelation functions @ 8.2GHz: single sample vs. ensemble functions

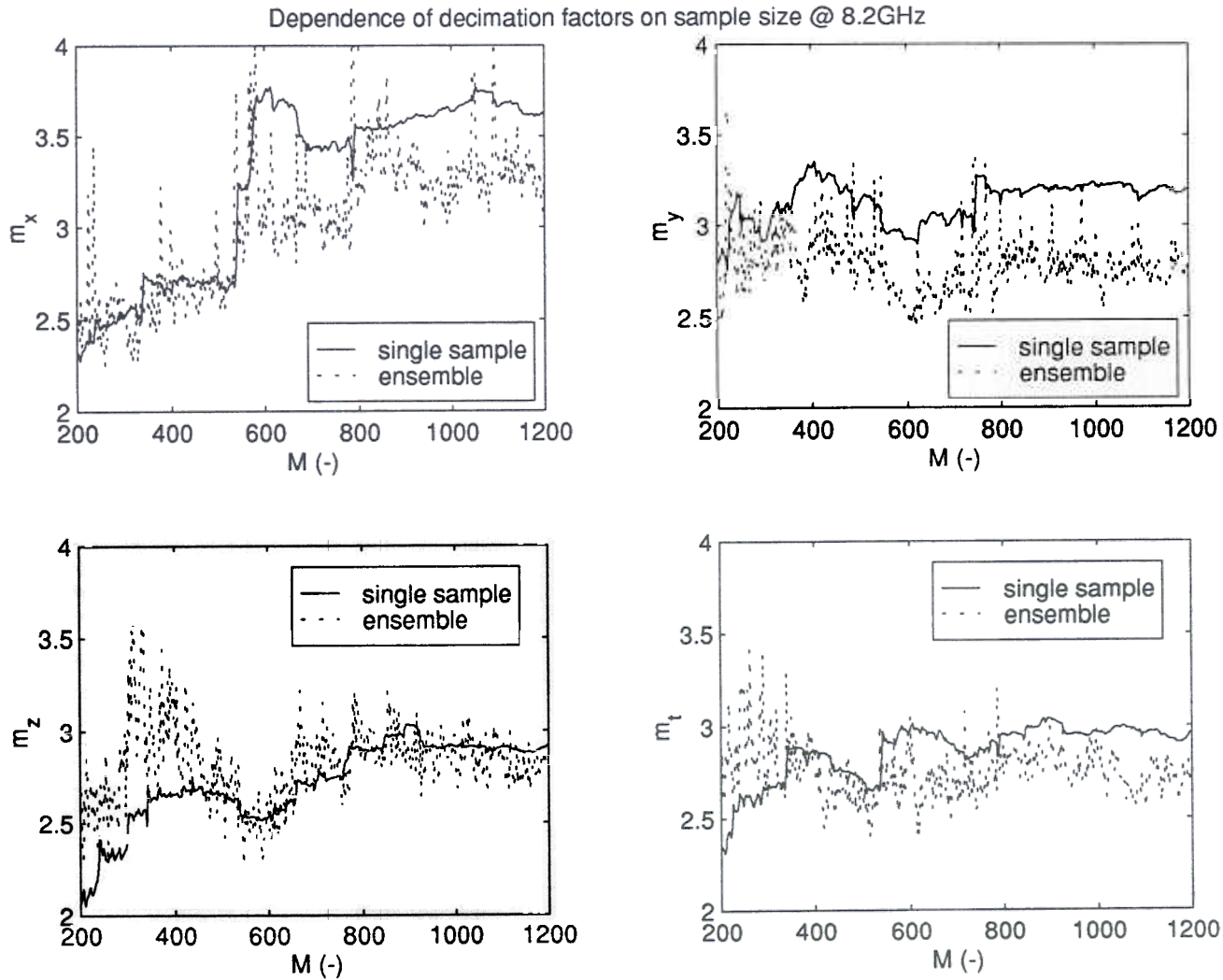


Figure 4: Dependence of decimation factors m_α on sample size M @ 8.2GHz: single sample vs. ensemble values