

REPORT

Evaluation of the *NPL* Untuned Stadium Reverberation Chamber Using Mechanical and Electronic Stirring Techniques

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Abstract

Measurement results, statistical data analysis and chamber characteristics are reported for the NPL untuned stadium reverberation chamber. The chamber has been tested at 2.5, 8.2 and 18GHz using wall stirring, mechanical source stirring and frequency stirring. Results obtained using both mode stirred and mode tuned operation have been intercompared. The levels of planar field anisotropy and field inhomogeneity have been quantified. First- and second-order spectral analysis is used to calculate the normalized spectral bandwidth for various locations inside the cavity and to obtain an autoregressive model for the stirring process. Some theoretical results for general reverberation chambers are also derived.

Keywords: reverberation chamber, mode-stirred cavity, untuned cavity, ergodicity, EMC test environments, EMC immunity and vulnerability testing, measurement uncertainty, spectral analysis.

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Approved on behalf of the Managing Director of NPL by
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1 Introduction

In a conducting cavity, the electromagnetic (EM) field is deterministic, inhomogeneous, anisotropic and has a fixed polarization state. A properly designed and operated reverberation chamber provides an EM test environment that exhibits a statistically homogeneous, isotropic and randomly polarized field distribution, within acceptable uncertainty, confidence and tolerance limits. These limits are largely determined by the number of independent boundary conditions that can be generated inside the chamber at the frequency of operation. In the limit of this number approaching infinity, the chamber has the same *statistical* EM properties at *all* locations, in *each* direction and with *every* possible direction of field polarization being *equally* probable, with zero uncertainty. As a direct consequence, information on the polarization and directional characteristics of the equipment under test (EUT) is lost in reverberation chamber measurements. Likewise, the polarization and directional properties of the receiving and transmitting antennas are not relevant in the measurement and analysis either. Apart from generating a complex EM environment, reverberation chambers are particularly useful in EMC immunity and vulnerability testing because of the very high field strengths ($>100\text{V/m}$) that can be generated using relatively low levels of input power. Reverberation chambers are also useful for emission testing of highly directive sources or receivers.

The statistical field isotropy and randomness of polarization in a reverberation chamber are obtained by applying statistical changes to a given complex cavity field distribution. Such a complex field is obtained through the coexistence of a large number of cavity modes which are excited within the bandwidth of operation, resulting in a strongly overmoded cavity. Owing to the central limit theorem, this large number of modes gives rise to each one of the three Cartesian components of the cavity field exhibiting a complex Gauss normal distribution with zero mean and equal variance σ^2 [$N(0, \sigma)$], at each location [6, 12, 40, 47]. The sum of n independent Gauss normal distributed random variables with such mean and variance forms a χ^2 probability density function (pdf) with n degrees of freedom, denoted as χ_n^2 . This distribution has a mean value $n\sigma^2$ and variance $2n\sigma^4$. When measuring power in one direction, or any other quantity that is proportional to component of the squared field, the measured value is the sum of squares of the in-phase and the quadrature components, hence $n = 2$. Hence for ideal reverberation operation the power density associated with one direction (component) of the field exhibits a Poisson χ_2^2 , i.e. exponential, pdf [12, 19, 21, 47, 53]. In standardized form, i.e. after replacing the field components E_i by $(E_i - 0)/\sigma$, the pdf reads:

$$\chi_2^2(x_i) = (1/2) \exp(-x_i/2), \quad (x_i > 0), \quad (1)$$

where x_i is the standardized measured (received) power for the direction i . For comparison

between theory and experiment, it is convenient to normalize the random variable by its own mean μ_i (nominal value 2):

$$\chi_2^2(x_i/\mu_i) = \exp(-x_i/\mu_i), \quad (x_i/\mu_i > 0), \quad (2)$$

thus making the distribution independent of the input power. More generally, if a p -axis antenna or probe is used to measure the received power or field strength, then a χ_{2p}^2 or χ_{2p} pdf applies, respectively, in ideal reverberation conditions ($p = 1, 2, 3$).

The homogeneity of the chamber is assessed separately, by measuring the power or field strength at different locations inside the chamber.

In spite of the statistical analysis being used for a single realization, the reverberation process itself is of course fully deterministic¹: the time evolution of the process, i.e. the EMC test, should be completely repeatable and reproducible, when keeping the initial and boundary conditions constant for two realizations of it. In other words, given the same exact time evolution of the excitation field, stirrer positions, EUT locations, . . . , the same tuner sweep data should be obtained, in principle, leading to an identical² realization. It is only *within* one realization of the test that the stirrer data appear as pseudo-random, because of the multitude of apparently independent boundary conditions and spatial field distributions generated during one sweep. It is because of this apparent randomness within one realization of the experiment that statistical techniques can be applied. An obvious limitation of a statistical approach is that it does not enable zero uncertainty for the results to be achieved. However, the very many degrees of freedom of complex EM systems tend to diminish the fluctuations in the predicted response. Hence the problem of uncertainties is likewise reduced.

2 Measurement set-up

Conventional EM reverberation chambers are typically prismatic cavities in which one or more rotating paddle wheel(s) ('mode stirrer(s)') operate(s) to effect changes in the EM boundary conditions and, hence, in the field pattern (mechanical stirring; e.g. [76, 20, 38]), or in which a CW source is swept in phase or frequency to the same effect (electronic stirring; e.g. [5, 48]). The NPL untuned 3D stadium reverberation chamber (Figs 2.1–2.3) was built following a design by Kremer and Genzel for spherical untuned cavities [31], combined with results that have emerged from advances in the theory, simulation and measurement of quantum chaos in ergodic microwave and quantum billiard systems in complex cavities (e.g. [17, 21, 40, 45, 74]).

¹in the absence of noise

²within the measurement uncertainty

Implementations with structured cavity surfaces, based on acoustic analogies, have also been investigated recently using numerical simulation [58, 65]. These studies may give the impression that improving field homogeneity always requires some form of diffracting surface. However, perfectly smooth but asymptotically ergodic cavities may equally be used under certain conditions and are often much simpler to implement in practice.

To first approximation, our chamber can be considered as a spherical cavity with corrugated aluminium inner wall. The average inner (outer) diameter ($2R$) of the two hemispheres is about 0.70m (0.74m), providing a working volume $V = 0.18\text{m}^3$. On the inner wall, smaller hemispherical corrugations of various sizes have been applied which are convex (i.e. having opposite, positive curvature compared to that of the hemispheres), with radii ranging between 5mm and 40mm (Fig 2.3). These corrugations establish the inner radius of the cavity as a spatially random variable ('untuning'). As a result, they provide a stirring capability (cfr. infra). As convex diffractors, they also avoid the formation of caustics inside the cavity that would result from wave focusing by the negative curvature of the large hemispheres. The wave dispersing properties of the corrugations further enhance the ergodic (chaotic) properties of the chamber considerably, because chaos in dispersing billiards is known to be larger than in 3D focusing billiards [61] (cfr. infra). Finally, the randomized positions and sizes of the corrugations help to preserve the ergodicity in 3D. The ergodic properties would be destroyed if an axially symmetric 3D stadium were constructed, e.g. through the simple rotation of a 2D ergodic stadium around the vertical long axis, leading to a continuous symmetry group.

At shorter wavelengths, the effect of the straight octagonal cylindrical section of height 0.175m (Fig 2.2), connecting the two hemispheres, must be taken into account. The chamber is then to be considered as a *3D nonintegrable and untuned Bunimovich stadium* [17, 40, 45, 49, 50, 61, 74]. The stadium parameter γ , defined as the ratio of the height of the cylindrical section to the cavity diameter, is 0.25 in our case i.e. relatively small. However, this ratio can easily be increased by inserting an extra cylindrical section at the equator, as the upper hemisphere is demountable.

Energy inside the chamber is injected and measured using two identical horn antennas, which are mounted on rectangular waveguides that penetrate the cavity at its equator, through any one of eight flat detachable coupling plates. In order to avoid direct illumination, the input horn is made to point 45deg downwards using a waveguide bend; the receiving horn is pointing 45deg upwards. The latter is also displaced along the equator by an azimuthal angle $\Delta\phi = -90\text{deg}$ and with a direction of polarization being 90deg rotated in elevation with respect to that of the input horn. By inserting waveguide bends the antenna apertures become slanted with respect to the cylindrical section of the cavity wall. Such orientations are known to give a higher weight to modes whose pattern shows an inclination with respect to any cavity wall as compared to

other modes whose pattern is parallel to one or more cavity walls [77].

Measurements for this configuration at 8.2GHz and 18GHz were found to show a good stirring performance.

At 2.5GHz, a somewhat poorer reverberation performance was noticed when using this set-up. This was put down to be mainly due to the fact that no horns were used at this frequency (impractical), with the waveguide aperture pointing towards the flat equatorial plates. This results in a 1st order reflection path between Tx and Rx, which can be considered as a quasi-direct³ illumination. To avoid such a path, the waveguides were mounted in the top of the cavity. Hence the waveguides apertures are then facing a corrugated surface. Better, i.e. more structured sweep data were thus obtained, even though the depth of the corrugations is already relatively small compared to the wavelength in this case.

3 Cavity ergodicity and mode tuned wall stirring

The central cylindrical section plays a key role in the design and operation of the chamber. It transforms the integrable spherical chamber into a nonintegrable but ergodic cavity, when considered in the geometric optics (GO) limit. This gives rise to the suppression of caustics inside the chamber and, more generally, to the reduction of field inhomogeneity (spatial nonuniformity). It also yields fundamentally different characteristics with respect to the average local field strength and spatial correlation properties, as compared to a simple integrable cavity such as a sphere or prism [21, 40]. Consequently, the stadium parameter γ becomes an extra design parameter. It can be optimized for maximum intrinsic ergodicity, for example, from the calculation of the Lyapounov exponent (e.g. [61]) which measures the rate of local divergence, from the Kholmogorov entropy for the entire cavity, from the matching of the cumulative distribution function (cdf) of the nearest neighbour spacing [4] to a Wigner cdf for Gaussian orthogonal ensemble (GOE) statistics [33], from the examination of number invariance and spectral rigidity [11], or by investigating the mode fluctuation distribution [57, 62] which also enables one to explain the ergodicity of more complex systems, such as hyperbolic biliards, and thus appears to be a more robust measure of chaos. At a few⁴ isolated eigenfrequencies, a (2D) stadium still displays classical resonances, giving rise to isolated region(s) of higher field strength ('scars') along certain trajectories [34]. Cavity modes with profound spatial inhomogeneity, also known as 'bouncing ball' and 'whispering gallery' modes, have been shown to exist at isolated frequencies [40, 45, 74]. They do not satisfy the Shnirelman ergodicity theorem [16] and should be

³Only repeatedly reflected waves with equal contributions to the total field can establish a reverberation field.

⁴It can be shown that a 2D Bunimovich stadium has a sparser density of bouncing ball states than e.g. a cosine billiard or a Sinai billiard, hence the choice of the stadium in our design.

avoided for our purposes. At microwave frequencies, the GO limit breaks down. Consequently, the cavity then shows only partially chaotic field distributions, i.e. with a distribution of the nearest-neighbour spacing which holds the middle between a pure Poisson distribution for an integrable cavity and a pure Wigner distribution for a nonintegrable cavity [35]. Thus, the untuning and asymptotic cavity ergodicity at lower frequencies yield a partially homogenized cavity field [73]. In our set-up, this partial uniformity is further improved upon through mechanical or electronic stirring.

Reverberation is achieved by rotation of the lower corrugated hemisphere (wall stirring) around an imaginary vertical axis, rather than using one or more rotating paddle wheel(s) inside. The upper hemisphere, the coupling plates and the positions of the horns remain fixed at all times during a scan (sweep) of the lower hemisphere with respect to an inertial frame of reference. At sufficiently large frequencies, this wall stirring appears to produce an efficient change in the EM boundary conditions, coupled with relatively low insertion loss, as well as a larger usable volume relative to the physical volume because obstructive paddles are not required. It has been well appreciated in the literature that a large stirrer (with reference to the room dimensions and wavelength) is recommended in order to achieve efficient mechanical stirring (see e.g. [1, 29, 43]). Since a stirring wall has the maximum possible stirrer size, it has the potential of making the reverberation process very efficient compared to paddle stirring. The chamber optimization lies in deciding on the number, depth and shape of the wall corrugations.

The reverberation performance can be assessed from the analysis of the statistical properties of the insertion loss P_{rec}/P_{in} , as measured for each stirrer position i.e. considered as a random variable [27]. Here P_{rec} signifies the recorded local total received power or received power for a field component in the (rectilinear) direction defined by the orientation of the horn, at one location inside the cavity. It is recommended to monitor P_{in} for each stirrer position as well, not just P_{rec} : mechanical stirring involves a statistical change of the chamber geometry and, hence, a change of the wave input impedance for each mode. This is particularly, though not exclusively, noticeable when the horn is directly facing the stirrer. Therefore, the monitoring of P_{in} enables the statistical variations of the wave impedance [38] for the source and, hence, the insertion loss to be accurately accounted for, because in general $\langle P_{rec}/P_{in} \rangle \neq \langle P_{rec} \rangle / \langle P_{in} \rangle$.

The distribution of the received power was measured at a fixed position, about 10cm ($> \lambda/4$) away from the chamber wall [5, 30, 38]. The chamber statistics were assessed by operating the chamber in stepped mode regime ('mode tuning') and by measuring the input and output power associated with one field component as a function of the angular position of the lower hemisphere. A total of $M = 500, 1500$ and 3200 samples at equidistant wall steps of size 2.424mm,

0.909mm and 0.303mm were taken at 2.5GHz, 8.2GHz and 18GHz, respectively. The measured data for P_{rec}/P_{in} are shown in Figs 3.1, 3.2 and 3.3 for the three respective frequencies.

4 Autocorrelation and independent samples

4.1 Autocorrelation of measured data

To estimate the correlation between sampled data points, the autocorrelation function (acf) $\rho(\tau)$ was computed for each frequency of operation. As may be expected, the correlation between two sample points at a fixed distance apart decreases as the frequency increases: in the mean, reverberation characteristics of *nonregular* cavities are expected to improve a fortiori with increasing frequency [1]. For example, at 8.2GHz and with a step size $s = 0.909\text{mm}$ measured along the rim, the acf was found to remain within the widely used $[-\exp(-1), \exp(-1)]$ -band for 6 steps or more of the stirrer, i.e. the decimation factor m for the measured data is 6. The advantage of using a small step size, i.e. one which results in $m \gg 1$, is that the correlation length and, hence, the number of linearly independent samples can be deduced accurately. In addition, on the basis of visual inspection of the chamber, the chamber sweep data was expected to possibly show repetition after 90deg, i.e. after approximately 660 sample points. However, such repetition did not show up in the sweep data, nor in the acf. This is probably a result of the fact that some diffractors are not perfectly hemispherical, but show indentations (corrugations) which have a large effect on the resonant modes. Thus, a subset of independent samples in the measured chamber sector contains maximum $N \approx M/m = 250$ points. Stated differently, at 8.2GHz the horn antennas experience statistically independent⁵ boundary conditions for a minimum azimuthal step size $\Delta\phi_{min}$ of approximately 0.82deg (i.e. approximately $s = 5\text{mm}$ rim travel as measured along the outer shell) at this frequency. To confirm the estimated number of independent sample points, sweeps were taken for various step sizes and the associated acf was calculated. The above values of N and minimum step size Δs_{min} were largely confirmed.

Since the data series $X(\tau)$ is in general rather long, the direct computation of the acf using its definition becomes time consuming. To accelerate this computation, one can use the fast Fourier transformation of the data series, $\mathcal{F}[X(\tau)](\omega)$ coupled with its inverse (FFT-IFFT pair):

$$\rho[X(\tau), X(\tau + i\Delta\tau)] = \frac{2\pi}{(2M-1)\sigma^2} \sum_{p=-(M-1)}^{M-1} \left| \mathcal{F}[X(i\Delta\tau)] \left(\omega = \frac{2\pi p}{2M-1} \right) \right|^2 \exp \left[j \frac{2\pi p(i\Delta\tau)}{2M-1} \right] \quad (3)$$

⁵This calculation is based on the somewhat arbitrary ' $|\rho| < \exp(-1)$ ' criterion for uncorrelated sample points. An alternative definition for the correlation length has been suggested (see e.g. [14, 67]), viz. the distance between the starting position and the one for the first zero crossing of the acf.

with $X = P_{rec}/P_{in}$, $i = -(M-1), \dots, M-1$ and $\Delta\tau = \Delta s/R$. For large samples, such a procedure works considerably faster than building the acf matrix from the data itself, based on its definition. Eqn (3) is based on the Wiener-Khinchine theorem for the covariance function $B(\tau) = \sigma^2 \rho(\tau)$ of $X(\tau)$:

$$\mathcal{F}[B(\tau)](\omega) \cdot \delta(\omega - \omega') = \langle \mathcal{F}[X(\tau)](\omega) \mathcal{F}[X^*(\tau)](\omega') \rangle \quad (4)$$

where the asterisk denotes complex conjugation.

The measured acfs are shown in Figs 4.1–4.3. For all three frequencies, the Matérn lower bound [9] for the acf of 3D homogeneous isotropic fields, viz.

$$\rho(\tau \neq 0) \geq -1/3 \quad (5)$$

is seen to be satisfied everywhere. Tbl 1 summarizes the step sizes for independent samples at the three frequencies of operation, based on the computed acfs. Noninteger values for m were allowed for in order to obtain these step sizes, from interpolation.

The data correlation can be analyzed in a more general manner by considering the pdf of the standardized values r of $\rho(\tau)$ for $\tau \neq 0$. For a sufficiently large number of totally uncorrelated data points, the distribution of their acf tends to the distribution of the acf of white noise, viz. a Gauss normal distribution [79]. Thus, this additional analysis on r expresses correlation in terms of deviations from the theoretical standard Gauss normal distribution. Figs 4.4–4.6 compare the cdf for the measured acf to the theoretical cdf at the three frequencies of operation. The insets show the complete acfs, as measured. It follows from the Figures that by subsampling the original data series, the associated cdf indeed tends closer towards the standard Gauss normal cdf, in spite of the reduction in the number of available data points. This is indicative of the expected reduction of the data correlation for increasing m . Further detailed study has revealed that the deviations from the theoretical cdf are almost entirely due to the large values of $\rho(\tau)$ near $\tau = 0$: by truncating the first few values of the acf, a nearly perfect match is obtained between both curves in all cases. The advantage of analyzing the distribution of the acf, rather than only the acf itself, is that it enables confidence and tolerance limits for the correlation and correlation length to be established, together with the use of tests for goodness-of-fit, and that it does not depend on the ambiguity of the measure of correlation being adopted.

4.2 Principal component decomposition

As mentioned before, the definition of the correlation length depends on the tolerated level of residual correlation in the stirrer data series beyond the correlation length. Consequently,

the degree of correlation of a measured sample of values for received power or field strength is somewhat arbitrary. In order to reduce the data correlation, the measured data can be preprocessed so as to yield a set of M asymptotically uncorrelated, i.e. linearly independent values using a Karhunen–Loève (K–L) transformation, also known as a principal component or eigenvalue decomposition, as follows.

The correlation matrix R associated with the M standardized tuner sweep data points $x_0, x_1, \dots, x_M$, is a real symmetric circulant $M \times M$ -matrix. For a general circulant matrix $\underline{\underline{C}}$, characterized by:

$$\underline{\underline{C}} = \begin{bmatrix} C_0 & C_1 & \dots & C_{M-1} \\ C_{M-1} & C_0 & \dots & C_{M-2} \\ \vdots & & & \vdots \\ C_1 & C_2 & \dots & C_0 \end{bmatrix}, \quad (6)$$

the M eigenvalues λ_m :

$$\lambda_m = \sum_{k=0}^{M-1} C_k \exp(jk\omega_m) \quad (7)$$

are complex and have associated orthonormal eigenvectors ('principal component vectors') $\underline{v}_m = [1; \exp(-j\omega_m); \exp(-j2\omega_m); \dots; \exp(-j(M-1)\omega_m)]/\sqrt{M}$, with $\omega_m = 2\pi m/M$ and $m = 0, 1, \dots, M-1$. For a real symmetric circulant $\underline{\underline{C}}$, an equivalent set of real eigenvectors can be constructed. Be $\underline{\underline{V}} = [v_{mn}]$ the transformation matrix constructed from the principal component vectors, then the transformed stirrer data $\underline{y}$:

$$\underline{y} = \underline{\underline{V}} \cdot \underline{x} = M^{-1/2} \begin{bmatrix} \sum_{m=0}^{M-1} \exp(-j2\pi m\omega_0/M) x_m \\ \sum_{m=0}^{M-1} \exp(-j2\pi m\omega_1/M) x_m \\ \vdots \\ \sum_{m=0}^{M-1} \exp(-j2\pi m\omega_{M-1}/M) x_m \end{bmatrix} \quad (8)$$

are approximately uncorrelated. Moreover, these eigenvalues become asymptotically *totally* uncorrelated for large M , i.e. $\underline{\underline{R}}$ approaches a diagonal matrix as M increases. Thus, the original stirrer data can be replaced by an equivalent set of weighting coefficients for a linear combination of M complex time-harmonic and orthonormal basis eigenfunctions that approximate the original data series. This provides a link to the spectral analysis of the data, to be discussed in

detail in Sec 12.

A clear advantage of the K-L transformation is that it always enables a set of asymptotically uncorrelated data to be constructed a posteriori, provided M (but not necessarily N) is taken to be sufficiently large, for example if the stirrer size is so small or if the operating frequency so low that the correlation between data points of the original set is high. Also, as a result of the reduced correlation, the application of order statistics for sampled χ_n^2 and χ_n pdfs defined from sums of N independent samples can be better justified when using K-L transformed data. The disadvantage of this orthogonal transformation is that the basis functions, i.e. the transformed stirrer positions, no longer carry an obvious physical interpretation and that the data are now complex numbers.

Figs 4.7–4.9 show the acf for the stirrer data, before and after K-L transformation. The somewhat unconventional double logarithmic format has been chosen to emphasize the salient features of the profound effect of the transformation at both ends of the scale. For small offsets the autocorrelation is much smaller than for the original data: with the $|\rho| < \exp(-1)$ criterion, a K-L decimation factor $m_{KL} = 1$ is now obtained for all three cases. As a bonus, for large offsets too the $|\text{acf}|$ is much smaller and steadily decreasing, in the mean, for the transformed data in contrast to the $|\text{acf}|$ for the original data.

An alternative and more general method to obtain the transformation matrix for the orthogonal transformation is to find the Cholesky factorization of the correlation matrix, viz. $\underline{R} = \underline{S} \cdot \underline{S}^T$, where $\underline{S}$ is a lower triangular $M \times M$ -matrix. The K-L transformation matrix then follows as $\underline{S}^{-1}$. However, the sample size obtained from a reverberation chamber measurement is usually quite large ($M \geq 100$), making a Cholesky factorization or even a singular value decomposition computationally intensive. In this case, one should take recourse to recursive techniques, such as the stochastic approximation [23].

5 Test of field isotropy and random polarization

5.1 Operation at 2.5GHz

At an operating frequency of 2.5GHz:

- visual inspection of the measured cdf compared to the theoretical χ_2^2 cdf shows reasonably 'close' agreement (Fig 5.1);
- the stirring ratio (SR) ('peak-to-valley' variation) is well above the minimum value required for proper reverberation [38]:

$$\text{SR} = 34.59\text{dB} > 20\text{dB} \quad (9)$$

- experimental max-to-mean ratio is within bounds of acceptable values [38, 77]:

$$5.0 < (max/mean = 5.38) < 6.3 \quad (\text{lin}), \quad (10)$$

$$i.e. \quad 7 < (max/mean = 7.32) < 8 \quad (\text{dB}) \quad (11)$$

- coefficient of variation $\sigma_n \triangleq \sigma/\mu$ is within acceptable range [63]:

$$0.9 < (\sigma_n = 0.928) < 1.1 \quad (12)$$

- goodness-of-fit test for Type I hypothesis (probability to reject correct hypothesis for cdf): using Kholmogorov–Smirnov test at the 10% significance level [60, 78]:

$$D_n \triangleq \frac{\max(|cdf_{exp} - cdf_{theor}|)}{1.22/\sqrt{N}} \stackrel{?}{<} 1 \quad (13)$$

yields $D_n = 0.044/0.109 < 1$, i.e. the hypothesis should not be rejected at this significance level. The Type II hypothesis, i.e. probability to accept an incorrect hypothesis, could not be tested;

- comparison of experimental max-to-mean ratio to theoretically predicted value (using extreme value theory for order statistics) [66] (Fig 5.2): the cdf of the max-to-mean ratio of a χ^2_2 pdf for $N = 125$ yields:

$$6.26 < (max/mean = 7.33) < 8.22 \quad (\text{dB}) \quad (1\sigma \text{ (67\% confidence level)}) \quad (14)$$

Hence the measured value (7.32dB) differs by only 0.01dB from the predicted statistical value (7.33dB). The 2σ (95%) confidence interval for $N = 125$ is [5.48dB, 9.29dB].

5.2 Operation at 8.2GHz

At an operating frequency of 8.2GHz:

- visual inspection of the measured cdf compared to the theoretical χ^2_2 cdf shows 'close' qualitative agreement (Fig 5.3);
- the stirring ratio is well above the minimum value required for proper reverberation:

$$SR = 42.75\text{dB} > 20\text{dB} \quad (15)$$

- experimental max-to-mean ratio is within bounds of acceptable values:

$$5.0 < (max/mean = 5.74) < 6.3 \quad (\text{lin}), \quad (16)$$

$$i.e. \quad 7 < (max/mean = 7.60) < 8 \quad (\text{dB}) \quad (17)$$

- coefficient of variation σ_n is within acceptable range:

$$0.9 < (\sigma_n = 0.970) < 1.1 \quad (18)$$

- goodness-of-fit test for Type I hypothesis (probability to reject correct hypothesis for cdf): using Kholmogorov–Smirnov test:

$$D_n = \frac{\max(|cdf_{exp} - cdf_{theor}|)}{1.22/\sqrt{N}} < 1 \quad (19)$$

yields $D_n = 0.019/0.077 < 1$, i.e. the hypothesis should not be rejected. The Type II hypothesis, i.e. probability to accept incorrect hypothesis, could not be tested;

- comparison of experimental max-to-mean to theoretically predicted value (using extreme value theory for order statistics) (Fig 5.4): the cdf of the max-to-mean ratio of a χ^2_2 pdf for $N = 250$ yields:

$$6.78 < (max/mean = 7.85) < 8.30 \text{ (dB)} \quad (1\sigma \text{ confidence level}) \quad (20)$$

The measured value (7.60dB) is thus well within one standard deviation of the predicted value. The 2σ confidence level for $N = 250$ corresponds to the interval [6.25dB, 9.64dB].

Field anisotropy coefficient As a direct verification of the field isotropy at this frequency, we measured the power associated with two⁶ perpendicular field components, explicitly, at a single location inside the cavity. Using an open-ended waveguide, i.e. removing the Rx horn antenna and waveguide bend, the received power P_x associated with the field component E_x perpendicular to the equatorial plane was measured ($\underline{1}_\theta \triangleq \underline{1}_x$). After rotation of the waveguide by 90deg with respect to the waveguide axis, the power P_y associated with the component E_y parallel to the equatorial plane was measured at the same location ($\underline{1}_\phi \triangleq \underline{1}_y$). From the data sets for P_x and P_y , the average crosscorrelation coefficient $\langle \rho_{xy} \rangle$ for a total of $N = 600$ samples is obtained as -0.103 ± 0.086 at the 95% confidence level, i.e. very small (Fig 5.5). This shows that both directions may be considered as linearly independent.

The coefficient of planar field anisotropy $\langle A_{xy} \rangle$, defined as [70]:

$$\langle A_{xy} \rangle \triangleq \left\langle \frac{|E_x|^2 - |E_y|^2}{|E_x|^2 + |E_y|^2} \right\rangle = \left\langle \frac{(P_x/P_{in}) - (P_y/P_{in})}{(P_x/P_{in}) + (P_y/P_{in})} \right\rangle, \quad (21)$$

is a self-normalized dimensionless measure, making the expression of the level of field anisotropy independent of the level of the density of the local total power. For an arbitrary stirrer position,

⁶Measurement of the third component was impractical in the present set-up, but will be investigated in future.

the stirrer. If three mutually perpendicular field components are measured, then the total (3D) field anisotropy $\langle A \rangle$:

$$\langle A \rangle = \langle \sqrt{(A_{xy}^2 + A_{yz}^2 + A_{zx}^2)/3} \rangle \quad (22)$$

can be similarly defined and computed.

It can be shown [70] that A_{xy} exhibits a uniform pdf $f_{A_{xy}}(a_{xy}) = 1/2$, ranging between -1 and $+1$, if perfect field isotropy holds i.e. if $|E_x|$ and $|E_y|$ exhibit identical Rayleigh pdfs χ_2 . This corresponds to a cdf $F_{A_{xy}}(a_{xy}) = (1 + a_{xy})/2$, i.e. a straight line with unit slope. Fig 5.6 shows the theoretical and measured $F_{A_{xy}}$, demonstrating close agreement with perfect field isotropy conditions, at least as measured in the plane oxy at this location. The dependence of the measured $\langle A_{xy} \rangle$ on the number of stirrer positions N is also depicted in Fig 5.5. It shows the reverberation process at work: as the sample size N increases, $|\langle A_{xy} \rangle|$ decreases in the mean, i.e. the statistical planar field isotropy increases. The stirring process can then be terminated as soon as a predetermined mean level for $|\langle A_{xy} \rangle|$ is reached. For $N = 600$, $|\langle A_{xy} \rangle| = 0.0046 \pm 0.048$, corresponding to $[(-23.42) \pm (-13.16)]\text{dB}$, at the 95% confidence level. In a perfect reverberation chamber, the values of $|\langle \rho_{xy} \rangle|$ and $|\langle A_{xy} \rangle|$ tend to zero as $N \rightarrow \infty$. These values, together with their uncertainties⁷, are expected to decrease still further when the stirring would be extended to cover one complete revolution of the wall.

Fig 5.7 shows the residual Δ_x , defined as the difference between the measured cdf and the theoretical χ_2^2 cdf for P_x/P_{in} for various N . The residual Δ_y is not shown, but was found to be very similar to Δ_x . In general, larger residuals are found for lower values of the field strength. The cdf mismatch, $|\Delta_x|$, is taken as a single average value of the modulus of the residuals over all stirrer positions, and then averaged over all possible bootstrap replicas (wall sweep subsamples) that have equidistant steps for the stirrer for given N . Hence for every N each data point in the Figure is in fact the average of $3(600/N)$ different realizations of the reverberation environment, each realization with its associated bootstrap estimation in addition to this. All measured cdfs seem to show 'reasonable' visual agreement with the theoretical cdf, even for N as low as 30, as evidenced by $\langle |\Delta_x| \rangle$ and $\langle |\Delta_y| \rangle$ all having values between 0.009 and 0.029, i.e. varying by much less than one order of magnitude. However, when relating this mismatch to the directly measured field anisotropy, stark differences occur: it is seen from the Figure that an average⁸ cdf curve mismatch of 1.2% gives a value of $|\langle A_{xy} \rangle|$ that is more than ten times smaller than an

⁷The estimates for ρ_{xy} and A_{xy} and their standard errors were obtained using nonparametric Efron bootstrap estimation on the measured data with 100 replicas per data point [28].

⁸Alternatively, the cdf for $(P_x^2 + P_y^2)/P_{in}$ could be compared against a single theoretical χ_4^2 pdf with associated mismatch. However, this mismatch would not enable one to analyze the difference in the stirring performance for the two directions.

average curve mismatch of (only) 3.7%.

It is thus concluded that visual inspection of the 'closeness' of matching between measured and theoretical cdf for the statistics of the measured power or insertion loss should be treated with great caution, even if the average curve mismatch is less than a few percent. A direct evaluation of the field isotropy is recommended where possible.

5.3 Operation at 18GHz

At an operating frequency of 18GHz:

- visual inspection of the measured cdf compared to the theoretical χ^2_2 cdf shows 'close' qualitative agreement (Fig 5.8);
- the stirring ratio is well above the minimum value required for proper reverberation:

$$SR = 29.39\text{dB} > 20\text{dB} \quad (23)$$

- experimental max-to-mean ratio is within bounds of acceptable values:

$$5.0 < (max/mean = 5.54) < 6.3 \quad (\text{lin}), \quad (24)$$

$$\text{i.e.} \quad 7 < (max/mean = 7.43) < 8 \quad (\text{dB}) \quad (25)$$

- coefficient of variation σ_n is within acceptable range:

$$0.9 < (\sigma_n = 0.908) < 1.1 \quad (26)$$

- goodness-of-fit test for Type I hypothesis (probability to reject correct hypothesis for cdf): using Kholmogorov-Smirnov test:

$$D_n = \frac{\max(|cdf_{exp} - cdf_{theor}|)}{1.22/\sqrt{N}} \stackrel{?}{<} 1 \quad (27)$$

yields $D_n = 0.031/0.061 < 1$, i.e. the hypothesis should not be rejected. The Type II hypothesis, i.e. probability to accept the incorrect hypothesis, could not be tested;

- comparison of experimental max-to-mean to theoretically predicted value (using extreme value theory for order statistics) (Fig 5.9): the cdf of the max-to-mean ratio of a χ^2_2 pdf for $N = 400$ yields:

$$7.09 < (max/mean = 8.18) < 9.25 \quad (\text{dB}) \quad (1\sigma \text{ confidence level}) \quad (28)$$

The measured value (7.43dB) is thus within 0.62dB of the predicted statistical value and within one standard deviation of the predicted value. The 2σ confidence level for $N = 400$ corresponds to the interval [6.71dB, 9.85dB].

5.4 Summary

Tbl 2 summarizes the measured performance on field isotropy at the selected frequencies. Since it has been shown in [71] that the degree of field isotropy and polarization randomness are totally correlated in ideal reverberation environments, the measured results listed above also immediately reflect the extent of randomness of polarization.

Generally, a better reverberation performance is expected as the frequency is increased. Hence the apparent inferior⁹ reverberation, as suggested by the 'quick checks' at 18GHz as compared to 2.5 and 8.2GHz and by the larger mismatch between theoretical and predicted cdfs, requires further investigation, by measuring $\langle A_{xy} \rangle$ at 18GHz. In any case, the reverberation remains acceptable at 18GHz and the insertion loss increases with frequency, as expected.

As a comparison, the reverberation chamber at the Naval Surface Warfare Center (NSWCDD), Dahlgren, VA (dimensions: $L = 10.82\text{m}$, $W = 5.18\text{m}$, $H = 3.96\text{m}$; $V = 221.95\text{m}^3$) has the following measured performance (at 1GHz):

- SR: 57dB
- max/mean = 5.92 (7.6dB)
- $\sigma_n = 0.92$

The reverberation chamber at NIST, Boulder, CO (dimensions: $L = 4.6\text{m}$, $W = 3.1\text{m}$, $H = 2.7\text{m}$; $V = 38.50\text{m}^3$) shows a stirring ratio of 27dB at 1GHz.

6 Test of field homogeneity

6.1 Stirred cavity

Field homogeneity is defined as the degree of statistical uniformity of the average field strength, per field component, among all locations inside the cavity. Hence, strictly speaking, the testing for homogeneity requires multiple monitoring positions and 3-axis noncombinatorial field probes for measurement of the individual field components, in amplitude and phase. The measurement positions should all be within the working volume, i.e. at least $\lambda/4$ away from the walls¹⁰: close to the wall, at least two field components cannot have the same isotropy as compared to the

⁹A possible explanation is the increased contribution of scarred eigenfunctions [34, 40] as the stadium parameter γ becomes significant with increased frequency. 'Bouncing ball' states could be influencing the eigenmodes and may thus affect the measured results, as these are performed close to the equator plane. Scarring is of increasing significance as the frequency increases.

¹⁰This rule-of-thumb is expected to be inapplicable at frequencies close to any one of the 'whispering gallery' eigenstates of the chamber, which are characterized by large field variations very close to the boundary [40].

field inside the cavity due to the EM boundary conditions.

For rapid evaluation of chamber field uniformity, it is useful to know a priori the mean distance between maxima, in order to decide on suitable measurement locations. If one considers a plane wave making angles ϕ and θ with a reference direction, then $u = 2|\cos\phi|/\lambda$ maxima are observed along this direction. For an ensemble of plane waves the mean number of maxima is then obtained as $\int_0^\pi d\theta \int_0^{2\pi} u d\phi = \lambda^{-1}$. Thus, the mean spacing between consecutive maxima of the power density is one wavelength.

In [38], inhomogeneity levels as high as $\pm 8\text{dB}$ were observed at low frequencies (where the mode density may be insufficient to warrant acceptable stirring) with, typically, $\pm 2\text{dB}$ at higher frequencies. The presently adopted criterion for homogeneity [69] is that all measured $\langle P_{reci} \rangle$ for all positions i should be not more than 4.5dB apart:

$$[\max(\langle P_{reci} \rangle) - \min(\langle P_{reci} \rangle)] \leq 4.5\text{dB} \quad (29)$$

The NIST characterization (pending IEC 61000-4-21 standard) uses 16 three-axis probe measurements at different locations inside the working volume and requires less than 6dB variation over 32 out of all 48 measured values.

In our study we made measurements at different locations parallel to the equatorial plane by moving around the Rx horn antenna. This horn is looking 45° upward and thus measures a combination of r - and θ - field components. The receiver is placed at 20 different positions $(r, \phi; \theta = \pi/2)$ with respect to the cavity center $(0, 0, 0)$. The transmitter is located at $(r = 300\text{mm}, \phi = 0, \theta = \pi/2)$ at all times and is crosspolarized with respect to the receiver. Results are based on 600-sample data sets per location. Tbl 3 shows the measured results at 8.2GHz , depicted in Fig 6.1. The average insertion loss $\langle P_{rec}/P_{in} \rangle$ is found to have a spatial variation within a 2.2dB (i.e. $\pm 1.1\text{dB}$) band. As a further check, field isotropy and random polarization at each of the measurement points was measured. The Table shows that points 142, 144, 145 and 150 exhibit somewhat larger differences between predicted and measured max-to-mean ratios.

In summary, at 8.2GHz the measured insertion loss is -21.22dB (95% confidence interval $[-21.67\text{dB}, -20.77\text{dB}]$). The measured max-to-mean ratio is 7.42 ± 0.31 and the theoretical max-to-mean ratio is 7.25 ± 0.05 , all at the 95% confidence level. The standard errors were obtained using Quenouille-Tukey jackknife estimation [18] on the measured data.

In addition, the spatial power crosscorrelation function (ccf), ρ_{12} , was computed for a set of collinear points (file ref nrs 133-139 in the Table) and compared against the theoretical ccf for ideal reverberation conditions, $\text{sinc}^2[k(r_2 - r_1)]$, i.e. diffuse noise (see e.g. [7, 10, 13, 14, 21, 40, 53, 82]). This function is the equivalent of the coherence function in the spectral domain.

Since the power analyzer has a nonzero bandwidth, i.e. $\Delta f = 150\text{MHz}$ for a vertical sensitivity in excess of $20\mu\text{W}$ [86], the excitation is strongly multimodal (see Appendix A and Fig A.13 for an estimate of the number of modes) and a correction should be applied to the correlation function (cfr. Appendix B) with $\Delta k/k_c = 0.15/8.2 = 0.018$. As seen from Fig 6.2, the values of the measured spatial correlation appear to be significantly larger than the theoretical values. In fact, the measured spatial dependence resembles more a $\text{sinc}(kr)$ function, qualitatively and quantitatively, indicating nonideal reverberation conditions.

Similar to the quantification of field anisotropy, a field inhomogeneity coefficient $\langle I_{\alpha i} \rangle$ can be defined for E_{α} ($\alpha = x, y, z$ or *total*) at two different locations $\underline{r}_1$ and $\underline{r}_2$ defining a direction $\underline{i}_i$ parallel to $\underline{r}_2 - \underline{r}_1$ [70]:

$$\langle I_{\alpha i}(\underline{r}_1, \underline{r}_2) \rangle \triangleq \left\langle \frac{|E_{\alpha}(\underline{r}_1)|^2 - |E_{\alpha}(\underline{r}_2)|^2}{|E_{\alpha}(\underline{r}_1)|^2 + |E_{\alpha}(\underline{r}_2)|^2} \right\rangle = \left\langle \frac{[P_{\alpha}(\underline{r}_1)/P_{in}] - [P_{\alpha}(\underline{r}_2)/P_{in}]}{[P_{\alpha}(\underline{r}_1)/P_{in}] + [P_{\alpha}(\underline{r}_2)/P_{in}]} \right\rangle, \quad (30)$$

with P_{α} defined as in Sec 5.2. The total field inhomogeneity I_i along direction $\underline{i}_i$ can then be defined, similar to the definition for A , as

$$\langle I_i(\underline{r}_1, \underline{r}_2) \rangle = \langle \sqrt{(I_{xi}^2 + I_{yi}^2 + I_{zi}^2)/3} \rangle \quad (31)$$

or as $\langle I_i(\underline{r}_1, \underline{r}_2) \rangle = \langle [|E_{tot}(\underline{r}_1)|^2 - |E_{tot}(\underline{r}_2)|^2] / [|E_{tot}(\underline{r}_1)|^2 + |E_{tot}(\underline{r}_2)|^2] \rangle$. Finally the 3D field inhomogeneity, defined as an orientational average over three orthogonal directions $\underline{i}_i$, $\underline{i}_j$ and $\underline{i}_k$, reads

$$\langle I \rangle = \langle \sqrt{(I_i^2 + I_j^2 + I_k^2)/3} \rangle \quad (32)$$

The total field inhomogeneity is of no further concern here, because our present set-up only allows for measurement of $I_{xr}(r_1, r_2)$ or $I_{yr}(r_1, r_2)$. We choose to analyze $I_{xr}(r_1, r_2)$ and denote it further as I_{12} for brevity. Fig 6.3 shows $\langle \rho_{12} \rangle$ and $|\langle I_{12} \rangle|$ for two points $S_1(r_1 = 60\text{mm}, \phi_1 = -90\text{deg}, \theta_1 = 90\text{deg})$ and $S_2(r_2 = 88\text{mm}, \phi_2 = -90\text{deg}, \theta_2 = 90\text{deg})$, i.e. the radial field inhomogeneity for E_x between two points separated approximately $3\lambda/4$. Again this demonstrates the mechanical stirring process at work: as the sample size N increases, $|\langle \rho_{12} \rangle|$ and $|\langle I_{12} \rangle|$ decrease. For $N = 600$, $|\langle \rho_{12} \rangle| = 0.029 \pm 0.054$, compared to the theoretical value $\text{sinc}^2[k(r_2 - r_1)] = 0.043$ for an ideal reverberation field [14, 53] and $\cos^2[k(r_2 - r_1)] = 0.00925$ for a nonreverberating (i.e. totally coherent) field. The measured field inhomogeneity is $|\langle I_{12} \rangle| = 0.0029 \pm 0.020$, corresponding to $[(-25.42) \pm (-16.93)]\text{dB}$, at the 95% confidence level. In other words, for 600 stirrer positions the average power densities at S_1 and S_2 differ by only 0.3%, relative to the

sum of the powers at these locations. The values for $|\langle \rho_{12} \rangle|$ and $|\langle I_{12} \rangle|$ decrease still further, in the mean, as the separation distance and/or N increases. By way of comparison, for $N = 10$, $|\langle \rho_{12} \rangle| = 0.65$ and $|\langle I_{12} \rangle| = -1.89\text{dB}$, demonstrating a significantly larger field inhomogeneity for smaller N . Thus, the wall stirring is indeed useful in improving the field homogeneity of the stadium at 8.2GHz.

6.2 Unstirred cavity

To separate the effect of mechanical wall stirring on field homogeneity from the effect of cavity detuning due to the wall diffractors, a second set of field homogeneity measurements was made, now in the static (i.e. unstirred) chamber. Along a radial direction in the equatorial plane, at $\phi = -90\text{deg}$ and for 8.2GHz, nineteen different locations, all in the equatorial plane along a radial direction, were selected for positioning the Rx antenna. For each location, one hundred power measurements at 10-second intervals were made to eliminate the effect of drift and transients¹¹. Disregarding drift and transients, the cavity field is now of course only asymptotically¹² ergodic. The time-averaged values for each one of the 19 positions form a complex radial standing wave pattern, as shown in Fig 6.4 using cubic spline interpolation. This radial variation is to be compared to that for $\langle P_{rec}/P_{in} \rangle$ for the stirred cavity, listed in Tbl 3 and also shown in Fig 6.4. It is seen that the average received power varies by more than 10dBW over this distance, compared to only about 1.6dBW if wall stirring is applied. Large field variations are particularly found at a radial distance between 100mm and 150mm. However, the 10dBW level of field inhomogeneity in the untuned cavity is still several orders of magnitude smaller than what would be observed in an integrable tuned cavity (smooth sphere) without wall corrugations (typically 40dBW [39]).

7 Test of correlation between field isotropy and field homogeneity

From the above results, it may be concluded that field isotropy and field homogeneity resulting from reverberation are two correlated, although different, aspects of a common phenomenon, viz. field uniformity in spatial location and spatial orientation. To quantify this interdependence, the correlation coefficient $\rho_{\langle A_{xy} \rangle, \langle I_{12} \rangle}$ can be computed as a function of N , because $\langle A_{xy} \rangle$ and $\langle I_{12} \rangle$ are themselves random variables. Fig 7.1 shows this correlation, based on the measured power levels at the abovementioned locations inside the cavity and again obtained using Efron bootstrap estimation. It follows that both reverberation parameters are indeed considerably correlated: for $N = 600$, $\rho_{\langle A_{xy} \rangle, \langle I_{12} \rangle} = -0.64 \pm 0.14$ at the 95% confidence level. This value for the correlation coefficient is of course only indicative as it depends somewhat on the chosen

¹¹These effects were found to contribute less than 2%.

¹²with respect to frequency

locations and distance, but remains largely independent of the value of N because of the rapid decay of both $\langle A_{xy} \rangle$ and $\langle I_{12} \rangle$ as functions of N . The graph of $\langle A_{xy} \rangle$ vs. $\langle I_{12} \rangle$ converges as an irregular spiral towards the origin as the parameter N increases (Fig 7.1).

8 Estimated maximum and average strength of test field at EUT location

The average or maximum power density is related to the measured average or maximum received power by the effective aperture of the antenna:

$$p_{rec} = P_{rec}/A_{eff} \quad (33)$$

In an ideal (i.e. perfectly isotropic and randomly polarized) reverberation chamber, $A_{eff} = \lambda^2/(8\pi)$.

8.1 Operation at 8.2GHz

The input power was set at 50mW nominally and 1,500 samples were taken of which 250 were linearly independent. From the measured average $\langle P_{rec} \rangle = 0.328\text{mW}$ and approximating $\langle P_{in} \rangle \approx P_{in}$, the predicted ratio $P_{rec_{max}}/\langle P_{rec} \rangle$ is $7.85\text{dB} \sim 6.095$. Estimates for the average and maximum strength for a *single* field component E , as defined by the polarization direction of the horn, follow as¹³ [69]:

$$|E_i|_{max} \approx 187 \times 10^{-9} f \sqrt{P_{rec_{max}}/\eta} = 70.3\text{V/m} \quad (34)$$

$$\langle |E_i| \rangle \approx \sqrt{\eta_o \langle p_i \rangle} / \eta = 166 \times 10^{-9} f \sqrt{\langle P_{rec} \rangle / \eta} = 25.3\text{V/m} \quad (35)$$

where $\eta = 0.95$ is the efficiency of the horn antennas and $\eta_o = 120\pi\Omega$ is the free space impedance as an approximation for the statistically averaged wave impedance. Hence $|E_i|_{max}/\langle |E_i| \rangle \approx 2.77$. The measured $P_{rec_{max}}$, 1.89mW, is within 5% of the predicted value, 1.99mW, hence the measured $|E_i|_{max}$ is expected to agree with the predicted $|E_i|_{max}$ to within 2.5%. Note that the predicted values are implicit functions of N . Even if the theoretical estimated $P_{rec_{max}}$ is used instead of the measured value, as it should be, then this predicted value will somewhat increase as N increases. Thus, in field strength measurements it is advised that estimates are calculated for the maximum number of independent stirrer positions. The average field strength is less prone to such variations and is hence expected to give more accurate estimates.

¹³These formulae assume Cartesian field components ($i = x, y, z$) and the local high-frequency assumptions (plane wave approximation) to hold.

8.2 Summary

Tbl 4 shows the estimated average and maximum field strength at the respective frequencies of operation.

9 Other stirring techniques

9.1 Mechanical source stirring

By moving the Tx for fixed positions of the Rx and the stirrer, an alternative mechanical mode stirring technique is obtained [52]. It has the potential of creating a larger number of independent boundary conditions and, hence, of increasing the reverberation performance, possibly also at lower frequencies with low modal densities where wall stirring becomes ineffective.

The Tx was moved inside the chamber in the equatorial plane, in steps of 1mm from $r = 20\text{mm}$ to $r = 115\text{mm}$, i.e. the extremal positions for the waveguide in our set-up. (A longer waveguide would have allowed for more source locations.) Figs 9.1–9.2 show that the stirring data contain not much structure and that the associated cdf does not track the sigmoid curve very closely, although still within reason. The autocorrelation data (Fig 9.3) show that these source positions allowed for only about $N = 96/5 = 19$ independent samples. This is insufficient to guarantee the validity of the assumption of Gauss normality of the statistical field and, hence, the validity of χ^2_2 statistics for the power distribution. As a further confirmation, the theoretical and measured *max/mean* for $N = 19$ are 5.52dB and 5.68dB, respectively¹⁴, which are well below the recommended 7...8dB level. The measured stirring ratio is 21.35dB, but the measured σ_n is only 0.84, again indicating severe understirring [63, 70].

It is concluded that at 8.2GHz, source stirring as realized by moving the Tx along one single direction, does not yield a sufficiently large sample to generate proper reverberation in the present set-up. However, the results demonstrate that some stirring does occur and that measured values agree reasonable well with predicted theoretical ones. It may be expected that the reverberation quality can be further improved if the Tx is placed at other positions inside the cavity as well, using the other coupling plates and waveguide extensions, thus increasing N .

9.2 Mixed mechanical stirring

Combining several stirring techniques not only enhances the chamber performance at low modal densities but also helps in reducing the uncertainty levels for operation at high modal densities, by allowing for a larger number of statistically independent samples. The combination of wall

¹⁴It must be noted however that such a relatively small value for N does not guarantee the asymptotics of the theory of extreme values of order statistics to be sufficiently accurate.

and source stirring is expected to generate statistical fields of quasi unlimited high quality, provided sufficient flexibility exists in moving the source around inside the cavity.

As an example, 250 independent wall positions for each one of 4 independent source positions were used to generate a set of mixed mechanical stirring data of size $N_{tot} = 1,000$. The stirrer data in Fig 9.4 show a very high degree of structure. The overall set was verified to remain uncorrelated in total, to a high degree [$|\rho(\tau \neq 0)| < 0.3$] (Fig 9.5). The theoretical χ^2_2 cdf was very closely reproduced, as shown in Fig 9.6. A stirring ratio of 31.2dB, $\sigma_n = 1.008$ and $max/mean = 8.72$ dB were obtained. A comparison of cdfs for pure wall stirring vs. combinatorial stirring for $N_{wall} = 50$ is shown in Fig. 9.7.

Further measurements suggest that, for a fixed total number of stirrer positions, $N_{tot} = N_{source} \times N_{wall}$, combinatorial stirring gives superior reverberation quality as compared to pure wall stirring:

- for pure wall stirring ($N_{source} = 1$, $N_{wall} = 100$): $max/mean = 6.31$ dB;
- for mixed wall/source stirring ($N_{source} = 2$, $N_{wall} = 50$): $max/mean = 6.68$ dB;
- for mixed wall/source stirring ($N_{source} = 4$, $N_{wall} = 25$): $max/mean = 7.59$ dB,

whereas all cases have an identical theoretical value of 7.15dB for $N_{tot} = 100$. The latter case is even superior to simple wall stirring with $M = N = 250$ (measured $max/mean = 7.06$ dB). These results suggest that the value of N_{tot} does not unambiguously determine the stirring quality in combinatorial stirring, but that there may be an optimum ratio N_{wall}/N_{source} . This may come as a surprise, because all independent sets of boundary conditions are deemed equivalent, regardless of whether their independence is achieved via source stirring or via wall stirring. Hence the performance should not depend on N_{wall}/N_{source} for arbitrary N_{tot} . However, truly independent stirrer positions are usually not obtained in practice. From the above measured results it may thus be concluded that source stirring yields greater statistical independence than wall stirring, even after sample decimation (decorrelation), and that residual correlation may play a significant role.

9.3 Electronic source stirring: frequency stirring

Frequency stirring and phase stirring are two forms of electronic stirring which can be used as an alternative to mechanical stirring (e.g. [5, 48, 56]). This is made possible by the fact that, in a plane-wave spectral representation of the cavity field, the plane-wave fields which impinge onto and are reflected from the wall cavity at an angle θ show a $kz \cos(\theta + \varphi)$ -dependence, where φ is a uniformly distributed phase angle [44]. Hence stirring through averaging (i.e. integration) over angles θ (mechanical stirring) can be equally accomplished by averaging over the wave number

k (electronic stirring). In electronic stirring, the cavity and source remain mechanically static at all times but the frequency or phase is changed in steps ('electronic mode tuning') or swept continuously ('electronic mode stirring').

Figs 9.8–9.10 show data collected between 8.18GHz and 8.22GHz in steps of $\delta f = 0.5\text{MHz}$ (80 samples). Some settling time (about 5s) had to be observed in between frequency changes before registering the input and received power. From the measured frequency response only about $N = 27$ sample points could be retained as being statistically linearly independent (Fig 9.9). The statistical field thus generated is rather poorly stirred within the chosen bandwidth (stirring ratio = 19.7dB; $\sigma_n = 0.93$; $\text{max/mean} = 5.57\text{dB}$) (see Fig 9.11), confirming findings in [64]. The relative maxima observed in Fig 9.8 are not to be considered as resonance peaks of individual cavity modes, but rather as a result of in-phase superpositions for the multitude of modes excited within the operational bandwidth.

As a further test on the stirring performance, the average frequency spacing $\langle \Delta f \rangle$ between consecutive relative maxima for a frequency sweep taken in a properly stirred room can be calculated using the following Rice formula [3, Eqn (3.6–6)], applied to the spectral domain:

$$\langle \Delta f \rangle = \sqrt{\frac{\int_0^\infty t^2 \exp(-t/\langle T \rangle) dt}{\int_0^\infty t^4 \exp(-t/\langle T \rangle) dt}} = (2\sqrt{3}\langle T \rangle)^{-1} = \frac{\pi \langle f/Q(f) \rangle}{\sqrt{3}} \quad (36)$$

where the brackets signify averaging over all modes excited within the signal bandwidth of the source and Tx antenna. The exponential functions in the integranda represent the average power density, as follows from the autoregressive model to be derived in Sec 12.2 and application of the Parseval identity. One may approximate $\langle f/Q(f) \rangle \approx f_c/Q_c$, i.e. defining a composite quality factor Q_c as an alternative to calculating in detail the Q_{mnp} factors for all participating modes across the frequency band. Then the composite quality factor can be computed from a simple counting of the measured maxima within a given bandwidth (see Appendix A). Alternatively, one may measure directly the group decay time $\langle T \rangle = \langle Q/\omega \rangle$ from a measurement of the reverberation time (see Sec 10.2). Using the latter with $\langle T \rangle = 154.5\text{ns}$, we obtain $\langle \Delta f \rangle = (2\sqrt{3}\langle T \rangle)^{-1} \approx 1.9\text{MHz}$, corresponding to an estimated $40/1.9 \sim 21$ relative maxima in the 8.18–8.22GHz band. This is considerably larger than the observed number and mean spacing of these maxima (Fig 9.8), indicating understirring.

With the present sample, the limit for the maximum size of the frequency step is almost being reached: it follows that the frequency step δf for frequency sampling should be [69]:

- for chamber characterization: $\delta f \ll 2\text{MHz}$ (recommended $\delta f = 2\text{MHz}/500 \approx 5\text{kHz}$; minimum $\delta f = 100\text{kHz}$);
- for testing: $\delta f \geq 2 \dots 8\text{MHz}$, but not larger than 10MHz. Note also that in this case the

chosen δf must be within the bandwidth of the EUT, which is dependent on the pass/fail specification.

Although more independent samples could have been obtained by choosing δf closer to 2MHz, the frequency span for sampling would also become larger (i.e. close to 160MHz). This renders the meaning to the notion of 'testing at a given frequency' to be ambiguous, which should then be replaced by 'testing across a given bandwidth'. This dilemma of bandwidth vs. sample size is typical for frequency stirring.

It is concluded that narrow-band pure frequency stirring is of limited value for our chamber at 8.2GHz. Its benefit lies probably more in combining it with mechanical or mixed mechanical stirring, to yield hybrid mechanical/electronic stirring, i.e. stirring in 2D or 3D parameter space yielding $(N_{source} \times) N_{wall} \times N_{freq}$ independent samples.

As a corollary, we mention that (36) was derived in [3] for a time-dependent signal and can thus equally be used for analysis of statistics of spacings $\Delta\tau$ of maxima in the above sweep data for *mechanical* stirring. Thus, an estimate can be obtained for the composite quality factor directly from the density of such maxima in the sweep data, i.e. by simply counting the number of peaks.

10 Quality factor and reverberation time

10.1 Steady-state method

Spherical cavities have an intrinsically higher quality factor Q than comparable cylindrical or rectangular cavities [85]. This is mainly due to a sphere being the shape that yields the minimum enclosing surface area for a given cavity volume. Also, a spherical cavity does not possess sharp corners or edges, unlike rectangular or cubic cavities. Therefore, the surface area S is better utilized by a sphere, resulting in a more homogeneous surface current density and absorption and, hence, a higher Q . The disadvantage of a spatially more symmetric cavity is that the mode distribution is degenerate and hence spectrally less uniform as compared to e.g. a rectangular chamber with three unequal dimensions. However, for narrow-band or low-frequency stirring this may become an advantage for isolated parts of the spectrum.

From the sweep data, the theoretical quality factor for CW excitation is obtained as [55]:

$$Q_{t,tot} = [(Q_{t,wall})^{-1} + 2(Q_{t,antenna})^{-1}]^{-1} \quad (37)$$

with

$$Q_{t,wall} = 0.003 (V/S) \sqrt{f\sigma/\mu_r} \quad (38)$$

$$Q_{t,antenna} = 5.85 \times 10^{-24} f^3 V / \eta \quad (39)$$

The experimental steady-state quality factor for an ideal reverberation chamber has been obtained as [27]:

$$Q_e = (16\pi^2/\lambda^3) V \langle P_{rec}/P_{in} \rangle \quad (40)$$

A theoretical composite Q_c factor can be formulated (see Appendix A), Eqn (77). The values of the respective Q -factors are given in Tbl 5. Parenthesized values indicate approximate values, due to the unknown value of V/S as a result of the wall diffractors and corrugations, here taken as $V/S = 0.125$. The various Q -factors are shown as a function of frequency in Fig 10.1. The calculated values assumed a smooth noncorrugated cavity, thus overestimating the actual value of V/S . The obtained values of the correction factor $K \triangleq Q_t/Q_e$ are well outside the commonly accepted range [63]. This discrepancy may indicate the presence of additional loss mechanisms and uncertainties (leakage through apertures in aluminium shield or coupling plates, instrumentation, ...) and needs further investigation. Secondly, in the calculation of Q_{wall} a smooth noncorrugated sphere was assumed, having a somewhat higher V/S -ratio than the actual cavity. Therefore the actual K is certainly smaller than the estimated values. The Q -bandwidth BW_Q , calculated on the basis of the measured Q , appears not to be changing appreciably from 8.2 to 18GHz.

The rather large discrepancy between theoretical and experimental Q -values, in this study and as well in findings by others (e.g. [63]), deserves further attention. It is possibly due to the nonuniform distribution of the total excitation energy over the excited modes, or due to the nonideal (i.e. locally anisotropic, polarized and/or inhomogeneous) field distribution in the chamber. More importantly, however, it may be caused by a nonuniform distribution of the absorption properties of the walls. For example, corners, joints, general discontinuities, surface roughness, ... may easily distort the distribution of the induced surface currents of the walls, when compared to those for a perfect cavity, giving rise to an inhomogeneous absorption coefficient $\alpha(\underline{r})$.

10.2 Reverberation time method

10.2.1 Introduction

An alternative, time-domain technique for measuring Q is based on the measurement of the mean decay or reverberation time for a carrier modulated pulsed input signal [36]. This technique is the

EM analogue of a classical method used for assessing sound absorption in acoustic reverberation chambers (see e.g. [77]). The modulated pulse signal is applied with the pulse length chosen to be sufficiently long to ensure that steady-state conditions for the cavity field are being reached. When the signal is subsequently switched off at time $t = t_0$, the received signal will extend beyond t_0 because of reverberation ('ringing'), but will eventually fade totally because of damping due to cavity wall losses. Assuming that all excited modes decay exponentially and at more or less the same rate, an effective (group) decay constant T can be defined from the measured received signal in the time domain. The received signal is a distorted pulse, partially because of differences in the decay constants for the individual participating modes and the differences in the associated path lengths l , and partially because of the 'skirts' of the excitation band which weigh the individual modes differently. In order to obtain an ensemble average, the reverberation curves for different stirrer positions are superimposed, after self-normalization, which is justified provided the excitation chamber remains linear for each realization. An average exponential decay time (reverberation time) $\langle T \rangle$ can be determined for the ensemble averaged stored energy to drop to a fraction $\exp(-1)$ of its initial value at t_0 . The average decay time is given by:

$$\langle T \rangle = \omega Q_d = \left\langle -\frac{\bar{l}}{c \ln(1 - \bar{\alpha})} \right\rangle, \quad (41)$$

with $c = 2.9979 \times 10^8 \text{ m s}^{-1}$. For each stirrer position, the mode averaged¹⁵ energy absorption coefficient $\bar{\alpha}$ is:

$$\bar{\alpha} = \int_0^{\pi/2} \alpha(\theta) \sin(2\theta) d\theta \quad (42)$$

where θ is the angle of incidence, as well as the decay composite quality factor Q_d . For each stirrer position, the mean free path length $\bar{l}$ is the average distance between two reflections in a 'travelling wave' ('bouncing ball') model for the cavity field, in the GO limit. For an ergodic system, $\langle \bar{l} \rangle = 4V/S \approx 0.51$. The longer this path, the fewer reflections and the smaller the time-averaged absorption for cavities with finite Q_d . The value of $\langle T \rangle$ is dominated by the contribution of the 'axial' eigenmodes, i.e. the modes mnp whose spatial pattern is parallel or almost parallel to the vertical axis of the stadium cavity, for which l_{mnp} and, hence, Q_{mnp} are largest and the ringing longest. In practice, the decay is not smooth due to interference between participating eigenmodes with different decay constants. This interference is a result of the small spectral spacing. It is worth noting that the convex diffractors on the inner surface are known to reduce the differences between the decay times, compared to a smooth 3D cavity, as they reduce

¹⁵Note that the average chamber absorption differs from the absorption at normal incidence as a result of the random field incidence due to reverberation. The average here is taken over all participating modes, at a single location and for an arbitrary stirrer position and represented by an overbar.

the decay rate of strongly damped modes and increase the rate for weaker damped ones [1]. Strictly, the applicability of an average exponential curve fit should be verified by an analysis of the time dependence of the reverberation for each individual participating eigenmode.

For EM measurements, it appears most practical to measure the decay time T_{20} , i.e. the elapsed time for the received power to drop¹⁶ by 20dB, from which to estimate T as:

$$T = \frac{T_{20}}{2\ln 10} \approx \frac{T_{20}}{4.605} \quad (43)$$

for each realization, or $\langle T \rangle$ for the ensemble average. Eqns (41)–(43) form the EM analogue of the Sabine–Fokker–Norris–Eyring equation for the acoustic reverberation time T_{60} , viz.[77]:

$$T_{60} = (6\ln 10)T = \frac{0.163V}{S\ln[(1 - \bar{\alpha})^{-1}]} \quad (44)$$

Further refinements to this formula are required and have been developed in the literature for the case where $\bar{\alpha}$ is not small.

An important assumption in the derivation and applicability of the Sabine–Fokker–Norris–Eyring equation is the randomness and homogeneity of the cavity field. Hence measurement of reverberation times for various pulses of different energy and at different locations provides a simple way to assess the degree of chaos in the cavity, after comparing the reverberation curves with an exponential fit. This method is much easier to implement experimentally than the measurement of spectral level spacing statistics and GOE characteristics for the cavity spectrum mentioned as in Sec 3, which requires a great many number of sufficiently sharp resonances to be identifiable. Both methods are rooted in the assumption that the cavity is mixing and that the absorption is weak.

10.2.2 Mode tuning

Using this technique, the reverberation curves for 35 virtually uncorrelated stirrer positions ($\Delta s = 10\text{mm}$) were measured and sampled using 50ns steps at 8.2GHz, at a fixed location inside the chamber. The input power was adjusted in between stirrer movements to maintain a constant level of +24dBm. The different realizations of the reverberation time process in Fig 10.2 are seen to yield markedly different reverberation curves. As a result, the standard deviation for the average mean normalized early decay is substantial (Fig 10.3). The effect is

¹⁶This 20dB drop is relatively small and is enforced by the large differences in the height of the received pulse which occur during stirring, for a constant input power. Stirrer positions for which the pulse height was smaller than -10dBm were discarded. In contrast, for larger pulse heights reverberation decays in excess of 30dB could be measured.

aggravated by considerable jitter, caused by the triggering on the complex-shaped slope of the leading edge of the pulse. Manual realignment of the time base is necessary. This makes the characterization of the important early decay response difficult. Note also the periodic deviation of the reverberation from a straight line during the early and intermediate decay, when compared to a 1st order fit. The late decay shows the typical 'sagging' of the reverberation curve, confirmed by a 3rd order curve fit and caused by the unequal damping of the individual modes (Fig 10.4). At still later times the time dependency changes, as witnessed before in complex cavities (e.g. [1]). In casu, the received power shows a t^{-1} -dependence of the decay, which has been attributed to the 'bouncing ball' states for reverberation in a 2D stadium [54]. The ensemble averaging yields $\langle T_{20} \rangle \approx 0.71 \mu s$, hence the quality factor and mean absorption coefficient as obtained with the decay time method are:

$$Q_d = \omega \langle T_{20} \rangle / 4.605 \approx 7,950 \quad (45)$$

$$\langle \bar{\alpha} \rangle = 1 - |\langle \bar{\tau} \rangle|^2 \approx 0.011 \quad (46)$$

The value for Q_d is significantly smaller than the theoretical values Q_c and $Q_{t,tot}$. Such may be expected, because the theoretical values presume a uniform distribution of the energy across the cavity modes within the BW_Q . The value for $\langle Q_d \rangle$ is also considerably larger than the above steady-state value Q_e . The steady-state method forces all participating modes to coexist over the full measurement time frame. In contrast, in the decay method these various modes will be damped unevenly because of their different Q_{mnp} . Therefore, at $t = \langle T_{20} \rangle$ only those modes with the longest ringing times will have survived, whose Q is larger than the composite (effective) Q of all modes that existed at $t = 0$. Hence the bias towards large values for Q when compared to the steady-state method. In spite of this fundamental difference between steady-state and decay quality factors, more theoretical and experimental work is needed to find an ensemble averaged value for Q with smaller uncertainty.

Also the reproducibility of reverberation curves was verified, by stopping and restarting the pulse train three times, and a fourth time after rotating and repositioning the cavity wall to its original position by eye ball (Fig 10.5). It is seen that the realizations of the reverberation curve can be closely reproduced in all cases.

10.2.3 Mode stirring

The averaging of the pulse reverberation curves can be realized more rapidly using mode stirring (continuous wall rotation). The response was measured for five different speeds of the rotating lower wall, whose values could be varied continuously and which were all well in excess¹⁷ of

¹⁷The actual speed was not measured accurately, but e.g. the lowest speed # 2 was estimated by eye ball to be approximately 120rpm. All values of the speed increase with their index number.

100rpm. As above, a pulse train with pulse repetition frequency (prf) 10kHz, pulse width $15\mu\text{s}$ and duty cycle 15% was applied. A time averaging over 2,048 pulses was performed and the result was monitored and sampled at 50ns or 10ns intervals. The input pulse for each case was found to be identical for all speeds, within a small margin. Compared to the input pulse for the static chamber, the fact that the EM environment now varies continuously owing to the wall rotation appears to increase the fall time of the input pulse to about 100ns (Fig 10.6). For the detected pulse, the height as well as the average decay time appear to depend somewhat on the values of the rotation speed (e.g. compare speed # 2 vs. speed # 4), because the latter is not negligibly small compared to the prf. The height also varies between realizations themselves (e.g. compare take # 1 to take # 2 for speed # 2), probably as a result of differences in the cavity position at the time of sampling between both pulse trains. Therefore, the speed averaged reverberation curve was obtained through averaging of the six responses. The early decay shows a very gradual decrease, unlike the sharp drops encountered for mode tuning. This is puzzling, but an effect akin to reverberation curves for coupled chambers, in which the source-free compartment shows a similar type of gradual early decay. For the intermediate decay, the speed averaged reverberation drops from -16dBm to -30dBm in $0.52\mu\text{s}$, yielding $\langle T_{20} \rangle \approx 0.74\mu\text{s}$, which is in reasonable agreement with the above position averaged reverberation time ($0.71\mu\text{s}$), obtained in mode tuned operation but using a much larger number of measurements. This yields $Q_d = 8,280$ for mode stirring.

By way of comparison, from measurement of the ratio $\langle P_{rec}/P_{in} \rangle = 0.0183$, obtained through averaging over the six pulse trains, the estimated value of $Q_e = (16\pi^2/\lambda^3) V \langle P_{rec}/P_{in} \rangle$ for mode stirring is now obtained as 10,610. However, taking into account that P_{rec} varies between -12.7dBm and -15.9dBm for the different speeds, this causes Q_e to vary between 6,190 to 15,670 for these speeds. Hence the agreement with the value for Q_d may be considered as reasonable for mode stirring. A larger number of measurements at different speeds is probably required to reduce the uncertainty on Q .

11 EUT test procedure

The mode tuning analysis provides insight in the maximum number of independent boundary conditions N_{max} and, hence, power measurements that can be generated within one revolution of the stirrer for a given frequency. Consequently, when the cavity is being used in mode stirred operation, the sampling rate is upper limited. The maximum rates are listed in Tbl 6, based on the minimum step sizes in Tbl 1 and assuming a rotational speed of v rps. The reciprocal of these sampling rates gives the maximum time available for one frequency sweep in the case of combined frequency/mechanical stirring, but should be kept as short as possible to

minimize time averaging affects (cfr. Sec 12.2). It is furthermore important to realize that the listed maximum sampling rates presume an instantaneous response of the EUT, requiring no dwell time per sample. If the EUT responds more slowly than the travel time per sample the sampling rate should be further reduced accordingly.

Mode stirring operation appears attractive because of the potential time savings obtained when compared to mode tuning (stepping), especially when using mechanical stirring. However, in an ergodic chamber the uncertainties on the average field strength and power density are larger for mode stirred operation as compared to mode tuned operation. Such follows from the duality between space and ensemble averages when applied to the analysis and results in [14] for space averaging, at least if all statistical dependencies are linear, i.e. expressible through the correlation function. If the statistical field for mode stirred operation is sampled at well-defined intervals, viz. at stirrer positions for which the resulting acf is zero, it then follows that the variance on the average field strength or power density is minimal and expressible as a hyperbolic function of the wavenumber [14].

12 Spectral analysis

12.1 Introduction

Complementary to the analysis of the stirrer data as a time series, the reverberation performance of an enclosure can also be characterized through spectral analysis of the tuner sweep data, i.e. in the Fourier domain. This is particularly attractive because stirring constitutes a random process, which results in noise-like characteristics for the measured power, in the time domain. Since our interest is mainly in mode tuned operation, we shall replace the time domain by the *stir domain* or *angular domain*. Clearly, it is the azimuthal angular position τ of the stirrer (in units rad), with associated total span and resolution $\Delta\tau$, which is of primary importance rather than the actual time¹⁸ t spent at or in between stirrer positions. The corresponding Fourier domain is then the spectral angular domain, characterized by the dimensionless spectral angular variable ω .

In our series of measurements, one unit $\Delta\tau$ for the wall position corresponds to the rotation angle between two consecutive stirrer positions, i.e. 6.926, 2.597 and 0.866 mrad at 2.5, 8.2 and 18 GHz, respectively. This angular resolution governs the bandwidth in the spectral domain: $[0, \omega_{max}] = [0, 2\pi/\Delta\tau]$. The maximum frequency for the spectral window can also be expressed as $\omega_{max} = 2\omega_N$, where $\omega_N = 453.6, 1209$ and 3628 are the Nyquist cut-off frequencies for 2.5, 8.2 and 18 GHz, respectively. The Nyquist frequency puts limitations on the occurrence of aliasing and can alternatively be expressed as $f_N = \omega_N/(2\pi) = (2\Delta\tau)^{-1} = 72.19, 192.5, 577.5 \text{ rad}^{-1}$,

¹⁸provided the dwell time is sufficiently long for the chamber and EUT to reach steady-state conditions in between stirrer rotations

respectively. Because of the spectral symmetry and periodicity, resulting from the periodic continuation of the angular data, only one half of the full spectral window needs to be considered (one-sided spectrum).

Mechanical reverberation is essentially a process of signal modulation: the 'carrier', i.e. the CW excitation wave as observed at an arbitrary location inside the cavity, becomes modulated in amplitude and polarization by the position of the stirrer, either in discrete time (for mode tuning) or in continuous time (for mode stirring). The use of CW excitation yields a considerable simplification for the spectral representation, as only the variation of field phasors, rather than field vectors, with angular position of the stirrer must be considered. For pulsed operation, a more elaborate full 4D space-time stochastic process must be considered. Such is also the case if the frequency dependence of the reverberation is to be analyzed.

12.2 Spectral density

The starting point for the spectral analysis of the stirrer data is the computation of the complete acf $\rho(\tau)$ or, in non-normalized form, the covariance function $B(\tau) = \sigma^2 \rho(\tau)$. These are shown for all three measurement frequencies in Figs 4.4–4.6 and 12.1–12.3, respectively. The deduced spectral densities (Figs 12.1–12.3; see below for definitions) as derived from the measured acf show rapid variations. Since the spectral densities $G^R(\omega)$ and $R(\omega)$ are derived from $G(\omega)$, it is recommended to re-derive them for a fitted model, i.e. for an estimate for $G(\omega)$ instead, as will be discussed below.

The Fourier transformation of $\rho(\tau)$ yields the normalized two-sided standard power spectral density function (psdf), $s(\omega)$, by definition. Figs 12.4–12.6 show the characteristics of three normalized one-sided sdfs for the measured data, viz. the normalized 1D psdf $g(\omega)$:

$$g(\omega) = \frac{2}{\pi} \int_0^\infty \rho(\tau) \cos(\omega\tau) d\tau, \quad (47)$$

the normalized radial spectrum $g^R(\omega)$:

$$g^R(\omega) = -\omega \frac{dg(\omega)}{d\omega}, \quad (48)$$

as measured along the direction R defined by the aperture of the Rx antenna, and the associated normalized radial spectral density function $r(\omega)$:

$$r(\omega) = \frac{\pi}{2} \omega^2 g^R(\omega) \quad (49)$$

Eqns (48)–(49) apply to fields in ideal reverberation conditions, i.e. a 3D homogeneous isotropic random field. Hence, they are translation invariant and are solely a function of the separation distance in both real and spatial Fourier space.

The psdf $g(\omega)$ indicates how the variance, i.e. the average power associated with the normalized field, is distributed over spatial frequency. As is well known, the sample psdf is not a consistent estimator for the statistical psdf, i.e. the former does not converge to the latter when increasing the sample size but remains noise-like. To avoid this, one can use a fitted sample psdf as an approximation for the statistical psdf, or use local or moving averaging or other filtering techniques to smoothen the sample psdf. In our case, however, no a posteriori autoregression or moving average (ARMA) filtering was applied to these spectra, as this would falsely increase the perceived level of field isotropy and homogeneity¹⁹, i.e. increase the apparent degree of randomness and chaos inside the chamber [82]. This is an important factor in assessing reverberation performance, particularly in mode stirred operation where measurement of stirring is performed in continuous time across a sampling window of nonzero width T , thus causing space *and* time-averaging *within* each sampled value ($\sigma_T \neq \sigma$). The effect of averaging can be quantified by computing the variance function γ_T [82], defined as

$$\gamma_T = \sigma_T^2 / \sigma^2 \quad (50)$$

where σ and σ_T are the standard deviation of the measured data for 'point' measurements and after local averaging over an interval $[\tau - T/2, \tau + T/2]$ for each measurement 'point' center τ , respectively. Figs 12.7–12.9 illustrate that, as the width T of the MA filter increases, σ_T decreases as is also the case when the reverberation performance of a chamber is being improved. If $T \rightarrow \infty$ then $\gamma_T \rightarrow 0$, which is a general characteristic for a process that is wide-sense ergodic. It appears to be difficult to fit γ_T to a single m th order exponential stochastic process, because the curvature of γ_T gradually changes as T increases. It was found that a composite exponential process of at least 6th order was needed to obtain a reasonable fit across the interval $[0, T/3]$. Hence local curve fitting across a limited interval of T -values of interest may be more advisable. In any case, values for T in excess of about $T_{max}/3$ should be disregarded because of the limited

¹⁹A similar increase results from the local spatial averaging due to the nonzero antenna aperture. This physically inherent averaging process is inevitable and fundamental, in that it places a Heisenberg uncertainty on the accuracy of field isotropy and homogeneity levels. A spatial variance function γ_S could be determined, for example, from the standard deviation of stirring data obtained with various lengths S of a Rx dipole antenna and extrapolating the results to obtain $\sigma_S \rightarrow \sigma_0$ as $S \rightarrow 0$. For sufficiently high frequencies of operation, γ_S and γ_T should be equivalent, as a consequence of chamber ergodicity. It is therefore recommended to always clearly state the type and (aperture) dimensions of the Rx being used, in order to correctly interpret the measured reverberation performance.

number of averaging intervals then available. Also shown in Figs 12.7–12.9 is the dependence of the scale of fluctuation θ_T , defined as [82]:

$$\theta_T = \lim_{T \rightarrow \infty} (T \gamma_T) \quad (51)$$

which describes the randomization of the statistical field through stirring. Owing to ergodicity, θ_T is also a measure for the rate of improvement of the field isotropy and homogeneity as N increases. Because of the finite sample size, it is difficult to find the actual limit in (51).

Returning to Figs 12.4–12.6, these Figures also show that $g(\omega)$ is an on average decreasing function as required, in general, for a 3D stochastic homogeneous and isotropic field [82]. Stated differently, the distribution of the normalized variance of the stirring data over spatial (Fourier) frequency gradually decreases with increasing frequency. A flattening of $g(\omega)$ towards the higher end of the spectra for 8.2 and 18GHz is noticed. This may possibly be due to the noise floor being reached. Note also that information on $\rho(\tau \rightarrow 0)$ and $\rho(\tau \rightarrow \infty)$ is lacking in the chosen mode of operation (mode tuning), hence $g(\omega \rightarrow \infty)$ and $g(\omega \rightarrow 0)$ are fundamentally ill defined, respectively. However, the high-frequency flattening of $g(\omega)$ indicates that aliasing due to the nonzero step width of the stirrer positions is effectively removed, i.e. that the stirrer step size has probably been chosen sufficiently small at 8.2 and 18GHz so that loss of high-frequency spectral information is avoided. Broader peaks appear near $\omega = 25, 35$ and 165 in Fig 12.4, which may be reflecting some irregular cyclic behavior with these periodicities in the stirrer data for 2.5GHz. Also shown are 3rd order exponential fits, i.e. estimates for the measured power spectra at the three frequencies of operation. Such estimates are in general unbiased, but not consistent with respect to an increase in sample size. Consistency is obtained by weighing the stirrer data using a suitable spectral window. Here we chose to precondition the angular data by applying a Bartlett–Priestley window W , viz.:

$$W(\tau) = \frac{3N}{(\pi\tau)^2} \left[\text{sinc}(\pi\tau/\sqrt{N}) - \cos(\pi\tau/\sqrt{N}) \right] \quad (52)$$

in order to obtain the estimated psdf, which then yields a consistent spectral estimate as $N \rightarrow \infty$. Nevertheless, it was found that a direct exponential curve fit, i.e. without preconditioning of the stirrer data, gave virtually the same result for the three samples question. The Figures show that $g(\omega)$ can in fact be represented, to good approximation, by an autoregressive 1st order exponential correlation model, i.e.

$$g(\omega) = g_0 \exp(\alpha\omega) \quad (53)$$

Values of the parameters g_0 and α as estimated from the measured data are given in Tbl 7. Coefficients for the more elaborate 3rd order exponential model, viz. $g(\omega) = \exp(d_0\omega^3 + d_1\omega^2 + d_2\omega + d_3)$, are given in the insets to Figs 12.4–12.6. The 3rd order fits for $g^R(\omega)$ and $r(\omega)$ are based on the fit for $g(\omega)$ and show higher order Gamma pdfs, i.e. in normalized form $\Omega \exp(-\Omega)$ and $\Omega^3 \exp(-\Omega)$ for the standardized Fourier frequency Ω , respectively.

12.3 Cumulative spectral density

Similar to the relationship between pdfs and cdfs, the use of the integrated spectrum or cumulative spectral density function (csdf) is an alternative for the sdf which, moreover, avoids data binning effects. In addition, the csdf is also useful in the study of correlation. A conventional analysis of data correlation is based on the acf, i.e. in the angular ('time') domain. This method enables one to detect local correlation, i.e. in the vicinity of the reference point for the correlation coefficients. A complementary analysis in the spectral domain, based on the csdf, can be more effective in detecting long-range correlation, i.e. the occurrence of periodicity patterns in the data [79]. The frequency at which a jump occurs in the measured csdf with reference to the linear csdf of white noise then indicates a spatial periodic component with that frequency.

For example, a mechanical mode stirrer may be overrunning one complete revolution and produce a noninteger²⁰ number N' of stirrer steps per revolution. In such a scenario, the data points beyond the first revolution are no longer resamplings of the data points in the first period, and may therefore be hard to detect through visual inspection, i.e. in search for a repeated pattern in the stirrer data. A correlation analysis in the spectral domain would show a peak or jump at a Fourier frequency corresponding to $(N'\Delta\tau)^{-1}$ and its multiples.

Figs 12.10–12.12 show the integrated spectra for the FFT based on the raw measured data, i.e. without curve fitting. As could be expected, these csdfs deviate significantly from the csdf for white noise (pure random process), the latter being a linear function of ω .

12.4 Spectral moments and spectral bandwidth

The spectral moments λ_k associated with the psdf are defined as:

$$\lambda_k = \int_0^\infty \omega^k g(\omega) d\omega \quad (54)$$

In the context of EM fields in reverberation chambers, λ_1 represents the covariance function between the in-phase component of the electric field and the quadrature component of the magnetic field constituting the normalized total complex field $E = E_{||} - jE_{\perp}, H = H_{||} - jH_{\perp}$,

²⁰The argumentation holds a fortiori for integer numbers.

because $\lambda_1 = E[E_{||}, dE_{\perp}/d\tau]$. Odd higher-order moments can be given a similar interpretation involving the higher-order derivatives of the electric and magnetic field components. The spectral moments of even order $2m$ represent the variance of the m th order derivate process $E^{(m)}$, i.e. $\lambda_{2m} = \sigma_{E^{(m)}}^2$ for $m = 0, 1, 2, \dots$. As an example, the spectral moments of (53) can be computed as

$$\lambda_k = \frac{g_0 k!}{\alpha^{k+1}} \quad (55)$$

In Figs 12.13–12.15, the values of λ_k for the measured psdfs are shown as a function of increasing sample size, because of their central importance to the calculation of statistics of spectral bandwidths, extrema and level crossings.

From the λ_k , the normalized spectral bandwidth δ can be obtained as:

$$\delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}} \quad (56)$$

Its upper bound $\delta_{max} = 1$ represents maximum randomness of the field, hence δ is a measure for stirring performance. A lower bound

$$\delta_{min} = \sqrt{2/3} \approx 0.816 \quad (57)$$

applies to a 3D homogeneous isotropic field [82].

A second normalized spectral bandwidth ϵ can be defined on the basis of the higher-order spectral moments, viz. [8]:

$$\epsilon = \sqrt{1 - \frac{\lambda_2^2}{\lambda_0 \lambda_4}} \quad (58)$$

Compared to δ , however, this parameter is more prone to numerical inaccuracy because of the abovementioned reduced accuracy of $g(\omega)$ and, hence, of λ_4 for large ω . The same lower bound as for δ , i.e.

$$\epsilon_{min} = \sqrt{2/3} \approx 0.816 \quad (59)$$

applies to a 3D homogeneous isotropic field [82]. For the 1st order exponential model (53), the theoretical normalized bandwidths for the measured data are $\delta = \sqrt{1/2} \approx 0.707$ and $\epsilon =$

$\sqrt{5/6} \approx 0.913$, respectively. These values are to be compared to those for a sinusoidal standing wave with random phase angle ($\delta = 0$), white or pink noise²¹ ($\delta = 0.5$), a 1st order Markov process ($\delta = 0.771$) and wide-band fractional $1/f$ -noise ($\delta \approx \sqrt{1 - 1/\ln(\sqrt{\omega_{max}/\omega_{min}})}$).

Values for δ and ϵ as a function of the number of stirrer positions N are shown in Figs 12.16–12.18 and listed, for the total sample, in Tbl 7. The values for the 1st order exponential process are being approached as $N \rightarrow \infty$. More importantly, all observed values of δ remain below the lower bound $\delta_{min} = \sqrt{2/3} \approx 0.816$. The less accurate ϵ , however, exceeds $\epsilon_{min} \equiv \delta_{min}$ for $N \geq 50$ in all cases.

Fig 12.19 shows the dependence of δ and ϵ for the measured data as a function of radial distance in the equatorial plane at 8.2GHz. The results suggest that, within this range of measurement positions, a somewhat inferior stirring performance exists, on average, towards the center of the cavity as compared to off-center locations.

12.5 Cross spectra for perpendicular orientations

The psdfs were also calculated for two perpendicular directions of the waveguide aperture, i.e. g_x and g_y associated with $|E_x|^2$ and $|E_y|^2$, as defined in Sec 5.2, at 8.2GHz for $N = 600$. The results are shown in Fig 12.20–12.23. The match between the cdf of the standardized crosscorrelation function (ccf) and the one for two pure white noise processes (Fig 12.20) is better than for the cdf of the acf, because of the absence of large values of ρ_{xy} near $\tau = 0$, unlike ρ_x and ρ_y . Thus, the two directions may be considered as almost perfectly uncorrelated, locally. Fig 12.21 indicates that the 3rd order exponential fits for both perpendicular directions are virtually identical. This is supported by the very similar values obtained for the spectral bandwidths ($\delta_x = 0.671$ vs. $\delta_y = 0.663$; $\epsilon_x = 0.889$ vs. $\epsilon_y = 0.892$). In addition, the cross psdf g_{xy} was obtained from $\rho_{xy}(\tau)$. The Figure suggests that, although g_{xy} is in general a complex function, the 3rd order exponential fit is a real function and similar to g_x or g_y . The phase of g_{xy} appears as being quasi-random, which is confirmed by comparing its cdf to the corresponding linear cdf for white noise (Fig 12.22). Hence 'long'-range correlation and periodicity are negligible. The integrated cross psdf $\Gamma_{xy}(\omega)$:

$$\Gamma_{xy}(\omega) = \int_0^\omega g_{xy}(\omega') d\omega' \quad (60)$$

can be used to measure the correlation between the sweep data for $|E_x|$ and $|E_y|$ (Fig 12.23). At 8.2GHz, this yields $\Gamma_{xy}(\omega_N) = -0.376 + j0.057$.

²¹Note that, because of this small value for δ , white or pink noise is unable to achieve perfect reverberation in 3D or even in 2D, unless it is applied to a chamber with a relatively low Q (overdamped harmonic oscillator) or used in combination with other field randomization techniques. As a possible alternative realization, a 3D homogeneous isotropic random field with Markov correlation properties, for example, must be of the order 1.5 at least, in which case $\delta = 0.830$.

12.6 Statistics of level crossing frequency and excursion length

The knowledge of the spectral moments also enables one to calculate several parameters that are of interest in the practice of reverberation chamber testing, such as maximum field strength, frequency of crossing a chosen field strength threshold, time spent in excess of a chosen field strength threshold, ... The values for these parameters are usually theoretically predicted by assuming an underlying normal Gaussian pdf for the process, such as e.g. for white noise. Since our interest is in expressing these parameters in terms of an *observable* mean normalized total field amplitude having a Rayleigh pdf, we have derived them in Appendix C for this case.

The upward threshold crossing rate ν indicates how often the mean normalized field strength jumps upwards across a chosen level b in the sweep data. Knowledge of this rate is useful in estimating *how many times* the test conditions for 'high' field strength are being created during the EMC test. Clearly, if either $b \rightarrow 0$ or $b \rightarrow \infty$, this rate tends to zero. Figs 12.24–12.26 show that this rate can be fairly accurately predicted by the results of Appendix C, especially when it is remembered that the theoretical statistics are only asymptotically exact, viz. they hold for arbitrarily high thresholds of field strength, and that this corresponds to the situation of greatest practical importance. At the same time, this correspondence serves as an indirect confirmation that the underlying distribution of the field strength is indeed close to that of an ideal random 3D homogeneous isotropic field, viz. a Rayleigh pdf.

A second parameter of importance is the overall threshold exceeding rate ν_{max} , which unlike ν disregards whether the threshold is being crossed upward during the latest stirrer step $\Delta\tau$ or before. Clearly, $\nu_{max} \rightarrow \nu$ as $b \rightarrow \infty$. Knowledge of ν_{max} is useful in estimating the *overall test time* during which the EUT is being exposed to a mean normalized field strength in excess of level b . Again good agreement between the measured (i.e. counted) and predicted asymptotic rates is observed from Figs 12.24–12.26.

Thirdly, the mean excursion length T above a given threshold is calculated as being the average width (in units τ) of 'islands' of 'high' field strength above the threshold. This parameter is of interest for EUTs which have a comparatively large response time, e.g. thermal processes, in which case the mean duration (in units τ) of *uninterrupted high exposure* may become important, in conjunction with the rate ν_{max} . Clearly, $T \rightarrow M$ as $b \rightarrow 0$. Again good agreement between the asymptotic values for predicted and measured excursion lengths is observed. For very 'high' field strengths, where islands have typical sizes of only a very few units τ , the agreement is somewhat less good because of the discretization error caused by the integer number of steps. Finally, the spatial equivalent of T is the 3D excursion 'length', D , which is the mean volume (in units τ^3) of locations on a τ lattice in which *all* lattice points are above level b . This parameter is important if the size of the EUT is not negligible with respect to the chamber volume, in which case the fact of whether the threshold has been exceeded or not may become a function

of the measurement position inside the EUT. It can only be estimated in our present set-up and this prediction is also shown in the Figures.

13 Summary

Measurement results, statistical data analysis and overall chamber characteristics have been presented for the NPL untuned 3D Bunimovich stadium reverberation chamber. The chamber was tested in mode tuned and mode stirred operation at 2.5, 8.2 and 18GHz, using wall stirring, source stirring, combinatorial mechanical stirring and frequency stirring techniques, and shows the following features:

1. The Matérn lower bound for the acf of 3D homogeneous isotropic fields was found to be satisfied at all three frequencies.
2. The Karhunen-Loève transformation for the stirrer data was found to maximally reduce the decimation factor of the original data, i.e. yielding $m = 1$.
3. The stirring ratio, maximum-to-mean ratio, variation coefficient and goodness-of-fit tests were found to yield results within the generally accepted limits for proper reverberation at all three frequencies.
4. At 8.2GHz the level of planar field anisotropy, as measured in a plane perpendicular to the equator plane for a sample of size 600, is less than 0.0046 ± 0.048 , i.e. $[(-23.4) \pm (-13.2)]\text{dB}$, at the 95% confidence level.
5. For the same sample size and frequency, the level of field inhomogeneity, as measured in a plane parallel to the equator, was found to be less than $\pm 1.1\text{dBW}$ at the 95% confidence level. This applies to a measured statistical and spatial average insertion loss of -21.2dB . For two points separated by a distance of three quarters of a wavelength, the field inhomogeneity coefficient for a sample of size 600 is less than 0.0029 ± 0.020 , i.e. $[(-25.4) \pm (-16.9)]\text{dB}$, at the 95% confidence level. In contrast, the unstirred untuned chamber has a measured level of field inhomogeneity which is in excess of 10dBW at 8.2GHz, though smaller than the values usually encountered in unstirred tuned cavities that are not asymptotically ergodic.
6. For 600 samples, the measured correlation between planar field isotropy and field homogeneity is $(64 \pm 14)\%$ at the 95% confidence level.
7. Visual inspection of the cumulative distribution function and goodness-of-fit tests were shown to be only indicative and sometimes deceptive in reaching conclusions on the stirring

- performance. The field anisotropy and field inhomogeneity coefficients should be measured and used where possible to quantify reverberation quality more reliably.
8. From the measured data, the maximum and average field strength at 8.2GHz for 50mW nominal input power were estimated to be about 70V/m and 25V/m, respectively, for 1,500 wall stirrer positions (250 uncorrelated positions).
 9. The number of uncorrelated source positions proved to be insufficient to achieve adequate source stirring at 8.2GHz. However, this can be easily remedied by moving the source along more than one direction.
 10. Combinatorial mechanical wall/source stirring was found to yield superior reverberation, compared to simple mechanical stirring. The maximum-to-mean ratio for the statistical power density was found to depend on the ratio of the number of wall positions to the number of source locations, for a fixed product of both.
 11. Frequency stirring across a 0.5% (40MHz) band centered at 8.2GHz yields insufficient reverberation. The band would need to be increased at least fivefold to obtain sufficient stirring, making the testing of EUTs more ambiguous.
 12. Relatively large uncertainties for the measured quality factor of the chamber appear to exist, depending on the method of evaluation being used.
 13. First- and second-order spectral analysis of the measured random process show a normalized spectral bandwidth δ of 0.646, 0.700 and 0.659 at 2.5, 8.2 and 18GHz, respectively. The spectral bandwidth appears to be somewhat smaller in the center of the cavity in comparison with off-center radial positions. At all three frequencies, it was found that the stirring process can be modelled, to good approximation, as a 1st order autoregressive stochastic process with associated spectral bandwidths $\delta = 0.707$ and $\epsilon = 0.913$.
 14. The cross power spectral density for mutually orthogonal directions in a highly reverberating field is a real quantity and can also be modelled as a 1st order autoregressive stochastic process.
 15. Statistics of threshold crossings and maxima have been derived for measurable (observable) quantities obtainable in EMC reverberation tests. Good agreement was found between theoretically derived and measured mean threshold upward crossing rates, mean threshold exceeding rates and mean 1D excursion lengths.
 16. Expressions have been given for the mode count and mode density for general cavities and spherical cavities in particular, as well as for the composite quality factor of spherical and stadium chambers.

17. A transcendental equation for the lowest usable frequency has been derived, with the number of modes within the excited band, chamber loading factor and chamber size as parameters. Limiting expressions for this frequency have been obtained for very large and very small cavities.
18. First-order corrections for the spatial correlation function have been derived for the case of nonzero excitation bandwidth and nonzero spatial extent of transmitting and receiving antennas.
19. Expressions for the predicted max-to-mean ratio as a function of sample size have been compared with measured values. No single expression appears to yield very accurate results. It is likely that these methods, which all presume a perfect reverberation field to exist, show different degrees of deterioration in accuracy when the field is a good but not perfect reverberation field.

14 Conclusions

The NPL untuned stadium reverberation chamber provides a compact and efficiently stirred cavity for EMC testing at microwave and millimeter-wave frequencies. Various statistical tests applied to the measured data suggest that the chamber is reverberating well (i.e. producing a reasonably isotropic, homogeneous field with random polarization, in the statistical sense, combined with reasonable field homogeneity) at 2.5GHz, 8.2GHz and 18GHz, even for small subsets of the maximum number of independent samples available. This is made possible by the fact that the mode density in this spherical cavity is very large at these frequencies (see Appendix A). The level of field homogeneity is comparable to reported values for a cubic reverberation chamber [26], taking into account the larger volume and higher test frequency of the latter in [26]. It is to be remembered that the compliance of the data with a χ^2_2 pdf is a direct consequence of the field isotropy and randomness of polarization (i.e. a zero mean and a variance independent of field component) for a Gaussian field distribution. In this situation the statistical postprocessing of the measured data enables the *maximum* field at an *arbitrary* location within the working volume to be estimated based on the uncertainty and confidence level from a *mean* value measurement at the location of the receiving antenna.

15 Proposed set-up modifications and further work

1. The full capabilities of the wall stirring have not been totally exploited yet: because of time limitations, the wall stirring was terminated as soon as a reasonable number of independent samples (a few hundred) was obtained, typically covering between 40% and 60% of one

complete revolution. The objective so far was to verify the stirring capabilities, rather than the maximum stirring performance. Measurements will be extended across *one full cavity rotation* to assess the maximum performance for field isotropy and field homogeneity.

2. At present, horn antennas are being used for transmission and reception which are relatively large compared to the size of the cavity. Thus, the spatial averaging in the measurements is not negligible, as compared to ideal 'point' measurements. This may mask a somewhat poorer actual reverberation. The use of electrically smaller antennas (e.g. *short dipole antennas*) should be considered, particularly to assess field homogeneity.
3. Only one field component can be measured at a time with the present set-up. Field isotropy can only be measured in two orthogonal directions with an open-ended waveguide. Because of the the present set-up (fixed horns), an average of the radial and azimuthal field components is excited and measured at each different angular position. Measurements using a *3-axis probe* with 3 LP antennas without internal power combination should be performed to verify the field isotropy at a fixed point inside the chamber. In addition, measurements using an isotropic probe (i.e. 3-axis with internal field combination) may be useful for comparison.
4. The current trend in the investigation of reverberation chambers is in new reverberation techniques for *low frequencies*, i.e. where only a few cavity modes are excited. Initial calculations (see Appendix A) demonstrate that measurements down to 1GHz or below for our chamber would serve this purpose well, not in the least to establish a benchmark value for reverberation performance at higher frequencies. Moreover, *hybrid reverberation* (i.e. the combination of surface diffraction, cavity rotation and source stirring) was shown to yield an increased number of independent samples and boundary conditions needed. It should be investigated to what extent hybrid reverberation can help in enhancing low-frequency stirring quality. In addition, a sphere – like a cube – yields a maximum mode degeneracy and minimizes the spread of the mode density. Therefore, spheroidal or even ellipsoidal inner shells should be considered if the low-frequency performance of the chamber needs to be improved.
5. Apart from mechanical and frequency stirring, *phase and band-limited white Gaussian noise stirring* should be investigated as alternative stirring techniques.
6. Estimates of the *Q-factor* appear to give largely different values, depending on the measurement and computation techniques used. More theoretical study of this important parameter is required to reduce estimation uncertainties. In this respect, a more accurate determination of the volume-to-surface ratio is also required for the corrugated stadium.

The leakage of the chamber could be tested for emission/immunity by using a nested-chamber approach (i.e. by emission/susceptibility testing of the present cavity inside a larger reverberation chamber) or using a hand-held radiation detector.

7. *Correlation* of the test results with measurements in *FARs, SARs and OATSs* is to be made to establish further confidence in the use of the reverberation chamber as an EM test environment.

References

- [1] Morse, P M and Bolt, R H: Sound waves in rooms; *Rev Mod Phys*, vol 16 no 2 (1944), pp 69–150.
- [2] Rice, S O: Mathematical analysis of random noise; *Bell Syst Techn J*, vol 23 (1944), pp 282–332.
- [3] Rice, S O: Mathematical analysis of random noise; *Bell Syst Techn J*, vol 24 (1945), pp 46–156.
- [4] Bolt, R H: Normal frequency spacing statistics; *J Acoust Soc Am*, vol 19 no 1 (1947), pp 79–90.
- [5] Bolt, R H and Roop, R W: Frequency response fluctuations in rooms; *J Acoust Soc Am*, vol 22 no 2 (1950), pp 280–289.
- [6] Schröder, M: Die statistischen Parameter der Frequenzkurven von grossen Räumen; *Acustica*, vol 4 (1954), pp 594–600.
- [7] Cook, R K, Waterhouse, R V, Berendt, R D, Edelman, S and Thompson (Jr), M C: Measurements of correlation coefficients in reverberant sound fields; *J Acoust Soc Am*, vol 27 (1955), pp 1072–1077.
- [8] Cartwright, D E and Longuet-Higgins, M S: The statistical distribution of the maxima of a random function; *Proc Roy Soc London A*, vol 237 (1956), pp 212–232.
- [9] Matérn, B: Spatial variation: stochastic models and their application to some problems in forest surveys and other sampling investigations; *Comm Swedish Forestry Res Inst*, vol 49 no 5 (1960), pp 1–144.
- [10] Cron, B J and Sherman, C H: Spatial correlation functions for various noise models; *J Acoust Soc Am*, vol 34 no 11 (1962), pp 1732–1736.
- [11] Dyson, F J and Mehta, M L: Statistical theory of energy levels in complex systems — IV.; *J Math Phys*, vol 4 no 5 (1963), pp 701–719.
- [12] Waterhouse, R V: Statistical properties of reverberant sound fields; *J Acoust Soc Am*, vol 43 no 6 (1967), pp 1436–1444.
- [13] Morrow, C T: Point-to-point correlation of sound pressures in reverberant chambers; *Shock and Vibration Bull*, No 39 (1969).
- [14] Waterhouse, R and Lubman, D: Discrete versus continuous space averaging in a reverberant sound field; *J Acoust Soc Am*, vol 48 no 1 (1970), pp 1–5.
- [15] Lukosz, W: Electromagnetic zero-point energy shift induced by conducting surfaces — II. The infinite wedge and the rectangular cavity; *Z Phys*, vol 262 (1973), pp 327–348.
- [16] Shnirelman, A I: Ergodicheskiye svoystva eigenfunktsiy; *Usp Math Nauk SSSR*, vol 29 (1974), pp 181–182.
- [17] Bunimovich, L A: Ob ergodicheskikh svoystvakh nekotorykh bilyardov; *Funkt Anal Prilozh*, vol 8 no 3 (1974), pp 73–74.

- [18] Miller, R G: The jackknife — a review; *Biometrika*, vol 61 (1974), pp 1–17.
- [19] Corona, P and Latmiral, G: Valutazione ed impiego normativo della camera riverberante dell'Istituto Universitario Navale, *1a Riunione Nazionale di Elettromagnetismo Applicato* (24–25 Jun 1976, Aquila, Italy), pp 103–107.
- [20] Corona, P, Latmiral, G, Paolini, E and Piccioli, L: Use of a reverberating enclosure for measurements of radiated power in the microwave range; *IEEE Trans EMC*, vol 18 no 2 (1976), pp 54–59.
- [21] Berry, M V: Regular and irregular semiclassical wavefunctions; *J Phys A*, vol 10 no 12 (1977), pp 2083–2091.
- [22] Balian, R and Duplantier, B: Electromagnetic waves near perfect conductors — I. Multiple scattering expansions. Distribution of modes; *Ann Phys (NY)*, vol 104 (1977), pp 300–335.
- [23] Ljung, L: Analysis of recursive stochastic algorithms; *IEEE Trans Autom Control*, vol 22 (1977), pp 551–575.
- [24] Blake, W K and Waterhouse, R V: The use of cross-spectral density measurements in partially reverberant sound fields; *J Sound and Vibration*, vol 54 no 4 (1977), p 589.
- [25] Bean, J L and Hall, R A: Electromagnetic susceptibility measurements using a mode-stirred chamber; *Proc IEEE EMC-S* (Jun 1978, Atlanta, GA), pp 143–149.
- [26] Corona, P, De Bonitatibus, A and Ferrara, G: Electromagnetic enclosures behaviour. Comparison data analysis — II. In: Dvořák, T (ed): *Electromagnetic Compatibility 1979*, 3rd Symposium and Technical Exhibition on Electromagnetic Compatibility (1–3 May 1979, Rotterdam, The Netherlands), pp 234–236.
- [27] Corona, P, Latmiral, G and Paolini, E: Performance and analysis of a reverberating enclosure with variable geometry, *IEEE Trans EMC*, vol 22 no 1 (1980), pp 2–5.
- [28] Efron, B: Bootstrap methods: another look at the jackknife; *Ann Statist*, vol 7 (1979), pp 1–26.
- [29] MIL-STD-1344A, Method 3008: *Shielding effectiveness of multicontact connectors* (10 Sep 1980).
- [30] Corona, P, De Bonitatibus, A, Ferrara, G and Paolini, E: The limits for anisotropy of the mixture of field-modes to be satisfied in the reverberation chambers with variable geometry used for power measurements; *Proc 5th Internat Wroclaw Symp on EMC* (17–19 Sep 1980, Wroclaw, Poland), pp 829–837.
- [31] Kremer, F and Genzel, L: Application of untuned cavities for millimeter wave spectroscopy; *Proc 6th Int Conf Infrared Millim Waves* (Dec 1981, Miami, FL), paper T-1-4.
- [32] Liu, B H, Chang, D C and Ma, M T: Eigenmodes and the composite quality factor of a reverberating chamber; *NBS Technical Note 1066*, US Government Printing Office, Washington, DC (1983).
- [33] Bohigas, O, Giannoni, M J and Schmit, C: Characterization of chaotic quantum spectra and the universality of the level fluctuation laws; *Phys Rev Lett*, vol 52 no 1 (1984), pp 1–4.

- [34] Heller, E J: Bound-state eigenfunctions of classically chaotic Hamiltonian systems: scars of periodic orbits; *Phys Rev Lett*, vol 53 (1984), pp 1515-1518.
- [35] Berry, M V and Robnik, M: Semiclassical level spacings when regular and chaotic orbits coexist; *J Phys A*, vol 17 (1984), pp 2413-2421.
- [36] Richardson, R E (Jr): Mode-stirred chamber calibration factor, relaxation time, and scaling laws; *IEEE Trans Instr Meas*, vol 34 no 4 (1985), pp 573-580.
- [37] Berry, M V: Semiclassical theory of spectral rigidity; *Proc Roy Soc London A*, vol 400 (1985), pp 229-251.
- [38] Crawford, M L and Koepke, G H: Design, evaluation, and use of a reverberation chamber for performing electromagnetic susceptibility/vulnerability measurements; *NBS Technical Note 1092*, US Government Printing Office, Washington, DC (1986).
- [39] Ma, M T: Understanding reverberation chambers as an alternative facility for EMC testing; *J Electromagnetic Waves and Applications*, vol 2 no 3/4 (1988), pp 339-351.
- [40] McDonald, S W and Kaufman, A N: Wave chaos in the stadium: statistical properties of short-wave solutions of the Helmholtz equation; *Phys Rev A*, vol 38 no 8 (1988), pp 3067-3086.
- [41] Price, R H, Davis, H T and Wenaas, E P: Determination of the statistical distribution of electromagnetic field amplitudes in complex cavities; *JAYCOR Report*, 88-JAL129 (Jun 1988); *ibid*, *Phys Rev E*, vol 48 no 6 (1993), pp 4716-4729.
- [42] Clarke, R N: The design and use of stirred mode cavities for the calibration of PFD monitors; *NPL-DES Review Report* (1988).
- [43] Wu, D I and Chang, D C: The effect of an electrically large stirrer in a mode-stirred chamber; *IEEE Trans EMC*, vol 31 no 2 (1989), pp 164-169.
- [44] Dunn, J M: Local, high-frequency analysis of the fields in a mode-stirred chamber; *IEEE Trans EMC*, vol 32 no 1 (1990), pp 53-58.
- [45] Sridhar, S: Experimental observation of scarred eigenfunctions of chaotic microwave cavities; *Phys Rev Lett*, vol 67 no 7 (1991), pp 785-788.
- [46] Tang, H and Izatt, J R: Untuned cavity resonator for use with pulsed infrared and far-infrared lasers; *Infrared Phys*, vol 32 (1991), pp 155-167.
- [47] Kostas, J G and Boverie, B: Statistical model for a mode-stirred chamber; *IEEE Trans EMC*, vol 33 no 4 (1991), pp 366-370.
- [48] Loughry, T A: Frequency stirring: an alternative approach to mechanical mode-stirring for the conduct of electromagnetic susceptibility testing; *USAF Phillips Laboratory Report*, PL-TR-91-1036 (1991).

- [49] Stein, J and Stöckman, H-J: Experimental determination of billiard wave functions; *Phys Rev Lett*, vol 68 no 19 (1992), pp 2867-2870.
- [50] Sridhar, S and Heller, E J: Physical and numerical experiments on the wave mechanics of classically chaotic systems; *Phys Rev A*, vol 46 no 4 (1992), pp R1728-R1730.
- [51] Huang, Y and Edwards, D J; An investigation of the electromagnetic field inside a moving-wall mode-stirred chamber; *Proc IEE Conf EMC* (Sep 1992, Edinburgh, UK), pp 115-119.
- [52] Huang, Y and Edwards, D J; A novel reverberation chamber: the source-stirred chamber; *Proc IEE Conf EMC* (Sep 1992, Edinburgh, UK), pp 120-124.
- [53] Lehman, T H: A statistical theory of electromagnetic fields in complex cavities; *USAF Phillips Laboratory Interaction Notes*, Note 494 (1993), Ch 3.
- [54] Mortessagne, F, Legrand, O and Sornette, D: Role of the absorption distribution and generalization of exponential reverberation law in chaotic rooms; *J Acoust Soc Am*, vol 94 no 1 (1993), pp 154-161.
- [55] Hill, D A, Ma, M T, Ondrejka, A R, Riddle, B F, Crawford, M L and Johnk, R T: Aperture excitation of electrically large, lossy cavities; *IEEE Trans EMC*, vol 36 no 3 (1994), pp 169-178.
- [56] Hill, D A: Electronic mode stirring for reverberation chambers, *IEEE Trans EMC*, vol 36 no 4 (1994), pp 294-299.
- [57] Aurich, R, Bolte, J and Steiner, F: Universal signatures of quantum chaos; *Phys Rev Lett*, vol 73 no 10 (1994), pp 1356-1359.
- [58] Marvin, A C, Angus, J A S, Dawson, J F and Clegg, J; Enhancements to stirred mode chambers by the use of pseudo-random phase reflection gratings; *Proc EMC'94* (13-16 Sep 1994, Rome, Italy), pp 218-221.
- [59] Deus, S, Koch, P M and Sirko, L: Statistical properties of the eigenfrequency distribution of three-dimensional microwave cavities; *Phys Rev E*, vol 52 no 1 (1995), pp 1146-1155.
- [60] Corona, P, Ferrara, G and Migliaccio, M: Reverberation chambers as sources of stochastic electromagnetic fields; *IEEE Trans EMC*; vol 38 no 3 (1996), pp 348-356.
- [61] Bunimovich, L, Casati, G and Guarneri, I: Chaotic focusing billiards in higher dimensions; *Phys Rev Lett*, vol 77 no 14 (1996), pp 2941-2944.
- [62] Alt, H, Gräf, H-D, Hofferbert, R, Rangacharyulu, C, Rehfeld, H, Richter, A, Schardt, P and Wirzba, A: Chaotic dynamics in a three-dimensional superconducting microwave billiard, *Phys Rev E*, vol 54 no 3 (1996), pp 2303-2312.
- [63] Hatfield, M O, Freyer, G J and Slocum, M: Reverberation characteristics of a large welded steel shielded enclosure; *US Naval Surface Warfare Center Technical Report*, NSWCDD/TR-96/149 (Oct 1996). Ibid; *Proc IEEE EMC-S* (18-22 Aug 1997, Austin, TX), pp 38-43.

- [64] (Various:) 97 NIST Mode-stirred chamber, anechoic chamber, and OATS Users Meeting Notes (28 April – 2 May 1997, Vail, CO).
- [65] Petirsch, M and Schwab, A: Investigation on different methods to improve shielded rooms for EMI measurements; *97 NIST Mode-stirred chamber, anechoic chamber, and OATS Users Meeting Notes* (28 April – 2 May 1997, Vail, CO).
- [66] Lehman, T H and Freyer, G J: Characterization of the maximum test level in a reverberation chamber; *Proc IEEE EMC-S* (18–22 Aug 1997, Austin, TX), pp 44–47.
- [67] Mitra, A K and Trost, T F: Statistical simulations and measurements inside a microwave reverberation chamber; *Proc IEEE EMC-S* (18–22 Aug 1997, Austin, TX), pp 48–53.
- [68] Tanner, G: How chaotic is the stadium billiard? A semiclassical analysis; *J Phys A*, vol 30 no 8 (1997), pp 2863–2888.
- [69] Hatfield, M O, Freyer, G J and Slocum, M: Reverberation chamber theory/experiment; *Short course*; 15–19 Jun 1998, Naval Surface War Center, Dahlgren Division, VA (1998).
- [70] Arnaut, L R: Field anisotropy, field inhomogeneity and polarization bias in imperfect reverberation conditions; *NPL-CEM-DGSS Technical Report*, No. R980614 (1998).
- [71] Arnaut, L R: Relation between field isotropy and degree of polarization in reverberation chambers; *NPL-CEM-DGSS Technical Report*, No. R980806 (1998).
- [72] Corona, P, Ferrara, G and Migliaccio, M: A spectral approach for the determination of the reverberation chamber quality factor; *IEEE Trans EMC*, vol 40 no 2 (1998), pp 145–153.
- [73] Bäcker, A, Schubert, R and Stifter, P: Rate of quantum ergodicity in Euclidean billiards; *Phys Rev E*, vol 57 no 5 (1998), pp 5425–5447.
- [74] Li, B and Hu, B: Statistical analysis of scars in the stadium billiard; *J Phys A*, vol 32 (1998), pp 483–504.
- [75] Courant, R and Hilbert, D: *Methoden der mathematischen Physik I*; Springer, Berlin (1931), Ch VI.4.
- [76] Sabine, P E: *Acoustics and Architecture*, McGraw-Hill, NY (1932), pp 98–101.
- [77] Brüel, P V: *Sound Insulation and Room Acoustics*; Chapman and Hall, London (1951), Chs 1, 3.
- [78] Fisz, M: *Probability Theory and Mathematical Statistics*; Wiley, NY (1963).
- [79] Jenkins, G M and Watts, D G: *Spectral Analysis and its Applications*; Holden-Day, San Francisco, CA (1968), Chs 5–8.
- [80] Mandl, F: *Statistical Physics*; Wiley, Chichester, UK (1971), App B.
- [81] Lyon, R H: *Statistical Energy Analysis of Dynamical Systems: Theory and Applications*; MIT Press, Cambridge, MA (1975), Chs 2, 4.

- [82] Vanmarcke, E: *Random Fields: Analysis and Synthesis*; MIT Press, Cambridge, MA (1983), Chs 3-5.
- [83] van Bladel, J: *Electromagnetic Fields*; revised printing, Hemisphere, NY (1985), Ch 10.
- [84] Jones, D S: *Acoustic and Electromagnetic Waves*; Clarendon, Oxford, UK (1986), Sec 4.11.
- [85] Balanis, C A: *Advanced Engineering Electromagnetics*; Wiley, NY (1989), Ch 10.
- [86] Hewlett-Packard, Inc.: *HP 8990A Peak Power Analyzer*; Programming Manual (1991), pp 10-6, 10-7.

A Spherical resonators

A.1 Tuned resonators

Smooth air-filled PEC spherical cavities of radius R have TE_{mnp} and TM_{mnp} eigenmodes with resonance frequencies:

$$f_{mnp}^{TE} = \frac{x_{np}}{2\pi R} \quad (61)$$

$$f_{mnp}^{TM} = \frac{x'_{np}}{2\pi R} \quad (62)$$

where x_{np} and x'_{np} are zeros of the spherical Bessel function $j_n(x)$ and $d[x j_n(x)]/dx$, respectively, and with $m \leq n$ [83, 85]. For a PEC sphere with $2R = 0.70\text{m}$, the first 23 modes are:

$$374\text{MHz} : 3 \text{ modes } (f_{011,e}^{TM} = f_{111,e}^{TM} = f_{111,o}^{TM}) \quad (63)$$

$$528\text{MHz} : 5 \text{ modes } (f_{021,e}^{TM} = f_{121,e}^{TM} = f_{121,o}^{TM} = f_{221,e}^{TM} = f_{221,o}^{TM}) \quad (64)$$

$$613\text{MHz} : 3 \text{ modes } (f_{011,e}^{TE} = f_{111,e}^{TE} = f_{111,o}^{TE}) \quad (65)$$

$$678\text{MHz} : 7 \text{ modes } (f_{031,e}^{TM} = f_{131,e}^{TM} = f_{131,o}^{TM} = f_{231,e}^{TM} = f_{231,o}^{TM} = f_{331,e}^{TM} = f_{331,o}^{TM}) \quad (66)$$

$$786\text{MHz} : 5 \text{ modes } (f_{021,e}^{TE} = f_{121,e}^{TE} = f_{121,o}^{TE} = f_{221,e}^{TE} = f_{221,o}^{TE}) \quad (67)$$

etc., where the subscripts e and o refer to even and odd modes, respectively. Filling the cavity with a gas or liquid characterized by ϵ_r , μ_r will lower these values by a factor $\sqrt{\mu_r \epsilon_r}$. The respective associated bandwidths per degenerate mode for the air-filled cavity are 154MHz, 85MHz, 65MHz, 108MHz etc.

For finite conductivities, the quality factors of the cavity modes are finite and given by

$$Q_{mnp}^{TE} = R/\delta_{mnp} \quad (68)$$

$$Q_{mnp}^{TM} = (R/\delta_{mnp}) \left[1 - \frac{n(n+1)}{x'_{np}{}^2} \right] \quad (69)$$

where $\delta_{mnp} = \sqrt{2/(\omega_{mnp}^{TE,TM} \mu \sigma)}$ is the skin depth of the cavity at the resonance $\omega_{mnp}^{TE,TM}$. An approximate value of Q for a general cavity of volume V and surface S is [83]:

$$Q_{approx} = (2/\delta)(V/S) \quad (70)$$

As a result of the finite Q , the resonance frequencies become complex:

$$\omega_{mnp}^* = \omega_{mnp}(1 - 1/2Q_{mnp} + j/2Q_{mnp}) \quad (71)$$

The lowering of the real part is only significant for relatively small values of Q_{mnp} , i.e. for the first few cavity modes. Because it is a lowering in frequency, reducing the Q_{mnp} of the cavity may possibly assist in extending the range of proper reverberation operation to lower frequencies.

In a reverberation chamber the energy radiated by the source is transferred to a large number of modes, which act as 'sinks' or 'bins' for the energy stored inside the cavity, i.e. the source energy less the dissipated energy. The equipartition of energy among the 'sinks' is sometimes assumed (cfr. infra), though not required in general. Energy transfer ('intermodal coupling') between these 'sinks' is possible if their respective bands show a sufficient degree of overlap, as governed by the ω_{mnp} and bandwidths (ω_{mnp}/Q_{mnp}). At present it is commonly accepted that for proper reverberation, the number of modes within the Q -bandwidth BW_Q should probably²² be at least 15 but not less than 10 [64], although other criteria exist [38]. In any case, the determination of such a 'minimum' value for N_{BW} is not clear cut because the increase of N_{BW} always results in a steady improvement of the field homogeneity and, hence, in reduced measurement uncertainty for the average field strength and power density [12]. We elaborate further on the issue of lowest usable frequency in Sec A.4.

Figs A.1–A.16 show theoretical predictions for mode degeneracy, mode count, resonance count, bandwidth per mode, mode density, Q -factor and Q -bandwidth between 0 and 3GHz and for a 100MHz-band centered at 8.2GHz. In the linear plots, curves have been fitted to the data using n th order polynomial regression, i.e. $y(f_s) = a_0f_s^n + a_1f_s^{n-1} + \dots + a_n$, where $f_s \triangleq (f - \mu_f)/\sigma_f$ is the standardized²³ frequency over the plotted frequency range (f_{min}, f_{max}). In the logarithmic plots, exponential curve fitting has been used, i.e. $\log_{10}[z(f_s)] = b_0f_s^n + b_1f_s^{n-1} + \dots + b_n$.

Two important parameters for reverberation chamber operation, the number of modes N and the mode density dN/df , are classically estimated using the Weyl formula, adapted to vector electromagnetic problems. For a resonator of volume V , surface area S and edge length L the extended Weyl formula predicts [15]:

$$N = \frac{8\pi V}{3} \left(\frac{f}{c}\right)^3 + \left[-\frac{4}{3\pi} \int_{S=\partial V} \frac{dS}{\bar{\rho}} + (6\pi)^{-1} \int_{L=\partial S} \frac{(\pi - \varphi)(\pi - 5\varphi)dl}{\varphi}\right] \left(\frac{f}{c}\right) + \text{const} \quad (72)$$

where $\bar{\rho}$ is the mean radius of curvature of dS and φ is the dihedral angle along an edge segment dl of the cavity. Note in particular that the curvature ρ of corners in otherwise prismatic

²²The number is not clear cut, partially because of the finite slope of the skirts of the resonance characteristics for a real cavity. For example, if an equivalent rectangular filter is defined with same height of the maximum impulse response and with the same response to a white noise process, then this is well known to result in an enlargement factor $\pi/2$ for the equivalent bandwidth, compared to the half-power 3dB bandwidth of the original 2nd order linear resonator [81].

²³Standardized values have been used to maximize the accuracy of the fitting constants.

enclosures can have a profound effect at lower frequencies. For a sphere, for which $\bar{v} = R$ and $\varphi = \pi$:

$$N = \frac{8\pi V}{3} \left(\frac{f}{c}\right)^3 - \frac{16R}{3} \left(\frac{f}{c}\right) + \text{const} \quad (73)$$

The absence of a quadratic term in (f/c) , which would carry a coefficient proportional to the surface area of the sphere, is due to the cancellation of a contribution for TE modes by the corresponding one for TM modes, for piecewise smooth boundaries. Such a term would survive, for example, for *acoustic* resonators, for which the *scalar* Helmholtz equation applies. Fig A.17 compares the exact mode count N with the Weyl approximation and with a non-normalized proper 3rd order fit to the exact count, i.e. $N_{fit} = c_0(f/c)^3 + c_1(f/c)^2 + c_2(f/c) + c_3$. As can be seen from the normalized residuals $|\Delta N|/N$ with respect to the exact mode count N , the Weyl approximation underestimates N typically by 10% at higher frequencies, whereas the error of the 3rd order fit is then less than typically 1%. This is indicative of a slight underestimation of the leading term by (73), as has been verified by checking the leading coefficient c_0 of the 3rd order fit for increasing frequency intervals. The actual N is thus increasing somewhat faster than cubically. Deviations from the cubic law have also been witnessed for rectangular cavities [69].

For the modal density, dN/df , both the 2nd order fit $[dN/df]_{fit} = c_0(f/c)^2 + c_1(f/c) + c_2$ and the extended Weyl formula for the mode density,

$$\frac{dN}{df} = \frac{8\pi V}{c} \left(\frac{f}{c}\right)^2 - \frac{16R}{3c}, \quad (74)$$

yield a qualitatively similar behaviour, as can be seen in Fig A.18. However, the error introduced by (74) is now of the same order as that of the 2nd order²⁴ curve fit, viz. typically 10%, on average, above 3GHz. The average frequency spacing between modes is then $\langle BW_m \rangle = (dN/df)^{-1}$ with $dN = 1$, but statistics of the eigenfrequency distribution can be studied more generally by defining a frequency spacing index ψ and its moments [4].

The total number of modes N can be broken down into contributions by several modal groups. For example, in rectangular rooms the eigenmodes whose standing wave pattern is parallel to q pairs of walls of the room ($q = 0, 1, 2$; so called 'oblique', 'tangential' and 'axial' modes, respectively) yield a leading-term contribution N_q to N that is proportional to $(f/c)^{(3-q)}$ [77]. As a result, 'oblique' modes ($m, n, p \neq 0$) are more numerous at higher frequencies, whereas 'axial' modes ($mnp = 00X, 0X0$ or $X00$) are dominant at lower frequencies.

²⁴A 3rd order fit was verified to give virtually no improvement.

A.2 Untuned resonators

Untuning of the cavity can be achieved by excising a small complex volume ΔV from a tuned (i.e. smooth) resonator. Using perturbation theory, it can be shown (see e.g. [84]) that the effect of inward pushing of the cavity surface results in an increase of ω_{mnp} if the local modal magnetic field dominates the local electric field in the original ΔV . The perturbed resonance frequency ω'_{mnp} is then:

$$\omega'_{mnp} = \sqrt{\omega_{mnp}^2 + \int \int \int_{\Delta V} [c^2 |\nabla \times \underline{E}_{mnp}|^2 - \omega_{mnp}^2 |\underline{E}|_{mnp}^2] dV} \quad (75)$$

Hence, because of the boundary conditions and field continuity, the resonance frequencies of TE modes are being shifted upward by small inward corrugations of the chamber wall, as $E_{tan} \approx 0$ in ΔV . For TM modes, the direction of the shift depends on the specific value of the local E_r , which in turn depends on the electrical size of the sphere ($k_{mnp}R$), in addition to the specific location (azimuth and elevation) on the surface. For a more detailed analysis and specific results, a computational-numerical analysis of the untuned cavity is necessary.

A.3 Reverberation chambers

As a further step, the cavity untuning can be made tunable, i.e. the perturbing volume ΔV can be made to vary in position or size, stepwise ('mode tuning') or continuously ('mode stirring'). This translates itself in each resonance frequency ω_{mnp} of the static cavity becoming a random variable with a particular (usually taken to be uniform) distribution function within the interval $(\omega - \Delta\omega_{mnp}, \omega + \Delta\omega_{mnp})$, where $\Delta\omega_{mnp} = \max(|\omega_{mnp} - \omega'_{mnp}|)$.

Composite quality factor For reverberation operation, an alternative 'average' Q -factor can be defined for the highly overmoded cavity which does not take the detailed mode-dependent variations into account. A composite Q -factor, Q_c , was proposed and calculated in [32, 39] for a rectangular overmoded cavity of size $a \times b \times d$, assuming a uniform energy distribution across all excited modes²⁵ ('equipartition of energy'):

$$Q_c^{rect} = \frac{3V}{2S\delta} \left[1 + \frac{3\lambda}{16}(a^{-1} + b^{-1} + d^{-1}) \right]^{-1} \quad (76)$$

²⁵Note the difference in weighing of individual modes in reality, as discussed in Sec 12.2. A correction for this nonuniformity should lead to more realistic values of Q_c .

A similar derivation for a smooth spherical cavity of radius R can be made by re-evaluation of the energy integrals. The result is:

$$Q_c^{sphere} = \frac{R}{2\delta} \left[1 + \frac{9\lambda}{16(4\pi/3)^{1/3}R} \right]^{-1} \quad (77)$$

In the high-frequency limit ($\lambda \rightarrow 0$), this is also the Q_c of a cubic cavity with sides $a = 2R$. Values for Q_c for the sphere are shown as a function of frequency in Figs A.8 and A.16. For a corrugated sphere, the theoretical value is significantly lower than that obtained from (77), as the wall corrugations reduce the value of V/S . The specific value of Q_c hence requires an accurate estimate of V/S .

A further complication arises when trying to extend the analysis to nonintegrable cavities, such as the 3D Bunimovich stadium. An analytic computation of Q_c is then not possible. As a first approximation, for small values of the stadium parameter $\gamma \triangleq a/(2R) \ll 1$ and for relatively low frequencies, one can assume the same frequency dependence as for the sphere, replace the first term by the corresponding value for the 3D stadium and add a γ -dependent correction factor α ($0 < \alpha < 1$):

$$Q_c^{stadium} \approx \frac{(2 + 3\gamma)R\alpha}{(4 + 2\gamma)\delta} \left[1 + \frac{9\lambda}{16(4\pi/3)^{1/3}R} \right]^{-1} \quad (78)$$

For $\gamma = 0.25$, the factor in front of the brackets is $(11R/18\delta)\alpha$.

Like the sphere, the 3D stadium is totally smooth because the dihedral 'angles' formed by the cylindrical section and either hemisphere are π , due to the continuous transition. It therefore follows that (72)–(74) remain formally valid for the stadium, i.e. the second term in these formulas is formally unaffected by the cylindrical section [22], although V is now of course enlarged by the volume bounded by the cylindrical section.

A.4 Lowest usable frequency

Returning to the question of the lowest usable frequency (LUF) of a given room for proper reverberation operation, it may be thought that (72)–(73) would suggest a straightforward theoretical value. Assuming that a minimum *average* number of N_{BW} participating eigenmodes within the BW_Q is required for the application of the central limit theorem and, hence, for the χ^2_2 pdf power statistics to be applicable requires for a general ergodic complex cavity $(\Delta N/\Delta f) \geq N_{BW}$ with $\Delta f \equiv BW_Q$ representing the excitation bandwidth. Hence, in the limit $BW_Q \rightarrow 0$:

$$f_{min} = \sqrt{\frac{N_{BW} c^3}{8\pi V} + \frac{c^2}{6\pi^2 V} \int \int_{S=\partial V} \frac{dS}{\bar{\rho}} - \frac{c^2}{48\pi^2 V} \int_{L=\partial S} \frac{(\pi - \varphi)(\pi - 5\varphi) dl}{\varphi}} \quad (79)$$

For a sphere of radius R , this yields:

$$f_{min} \rightarrow \sqrt{\frac{c^3}{8\pi V} \left[N_{BW} + \frac{16R}{3c} \right]} \approx \sqrt{\frac{N_{BW} c^3}{8\pi V}} \quad (80)$$

For any integer value of N_{BW} , this yields estimates for f_{min} that are much too conservative when compared with the actual performance of real reverberation chambers, mainly because the assumption $BW_Q \rightarrow 0$ is to be relaxed for realistic enclosures and frequencies. However, if one can estimate the composite quality factor $Q_c(f) = f/BW_Q$ sufficiently accurately over a sufficiently wide frequency range, then a more realistic estimate of f_{min} can be obtained as the solution of $dN/df = [Q_c(f) N_{BW}]/f$, viz.:

$$\frac{f}{c} \left[8\pi V \left(\frac{f}{c} \right)^2 + \kappa \right] - N_{BW} Q_c = 0 \quad (81)$$

where

$$\kappa = -\frac{4}{3\pi} \int \int_{S=\partial V} \frac{dS}{\varrho} + (6\pi)^{-1} \int_{L=\partial S} \frac{(\pi - \varphi)(\pi - 5\varphi) dl}{\varphi} \quad (82)$$

in units m. For a sphere, $\kappa = -16R/3$. In the pathological case where Q_c can be made or considered to be largely frequency-independent, the reduced cubic equation (81) can be solved with the aid of the Cardano formulas. The discriminant of (81), $[N_{BW} Q_c / (16\pi V)]^2 + [\kappa / (24\pi V)]^3$, is always strictly positive hence there is only one real root:

$$f_{min} = \left[\left\{ \frac{N_{BW} Q_c}{16\pi V} + \sqrt{\left[\frac{N_{BW} Q_c}{16\pi V} \right]^2 + \left[\frac{\kappa}{24\pi V} \right]^3} \right\}^{1/3} + \left\{ \frac{N_{BW} Q_c}{16\pi V} - \sqrt{\left[\frac{N_{BW} Q_c}{16\pi V} \right]^2 + \left[\frac{\kappa}{24\pi V} \right]^3} \right\}^{1/3} \right] c \quad (83)$$

In the more realistic case of Q_c having an implicit frequency dependence, (81) cannot be solved in closed form. If a series expansion for Q_c for the limiting cases $f_{min} \ll c$ and $f_{min} \gg c$ is substituted, then approximate limiting expressions for f_{min} can be found. To this end we write Q_c , inspired by (76)–(77), as

$$Q_c = Q_0 \sqrt{\frac{f}{c}} \left[1 + \zeta \frac{c}{f} \right]^{-1} \quad (84)$$

with

$$Q_0 = (3/2)(V/S)\sqrt{\pi\mu c\sigma} \quad (85)$$

and ζ a shape-dependent constant (units m^{-1}), roughly of order unity: e.g. for a rectangular cavity, $\zeta = (3/16)(a^{-1} + b^{-1} + c^{-1})$; for a sphere, $\zeta = [9(4\pi/3)^{-1/3}/16]/R$. If $f_{\min} \ll c$, then $Q_c \approx (Q_0/\zeta)\sqrt{(f/c)^3}$ and substitution in (81) yields:

$$f_{\min} \approx \left[\frac{\kappa\zeta}{N_{BW} Q_0} \right]^2 c, \quad (f_{\min} \ll c) \quad (86)$$

Since κ and ζ have inverse dependence on V for smooth cavities, to first approximation, their product is dimensionless. Hence, with $S \sim V^{2/3}$, we have $f_{\min} \sim V^{-2/3}$ and $f_{\min} \sim N_{BW}^{-2}$.

At the other extreme, if $f_{\min} \gg c$, then $Q_c \approx Q_0\sqrt{f/c}$. Substitution in (81) now yields:

$$f_{\min} \approx \left[\frac{N_{BW} Q_0}{8\pi V} \right]^{2/5} c, \quad (f_{\min} \gg c), \quad (87)$$

hence $f_{\min}$ is now independent of its shape ζ , proportional to $V^{-4/15}$, i.e. approximately inversely proportional to the quartic root of V and proportional to the square root of N_{BW} . Unfortunately, for most chambers of practical interest $f_{\min} \sim c$, in which case these limiting expressions cannot be applied. Therefore, (81) should be solved on the basis of a measured or estimated Q_c for a vector of frequency values in the vicinity of the anticipated value of $f_{\min}$. More precise estimates for $f_{\min}$ must be obtained from the empirical $(\Delta N/\Delta f)$ function.

We computed $f_{\min}$ for a spherical cavity as a function of N_{BW} by solving (81) with Q_c given by (77). The results were computed as a function of N_{BW} , the chamber loading factor K relative to the theoretical Q_c , and the chamber volume V . These are shown in Figs A.19, A.20 and A.21, respectively. The choice of N_{BW} should be subjected to further theoretical and experimental study. From our measurements at a limited number of frequencies, it is expected that N_{BW} should be in excess of 3 and probably need not exceeding 10. The latter figure may in fact be too conservative, not in the least because of the finite slope of the skirts of the operational frequency band which results in a nonzero contribution of adjacent modes 'outside' BW_Q . A different choice for the value of N_{BW} will scale the above results accordingly. This value is likely to depend on the particular cavity geometry and to yield in the first instance an operation 'island' in the frequency spectrum. Also the degree of 'mode overlap', defined as ratio of the bandwidth of the individual eigenmodes to their mutual spacing and degeneracy, could influence the value of $f_{\min}$ for it enables mode coupling. To verify the extent of this phenomenon for non-degenerate modes, we have calculated BW_Q/BW_m for the spherical resonator (Figs A.22–A.23).

The overlap between adjacent resonances is seen to be generally very small at low frequencies. For example, it is less than 1% for $f < 3\text{GHz}$ and shows only very occasionally a value in excess of 15% around 8.2GHz. Thus, although the average nondegenerate overlap shows a marked increase with frequency, it is not expected to influence the required value of N_{BW} in a significant way. Of course, the overlap of degenerate modes is always total and has a marked effect in highly symmetric cavities. The correct value for N_{BW} for a particular cavity could be determined with the aid of the statistical methods outlined in the main body of the text or using EM numerical simulation techniques.

B Spatial correlation function for nonzero excitation bandwidth and nonzero spatial extent

The spatial correlation function for ideal reverberation fields, viz. $\rho(kr) = \text{sinc}(kr)$ is a well known and long standing result (e.g. [7, 10, 13, 21, 40, 53, 82]). It holds for the idealized situation in which the excitation bandwidth is zero and the field measurement is concentrated in true points, i.e. for zero spatial extent of the antennas. However, real excitation fields always have a nonzero excitation bandwidth and are measured across a nonzero aperture, however small. Here we compute the 1st order correction to $\rho(kr)$ that results from these effects.

First assume that concentrated point measurements can be made but that the excitation bandwidth Δk is nonzero. For each wavenumber k within the excitation band $[k_0, k_1]$, the above expression for $\rho(kr)$ holds. Hence

$$\rho(kr) = (\Delta k)^{-1} \int_{k_0}^{k_1} \rho(kr) dk \quad (88)$$

with $\Delta k = k_1 - k_0$. Defining $k_c = (k_0 + k_1)/2$ then this can be rewritten as

$$\rho(\Delta k, k_c, r) = (\Delta k)^{-1} \int_{-\Delta k/2}^{\Delta k/2} \frac{\sin(k_c r + kr)}{k_c r + kr} dk \quad (89)$$

Expanding the numerator using the trigonometric sum formula and expanding $(k_c r + kr)^{-1}$ for small (k/k_c) as $[1 - (k/k_c) + \dots]/(k_c r)$, we obtain to first order:

$$\rho(\Delta k, k_c, r) = \text{sinc}(r\Delta k/2) \left[\text{sinc}(k_c r) - 2 \frac{\cos(k_c r)}{(k_c r)^2} \sin^2(r\Delta k/4) \right] \quad (90)$$

as the spatial correlation function with correction for a nonzero excitation bandwidth Δk . Since $|\text{sinc}(r\Delta k/2)| \leq 1$, the effect of the nonzero bandwidth for $k_c r \gg 1$ is essentially a stronger damping of the periodic behaviour of the correlation function as kr increases. If $k_c r \gg 1$, then the second term between brackets causes the correlation function to 'contract' along the r axis. Figs B.1–B.2 show ρ on two different scales for various values of $\Delta k/k_c$. Also shown is the spatial correlation function in the case where no reverberation occurs (i.e. for totally coherent waves), viz. $\rho = \cos(k_c r)$.

Since $\rho(kr)$ is symmetric in k and r , a similar correction can be made to account for the effect of spatial averaging for nonzero Δr by replacing $k_c r$ by kr_c and $r\Delta k$ by $k\Delta r$ in (90). Hence

$$\rho(\Delta r, r_c, k) = (\Delta r)^{-1} \int_{r_0}^{r_1} \rho(kr) dr = \text{sinc}(k\Delta r/2) \left[\text{sinc}(kr_c) - 2 \frac{\cos(kr_c)}{(kr_c)^2} \sin^2(k\Delta r/4) \right] \quad (91)$$

with $r_c = (r_0 + r_1)/2$ representing the geometric center of the antenna or field probe.

The combined effect of nonzero bandwidth and nonzero spatial extent can be found along similar lines as above, after a double integration.

C Statistics of extrema and threshold crossings for the Rayleigh distribution

C.1 Mean maximum-to-mean ratio

An important parameter in reverberation chamber testing is the maximum-to-mean ratio for the field strength inside the chamber, because this enables the maximum field strength to be estimated based on the mean of a limited number of field strength measurements. If ergodicity holds then this yields at the same time an estimate for the maximum field strength anywhere inside the cavity, for an arbitrary stirrer position. For a 'large' sample of size N , the theory of 'Type I' extreme value distributions can be used to find the pdf of the maximum-to-mean ratio w of a pdf with exponential decay in the limit $N \rightarrow \infty$, in particular the χ_2^2 and χ_2 pdfs. For the χ_2 pdf:

$$\begin{aligned} f_N(w) &= N \chi_2(w) \left[\int_0^w \chi_2(w') dw' \right]^{N-1} \\ &= N \frac{\pi}{2} w \exp(-\pi w^2/4) \left[1 - \exp(-\pi w^2/4) \right]^{N-1} \end{aligned} \quad (92)$$

We assume that the N measurements obtained by stepping the stirrer completely specify the response, i.e. that the dependence of the maximum field strength on the stirrer position has been sufficiently closely 'captured' by the stirrer data. Then for one particular value w_m to be the maximum max-to-mean field strength, the 1st order derivative of $f_N(w)$ must vanish at $w = w_m$:

$$\frac{d\chi_2(w_m)}{dw_m} \int_0^{w_m} \chi_2(w) dw + (N-1) \chi_2(w_m) \int_0^{w_m} \frac{d\chi_2(w)}{dw} dw = 0 \quad (93)$$

After calculation, this yields w_m ($w_m \geq 1$) as the solution of the transcendental equation:

$$(1 - N\pi w_m^2/2) \exp(-\pi w_m^2/4) = 1 - \pi w_m^2/2 \quad (94)$$

Its numerically obtained solution w_{m0} is shown as a function of N in Fig C.1. If $w_m \gg 1$, as is the case in a good reverberation chamber, we can approximate (94) by

$$\exp(-\pi w_m^2/4) \approx N^{-1} \quad (95)$$

whose solution

$$w_1 = \begin{cases} \sqrt{2\sigma^2 \ln(N)} = \sqrt{(4/\pi) \ln(N)}, & \text{(Neper)} \\ 10 \log_{10} [2\sigma^2 \ln(N)] = 1.049 + 10 \log_{10} [\ln(N)], & \text{(dB)} \end{cases} \quad (96)$$

is also shown in the Figure. The latter expression can be rewritten for a common logarithmic basis as $1.049 + 4.343 \ln[\ln(N)]$, or equivalently $4.671 + 10 \log_{10} [\log_{10}(N)]$. The accuracy of the approximation is seen to improve as N increases. Another expression can be obtained using 'Type I' extremum statistics for the logarithm of the mean normalized field strength [69]:

$$w_2 = 10 \log_{10} [\gamma + \ln(N)] \quad \text{(dB)} \quad (97)$$

where $\gamma \approx 0.57722$ is the Euler constant. This is equally shown as a function of N in the Figure. Another expression has been proposed for the case $N > 50$ and vanishingly small spectral bandwidth $\epsilon \rightarrow 0$ [8]:

$$w'_2 = \frac{\gamma}{2\sqrt{\ln(N)}} + \sqrt{\ln(N)} \quad \text{(Neper)} \quad (98)$$

and for wideband spectra ($\epsilon \rightarrow 1$) [8]:

$$w''_2 \approx \sqrt{2 \ln \left(\frac{N}{\sqrt{2\pi}} \right)} \quad \text{(Neper)} \quad (99)$$

The expression for more general $\epsilon \ll 1$ is more complicated [8, Eqn (6.15)]. Unfortunately, no general expression when $\epsilon \rightarrow 0.913$ seems available.

Figs C.2–C.4 compare the theoretical predictions w_1 and w_2 with the measured maximum-to-mean ratios, after Efron bootstrapping of the latter, for the three respective measurement frequencies, along with the mean $\langle w \rangle$ and standard deviation σ_w . Discrepancies are mainly due to the fact that the (96)–(97) do not contain ϵ as a parameter and vary considerably as a function of N (Figs 12.16–12.18). The prediction w_2 is somewhat less accurate than w_1 for small to intermediate values of N , though better for higher values of N ($N > 200$). A compromise expression:

$$w_3 = \frac{w_1 + w_2}{2} \quad (100)$$

can be used as a reasonable approximation over a wider range of values for N ($5 < N < 1,000$).

C.2 Thresholds and excursions

The statistics of threshold crossings and associated excursions are usually derived for the case of Gaussian stochastic processes, such as white noise [82]. Since the magnitude of the total electric field may be considered to be the envelope of the vector sum of the in-phase and quadrature field components with Gauss normal pdfs, it is in principle possible to express the statistics of the total field using crossing statistics for the envelope of the underlying Gauss normal stochastic processes. This envelope for the stochastic electric field E is defined as

$$\varepsilon \triangleq \sqrt{E^2 + [\mathcal{H}(E)]^2} \quad (101)$$

where $\mathcal{H}(E) \triangleq \pi^{-1} \int_{-\infty}^{\infty} [E(t-x)/x] dx$ is the Hilbert transform of E at time t . However, this requires knowledge of the spectral moments λ_k of both constituent normal field components. These components are in general unknown when making scalar field or power measurements, as in the particular case of low-cost reverberation EMC test facilities. Hence we here derive these statistics solely based on the knowledge of measurable quantities only, i.e. using the statistics for the field magnitude or power as the fundamental quantities.

Following Rice [3], the average rate of occurrence of upward threshold crossings of a level b for the normalized field magnitude $|E_n| \triangleq |E|/\langle |E| \rangle$ is $\nu^+(b) = f_{E_n}(\varepsilon_n = b) E[|\dot{E}_n|]$, where the overdot represents 1st order derivation with respect to 'time', i.e. angular position τ . If E_n has a Rayleigh pdf then $\dot{E}_n$ and $\ddot{E}_n$ have a Gauss normal pdf with mean zero and respective standard deviations $\sigma_{\dot{E}_n}$ and $\sigma_{\ddot{E}_n}$. By way of verification for our stirrer data at 8.2GHz shown in Fig 3.2, the cdfs of the 1st and 2nd order derivate process are shown in Fig C.5. These have been derived using numerical finite differences between adjacent points in the tuner sweep data. The agreement is reasonable, bearing in mind the accumulating effect of discretization errors for such differences as the order of differencing increases and the fact that the consequences of possible deviations of $f_{\ddot{E}_n}$ from Gauss normality are far less severe for the applicability of the following results than similar deviations for $f_{\dot{E}_n}$. With

$$E[|\dot{E}_n|] = \int_{-\infty}^{+\infty} |\dot{\varepsilon}_n| f_{\dot{E}_n}(\dot{\varepsilon}_n) d\dot{\varepsilon}_n = \sqrt{\frac{2}{\pi}} \sigma_{\dot{E}_n} \quad (102)$$

and $\sigma_{\dot{E}_n} = \sqrt{\lambda_2}$, $\sigma_{E_n} = \sqrt{\lambda_0}$, $\lambda_k = \int_0^\infty \omega^k g(\omega) d\omega$, $g(\omega) = \mathcal{F}[E_n](\omega)$, the upward threshold crossing rate follows as:

$$\nu^+(b) = \frac{\sqrt{\lambda_2}}{\sqrt{2\pi}\lambda_0} b \exp[-b^2/(2\lambda_0)] \quad (103)$$

Since the derivate of a Gaussian pdf is again Gaussian, the average rate of occurrence of local maxima for $|E_n|$ during a tuner sweep, generally given by $\nu_{max} = (2\pi)^{-1}(\sigma_{\dot{E}_n}/\sigma_{E_n})$, is obtained as:

$$\nu_{max} = (2\pi)^{-1} \sqrt{\frac{\lambda_4}{\lambda_2}} \quad (104)$$

The mean excursion length T above a threshold b is generally given by the expression $E[T(b)] = 2[1 - F_{E_n}(b)]/\{f_{E_n}(b)E[|\dot{E}_n|]\}$. With the cdf $F_{E_n}(\varepsilon_n) = 1 - \exp[-\varepsilon_n^2/(2\sigma_{E_n}^2)]$, we obtain:

$$E[T(b)] = \frac{\sqrt{2\pi}\lambda_0}{\sqrt{\lambda_2}b} \quad (105)$$

In order words, this is the mean width of the 'time' intervals $\Delta\tau$ of the operating stirrer for which the 'peaks' of the stirrer data exceed a level b .

The spatial rate of occurrence of local maxima of a level above a threshold b is the spatial equivalent of $\nu(b)$. Its mean is given by $\mu(b) = [1 - F_{E_n}(\varepsilon_n)]/\{E[D(b)]\}^3$, i.e.

$$\mu(b) = \left[\frac{\sqrt{\lambda_2}b}{\sqrt{2\pi}\lambda_0} \right]^3 \exp[-b^2/(2\lambda_0)] \quad (106)$$

In other words, on a spatial lattice of measurement locations inside the chamber, $\mu(b)$ is the mean rate of finding a location where the field strength exceeds b . The mean spatial excursion volume in a 3D isotropic homogeneous field follows as $E[D(b)] = \{E[T(b)]\}^3$. This is the mean volume of the (not necessarily simply connected) region for which the mean normalized field strength in all²⁶ points exceeds b .

²⁶of a τ lattice

D Tables

Frequency	2.5GHz	8.2GHz	18GHz
Δs_{min} (mm)	8.5	4.7	2.1

Table 1: Minimum step size for uncorrelated samples

Parameter	Range of nominal values	2.5GHz	8.2GHz	18GHz
Stirring ratio (dB)	> 20	34.6	42.8	29.4
σ_n (-)	0.9 ... 1.1	0.93	0.97	0.91
D_n (-)	< 1	0.41	0.25	0.51
Measured max/mean (dB)	7 ... 8	7.32	7.60	7.43
Predicted max/mean (dB)	7 ... 8	7.33	7.85	8.18
$\frac{ (max/mean)_{meas} - (max/mean)_{theo} }{(max/mean)_{theo}}$ (%)		0.2	5.9	15.8

Table 2: Measured performance for field isotropy

File ref nr	r (mm)	ϕ (deg)	$\langle P_{rec}/P_{in} \rangle$ (dB)	$\langle P_{in} \rangle$ (dBm)	SR (dB)	σ_n (-)	$[\frac{max}{mean}]_{meas}$ (dB)	$[\frac{max}{mean}]_{theo}$ (dB)
140	60	-45	-21.81	6.43	32.5	0.94	7.37	7.15
141	168	-45	-20.96	6.31	32.0	0.87	6.29	7.30
133	60	-90	-20.80	6.26	36.2	0.96	7.86	7.30
134	78	-90	-21.12	6.54	35.5	0.91	7.05	7.30
135	88	-90	-21.17	6.29	28.7	0.84	6.48	7.30
136	98	-90	-22.28	6.27	34.6	0.98	7.62	7.30
137	108	-90	-21.60	6.22	31.7	0.98	7.02	7.30
138	118	-90	-20.98	6.25	33.5	1.04	7.82	7.30
139	168	-90	-20.65	6.16	35.4	0.98	6.81	7.30
144	60	-135	-21.13	6.22	33.5	1.01	8.62	7.30
143	118	-135	-20.79	6.27	35.8	0.99	6.91	7.15
142	168	-135	-21.19	6.28	36.8	1.02	8.48	7.30
145	60	-180	-20.74	6.21	37.0	1.06	8.44	7.02
154	78	-180	-21.44	6.27	33.0	1.11	7.94	7.30
155	96	-180	-21.78	6.34	29.0	1.08	7.42	7.48
147	118	-180	-21.30	6.29	25.6	0.96	7.63	7.30
156	136	-180	-20.03	6.30	28.4	0.82	6.53	7.30
153	168	-180	-21.23	6.27	25.0	0.97	6.75	7.30
150	60	-270	-21.89	6.45	35.6	1.06	8.00	6.90
148	168	-270	-21.56	6.28	35.4	0.96	7.37	7.15

Table 3: Measured performance for field homogeneity at 8.2GHz, $\theta = \pi/2$, based on measured averaged insertion loss with check on field isotropy and random polarization

Parameter	2.5GHz	8.2GHz	18GHz
P_{in} (mW)	—	50	6
P_{recmax} (mW)	—	1.91	0.0539
$\langle P_{rec} \rangle$ (mW)	—	0.328	0.0085
$\langle IL \rangle \triangleq \langle P_{rec}/P_{in} \rangle$ (dB)	—	-21.2	-30.4
$ E _{max}$ (V/m)	—	70.3	24.7
$\langle E \rangle$ (V/m)	—	25.3	8.71

Table 4: Estimated field strength inside chamber

Parameter	Range of typical values	2.5GHz	8.2GHz	18GHz
$Q_e (-)$		—	4,380	9,610
$Q_d (-)$		—	7,950	—
$Q_c (-)$		(46,100)	(90,100)	(136,100)
$Q_{t,wall} (-)$		51,900	94,000	139,300
$Q_{t,ant} (-)$		—	644,000	6,800,000
$Q_{t,tot} (-)$		(51,900)	(72,800)	(133,800)
$Q_{approx} (-)$		(68,800)	(124,500)	(184,500)
$K \triangleq Q_{t,tot}/Q_e$	1 ... 5	—	(17)	(14)
BW_Q (MHz)		—	1.87	1.97

Table 5: Estimated and measured quality factors for mode tuned operation

Parameter	2.5GHz	8.2GHz	18GHz
N_{max} per revolution	270	495	1,100
Maximum sampling rate (Hz)	270ν	495ν	$1,100\nu$

Table 6: Estimated maximum sampling rates for mode stirred operation

Parameter	2.5GHz	8.2GHz	18GHz
$\Delta\tau$ (mrad)	6.926	2.597	0.866
$\omega_N (-)$	453.6	1209	3628
$g_0 (-)$	6.866	8.242	10.24
$\alpha (-)$	-0.0434	-0.0178	-0.0124
$\lambda_0 (-)$	2.500×10^2	7.500×10^2	1.600×10^3
$\lambda_1 (-)$	7.310×10^3	4.438×10^4	1.427×10^5
$\lambda_2 (-)$	3.669×10^5	5.148×10^6	2.250×10^7
$\lambda_4 (-)$	2.894×10^9	2.246×10^{11}	2.577×10^{12}
$\delta (-)$	0.646	0.700	0.659
$\epsilon (-)$	0.902	0.918	0.937

Table 7: Measured values of spectral parameters

E Figures

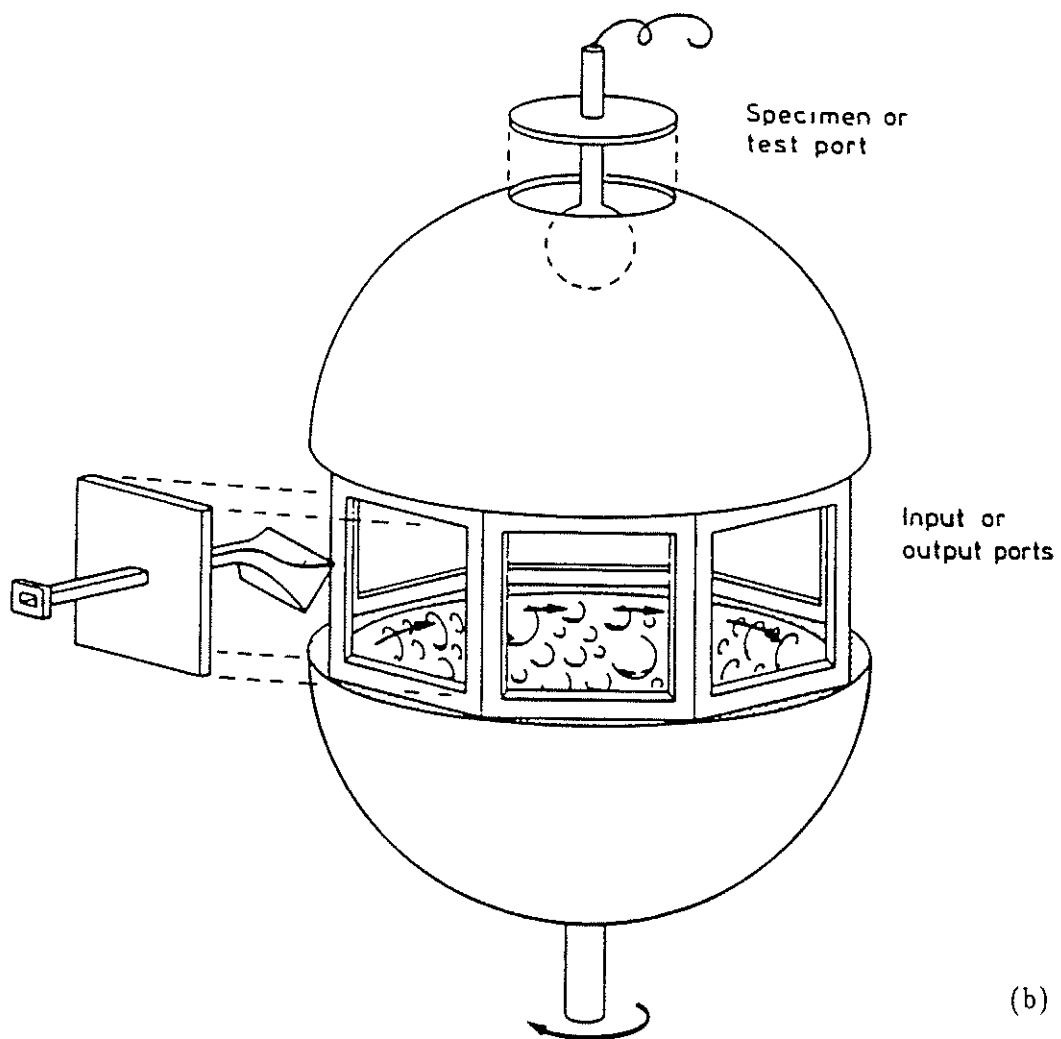
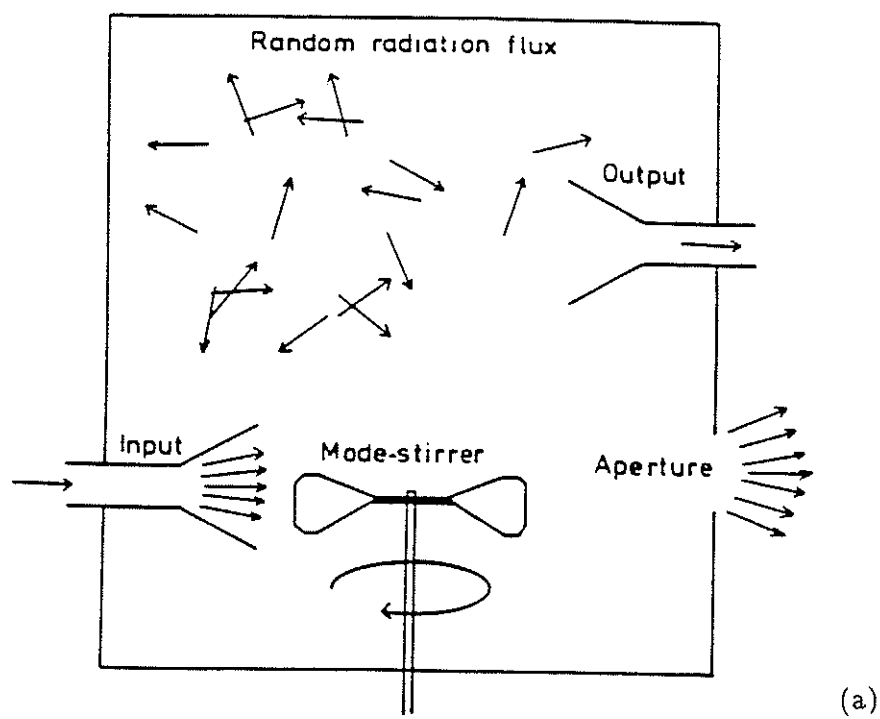
**Fig 2.1**



Fig 2.2

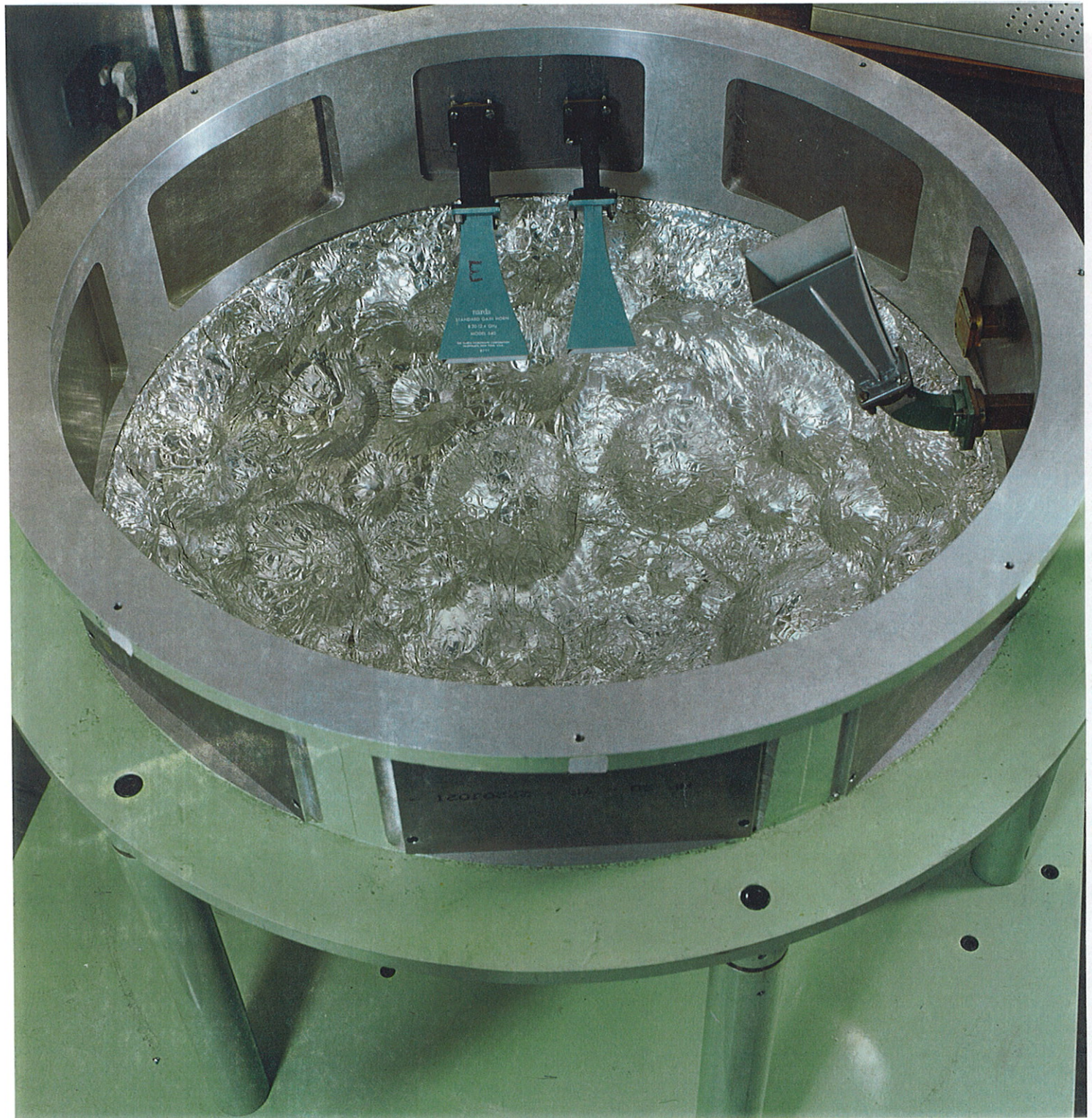


Fig 2.3

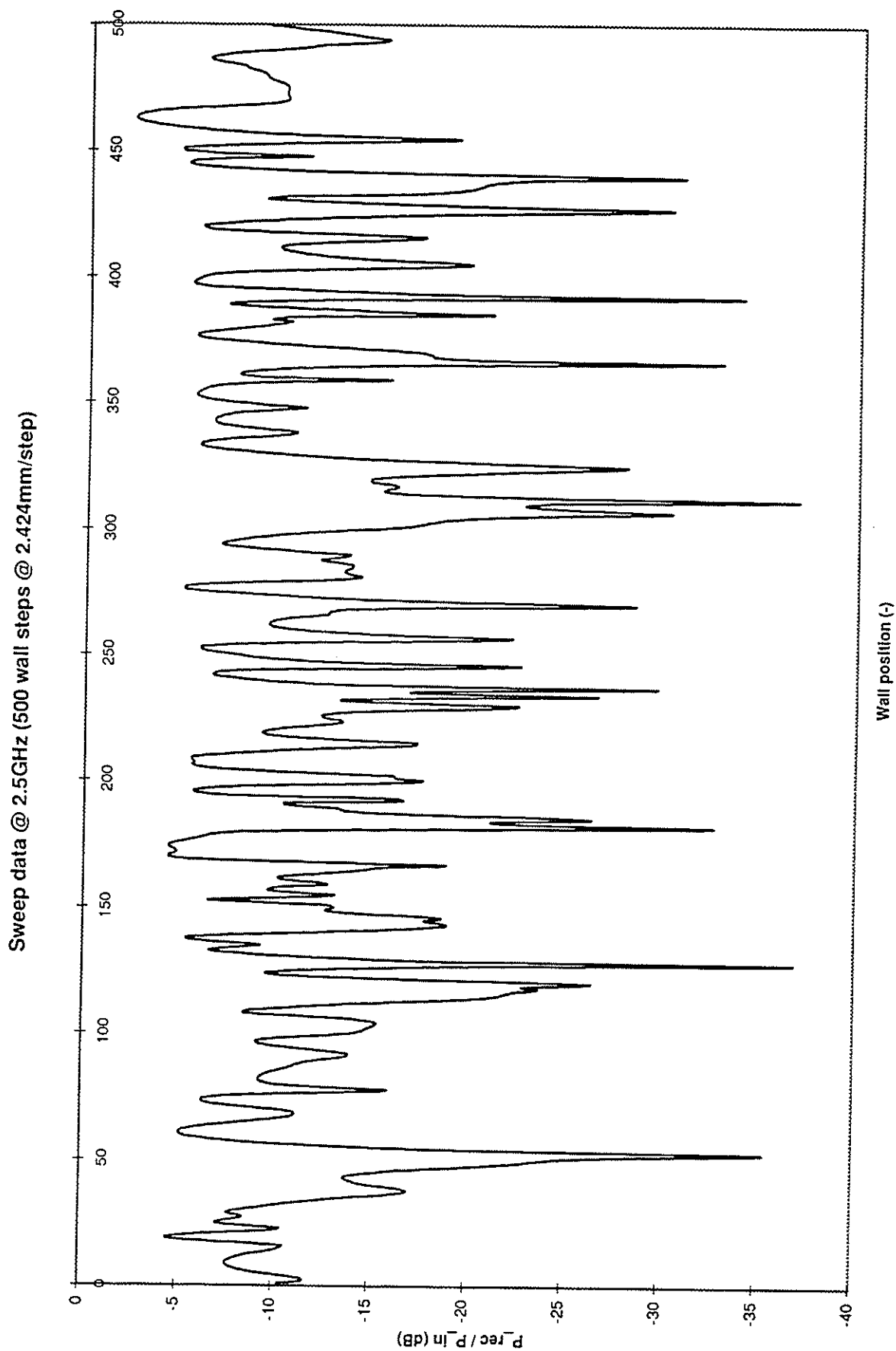


Fig 3.1

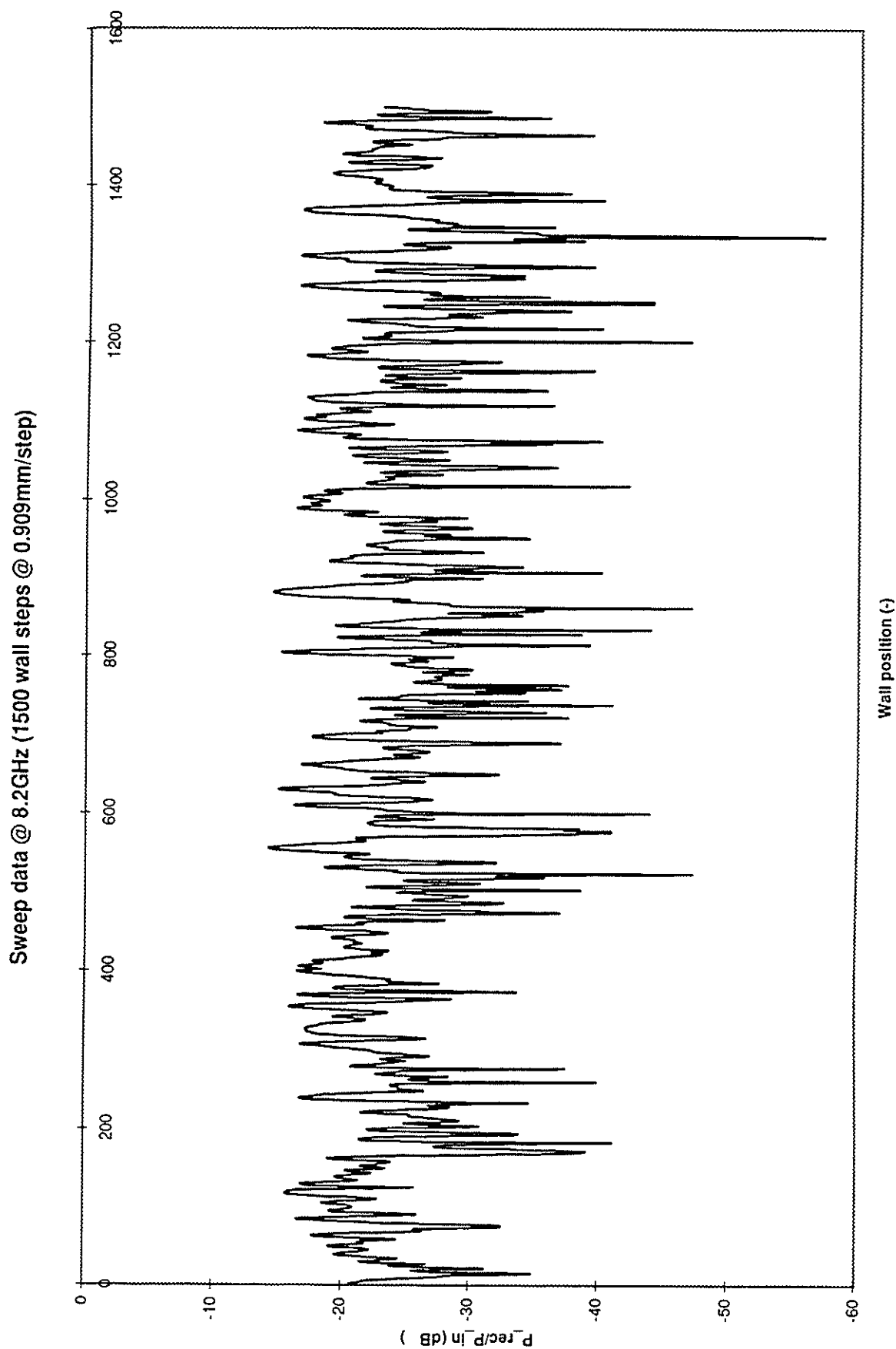


Fig 3.2

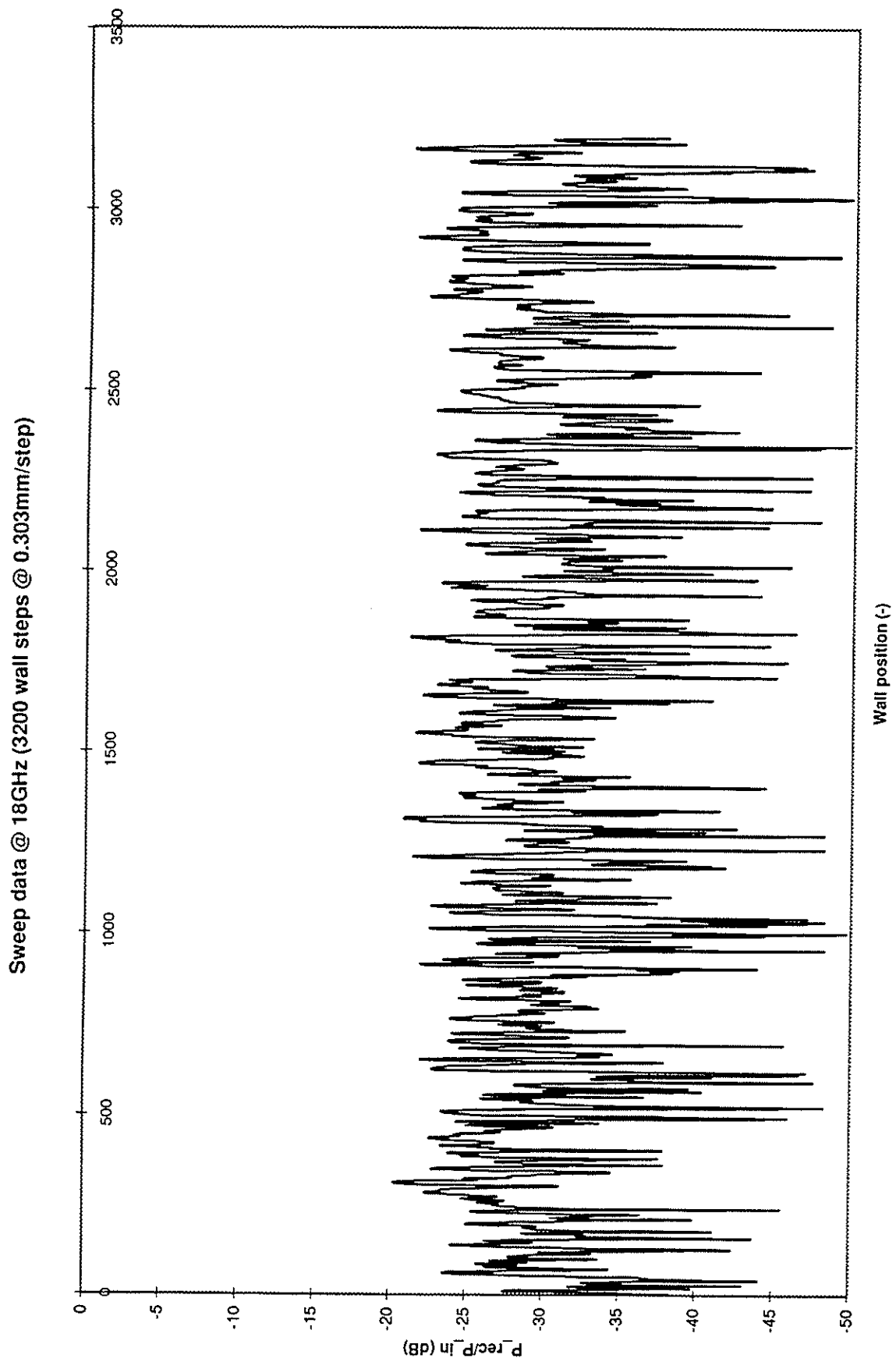


Fig 3.3

Autocorrelation function @ 2.5GHz (500 wall steps @ 2.424mm/step)

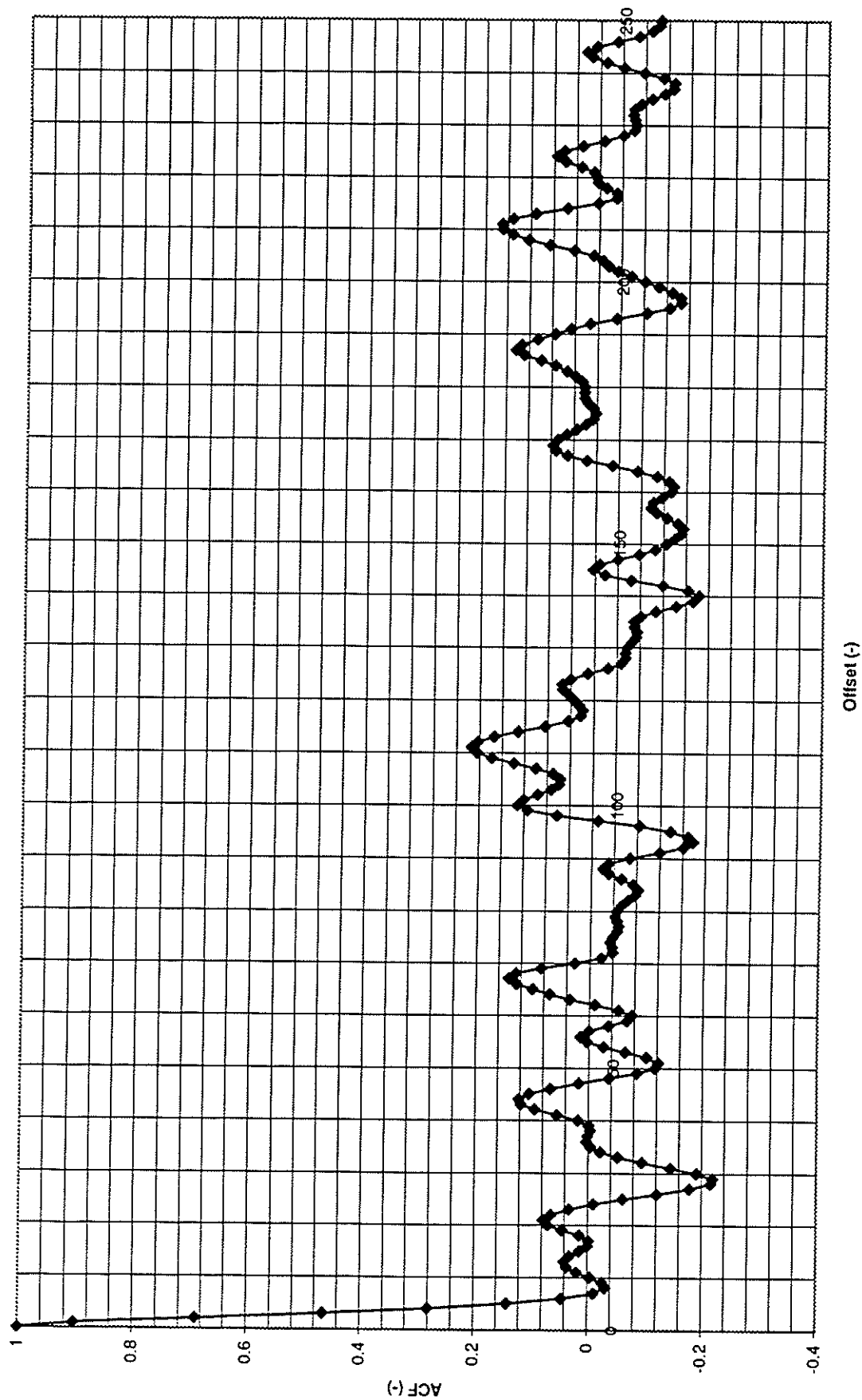


Fig 4.1

Autocorrelation function @ 8.2GHz (1500 wall steps @ 0.909mm/step)

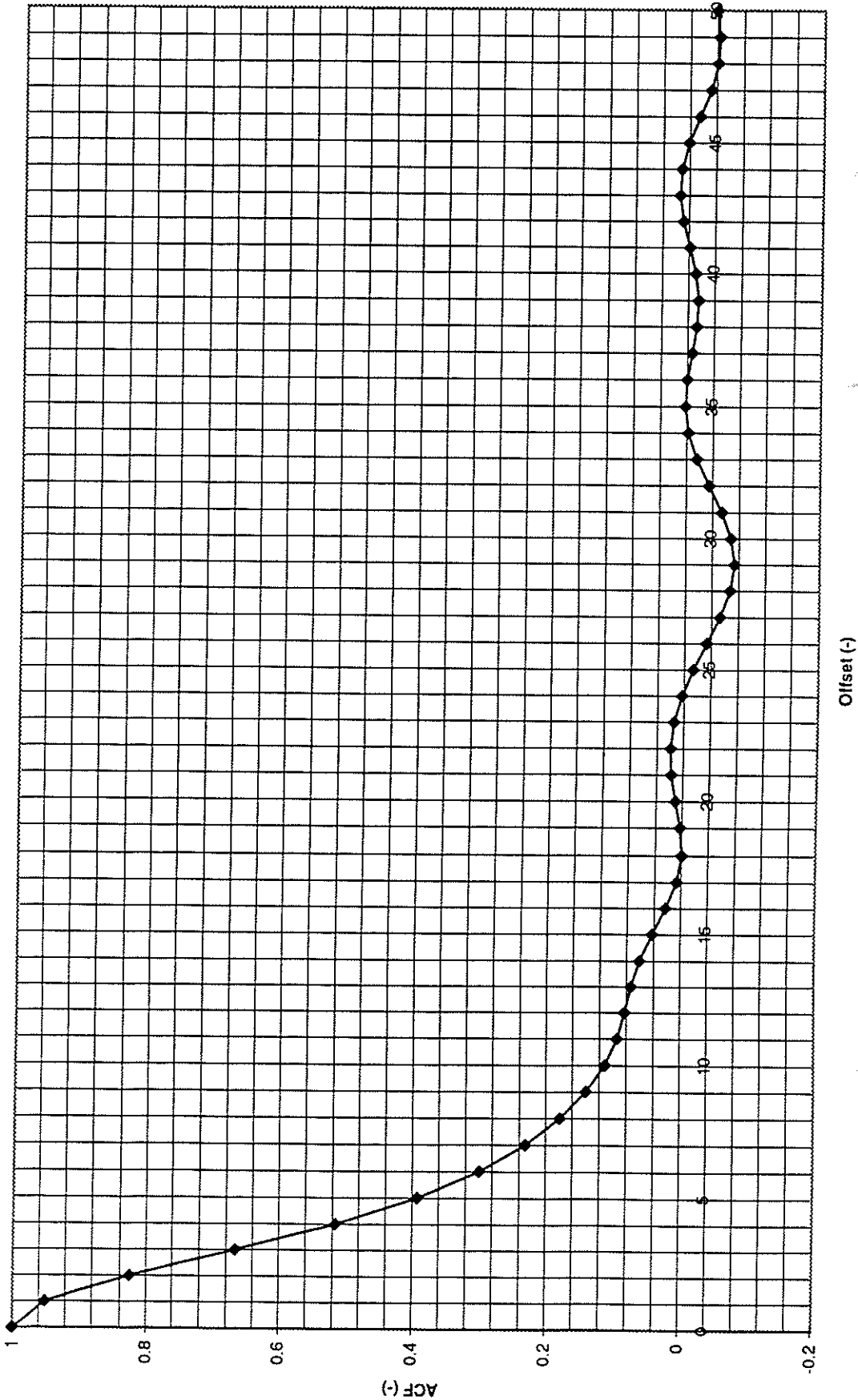


Fig 4.2

Autocorrelation function @ 18GHz (3200 wall steps @ 0.303mm/step)

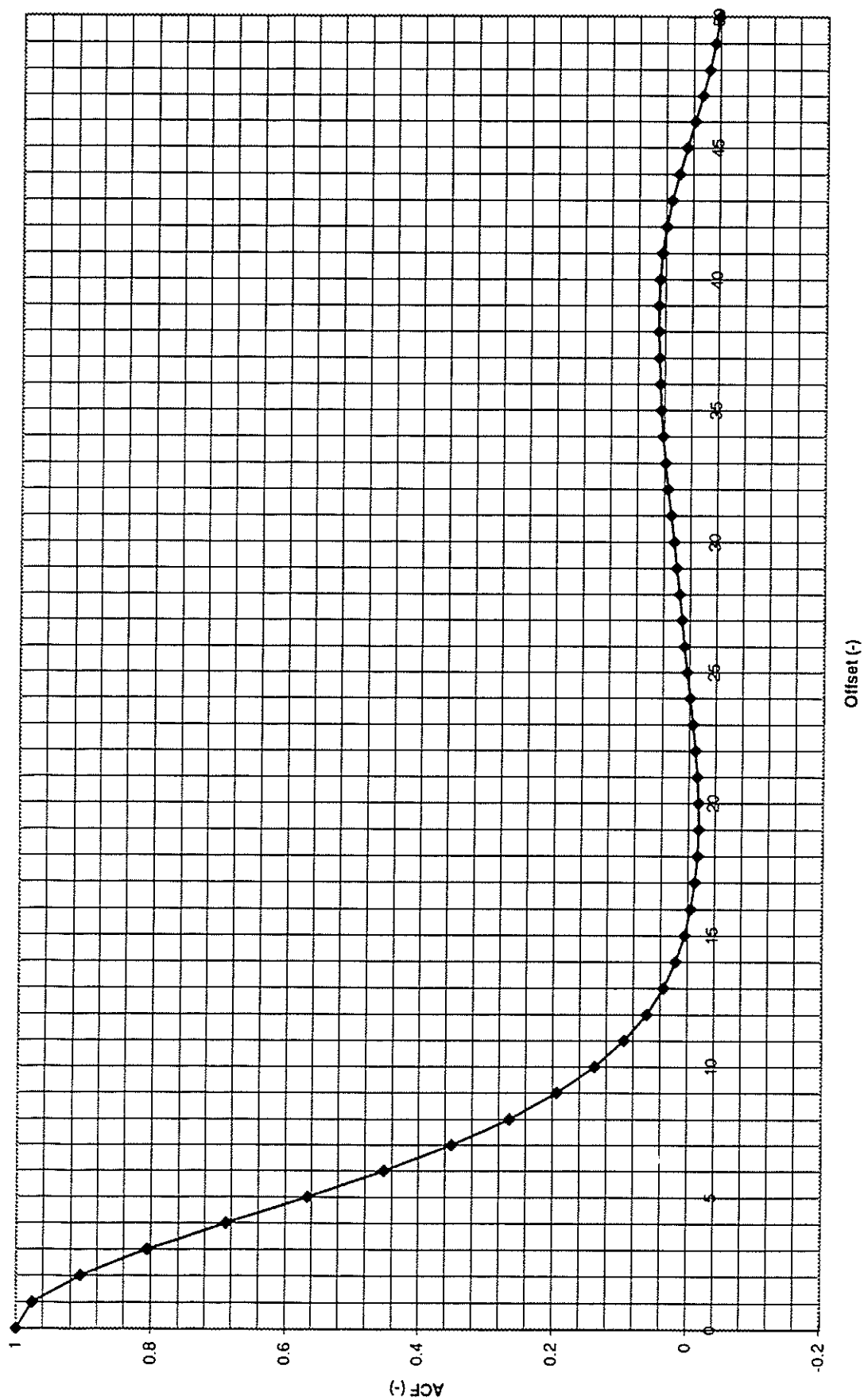


Fig 4.3

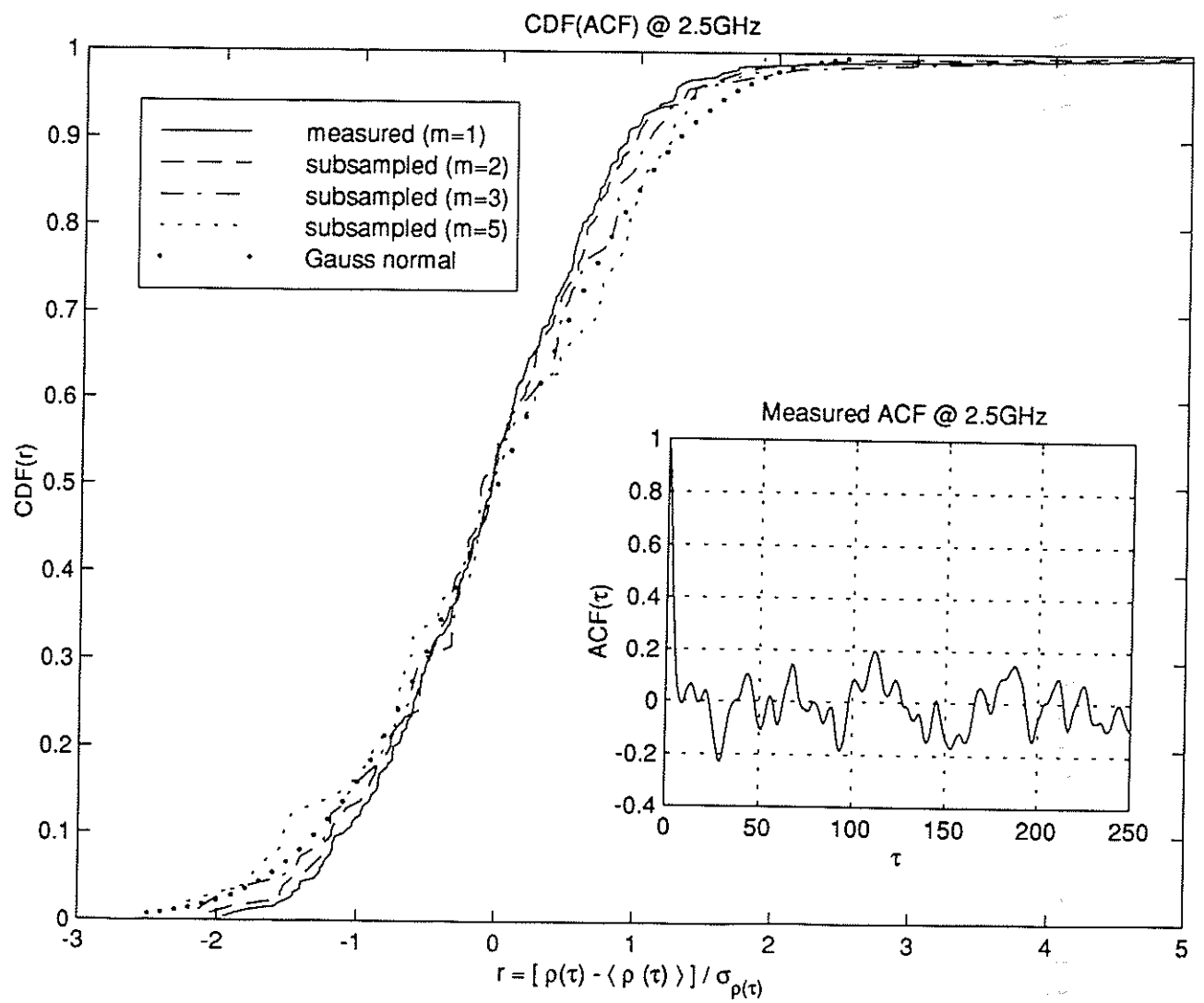


Fig 4.4

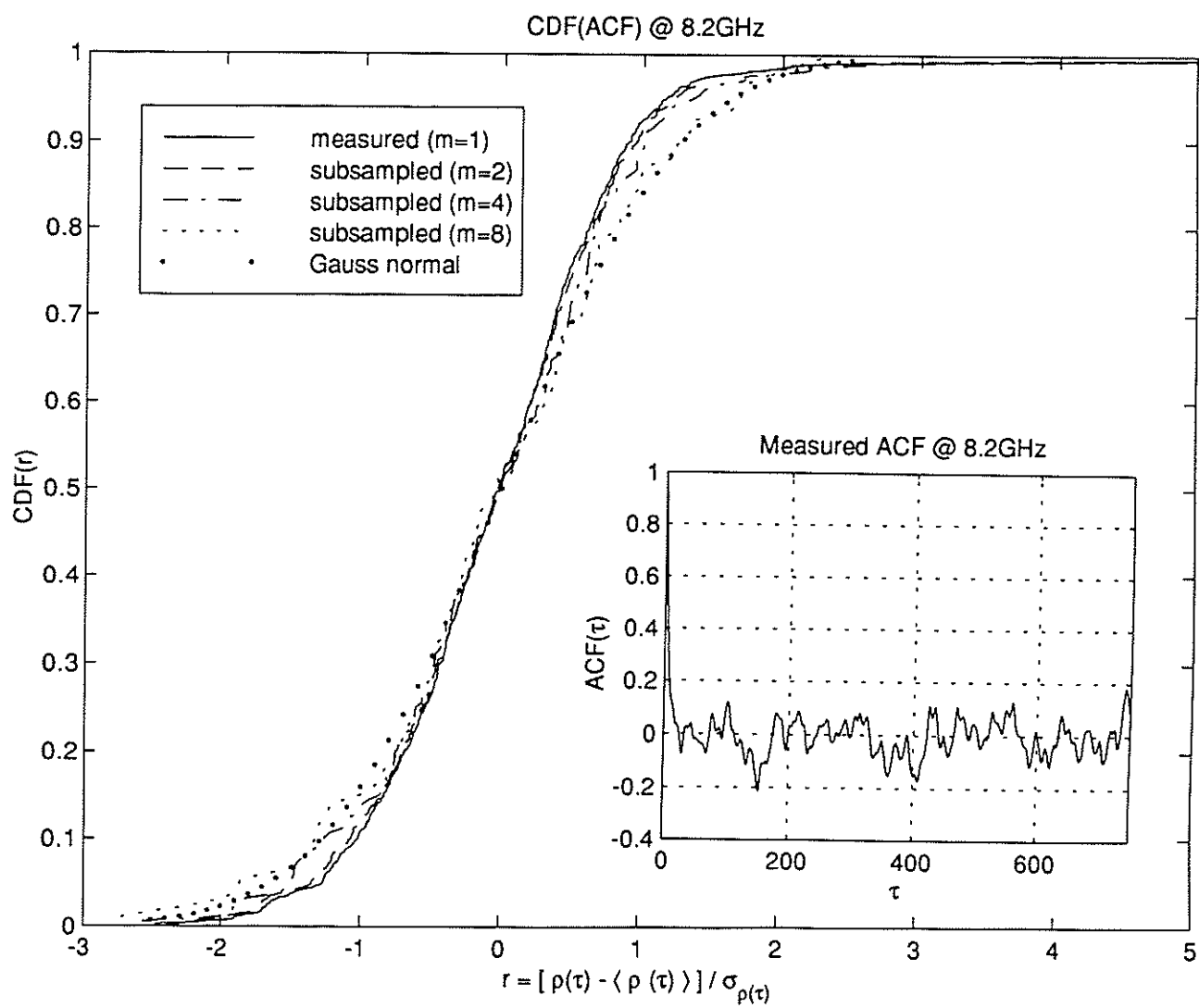


Fig 4.5

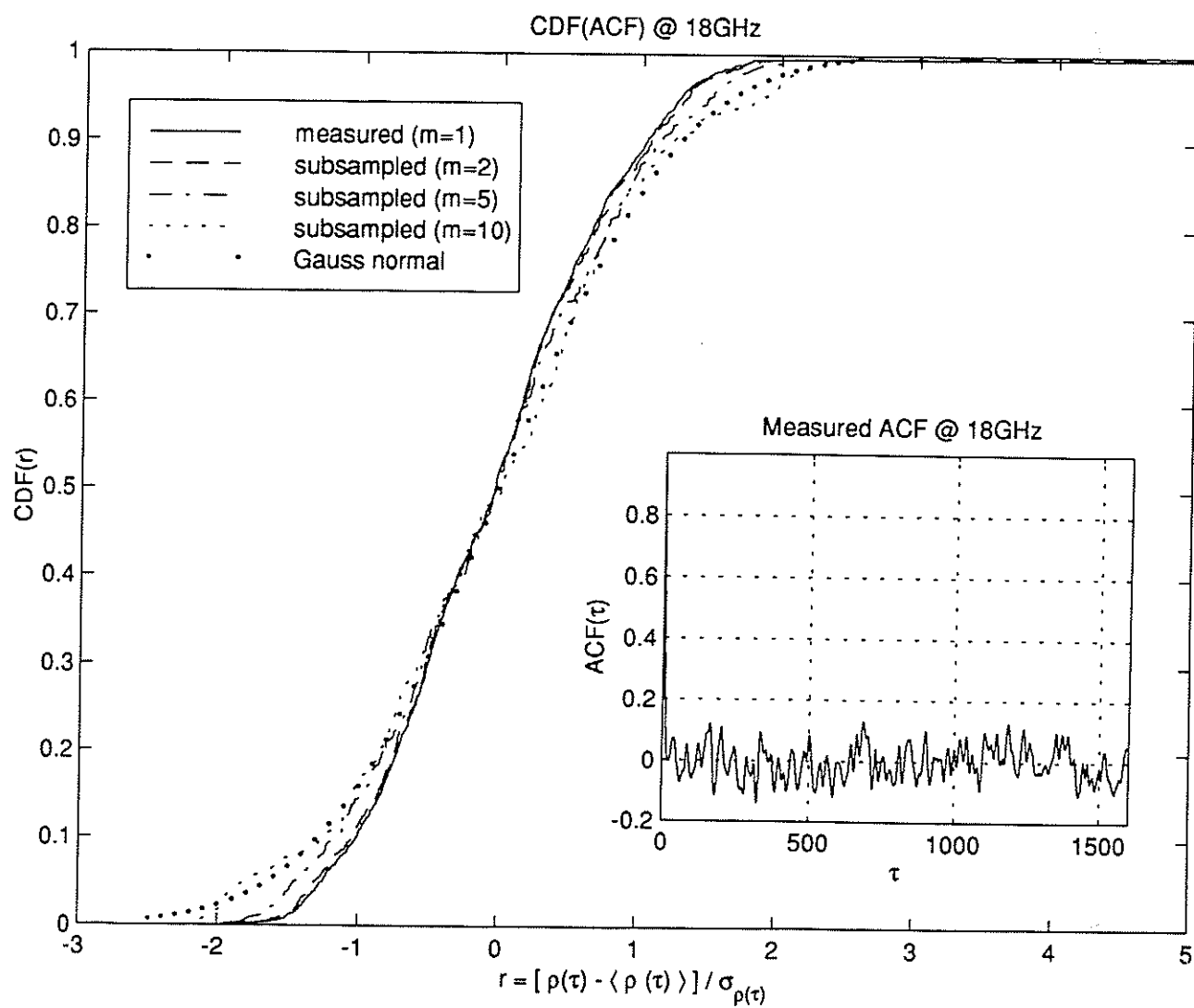


Fig 4.6

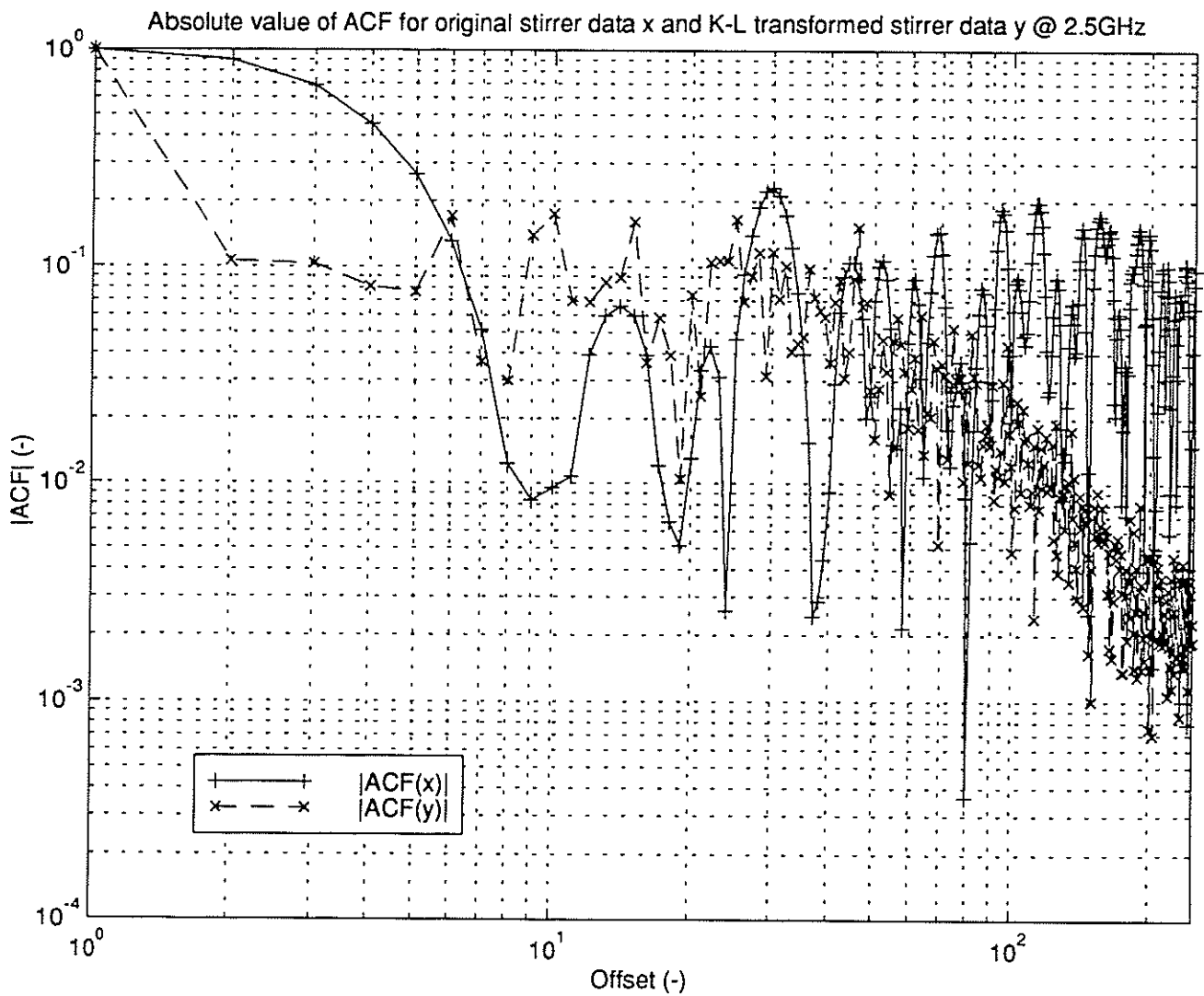


Fig 4.7

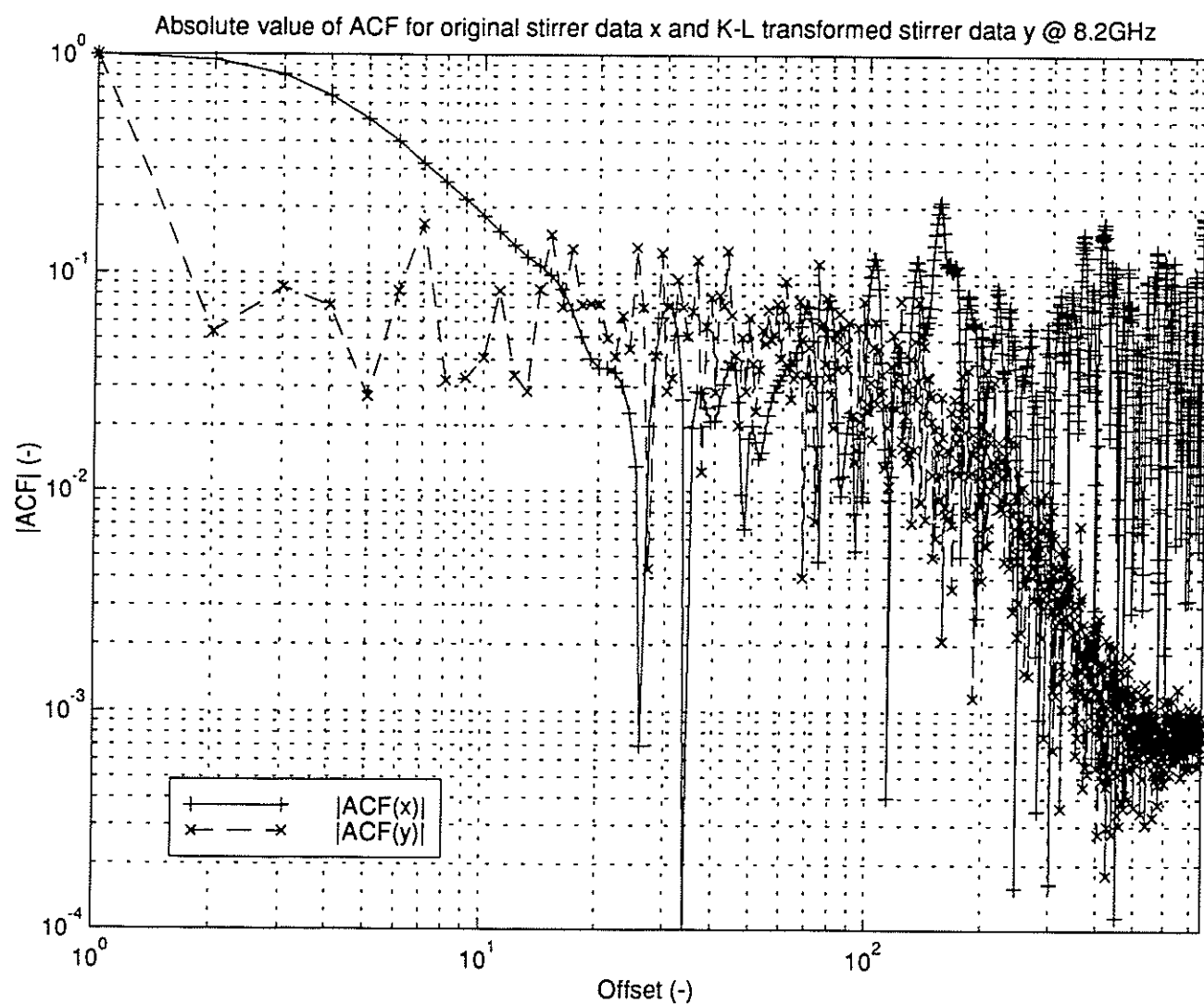


Fig 4.8

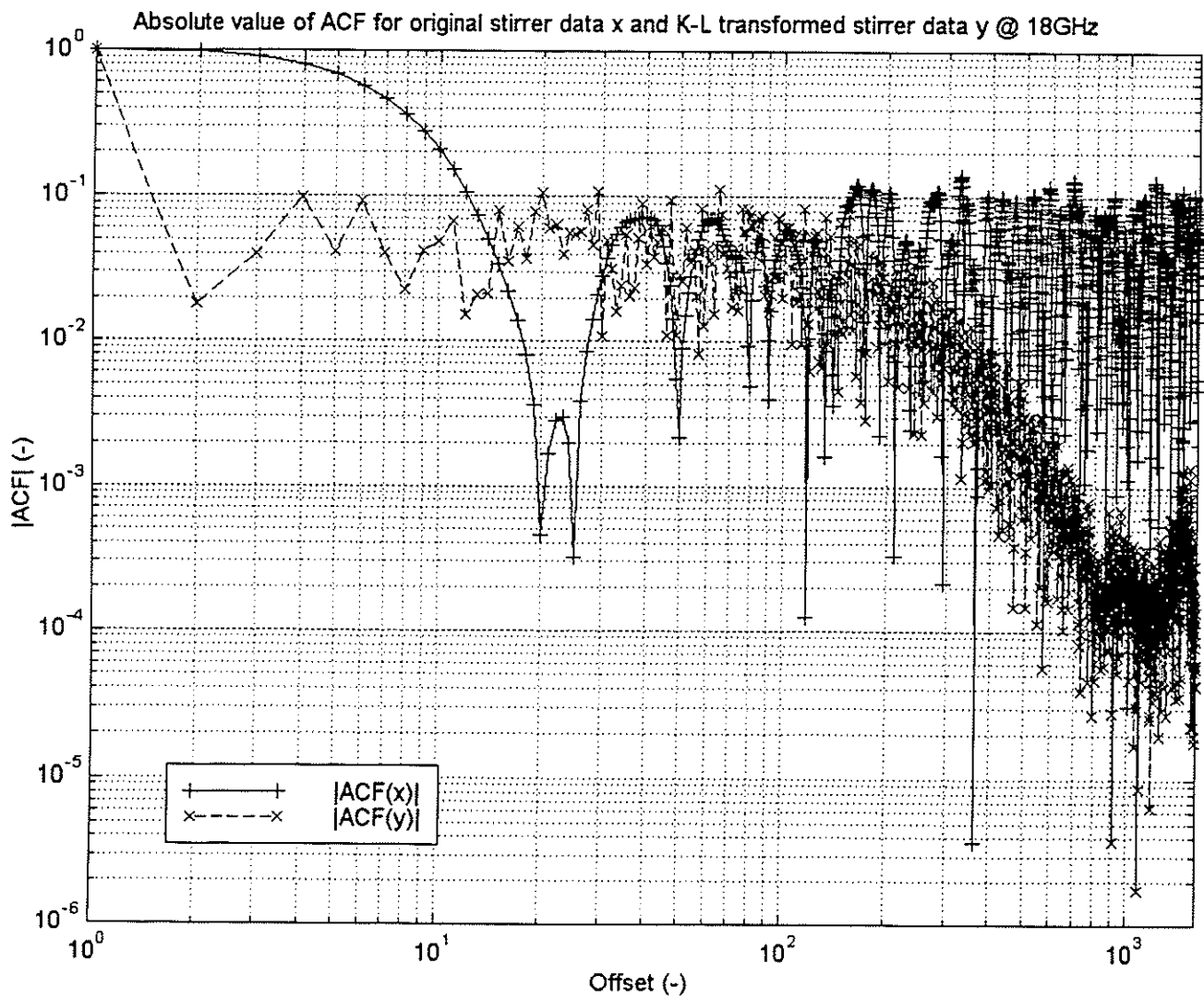


Fig 4.9

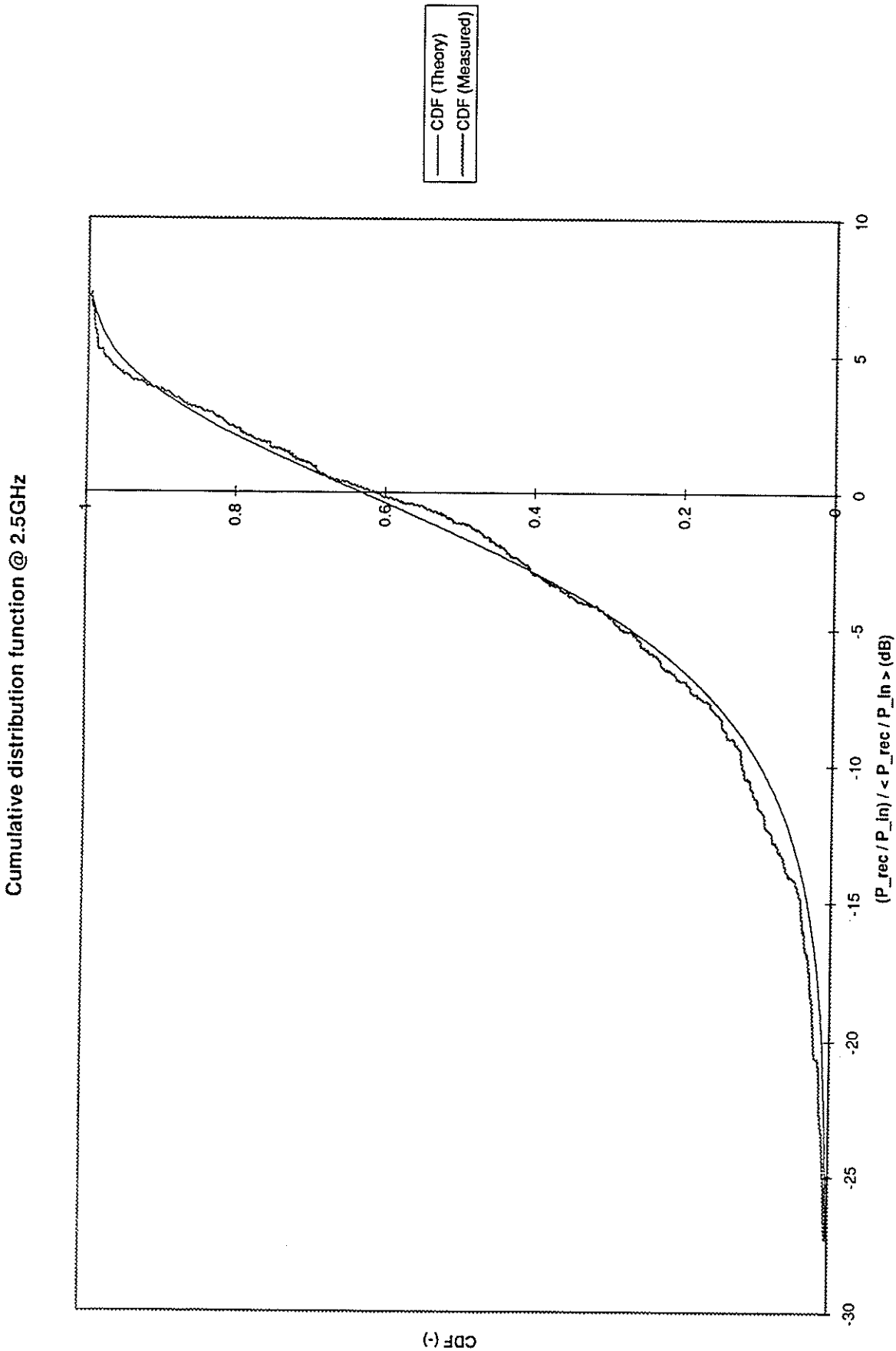


Fig 5.1

CDF for Max-to-Mean ratio (N=125)

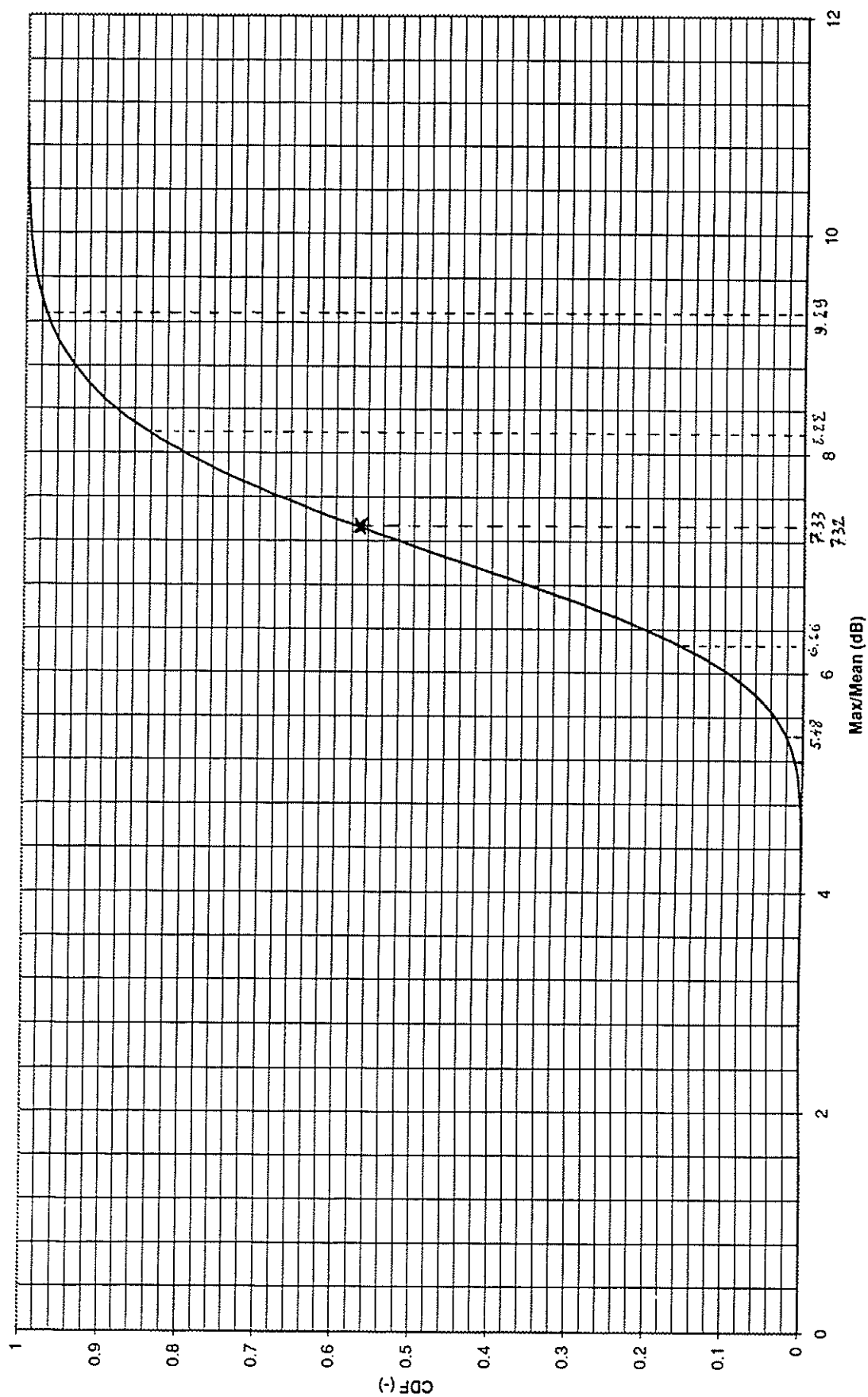


Fig 5.2

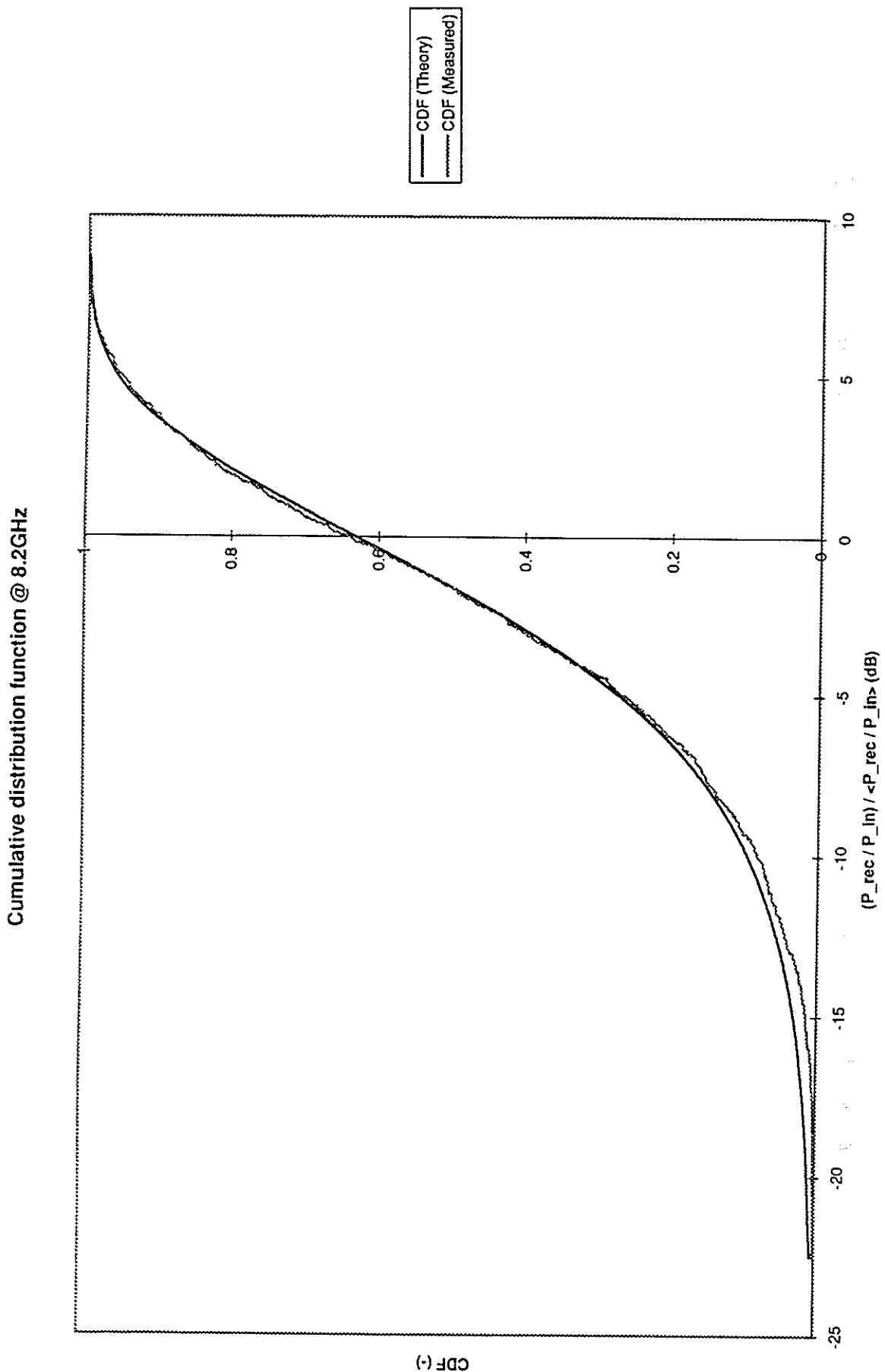


Fig 5.3

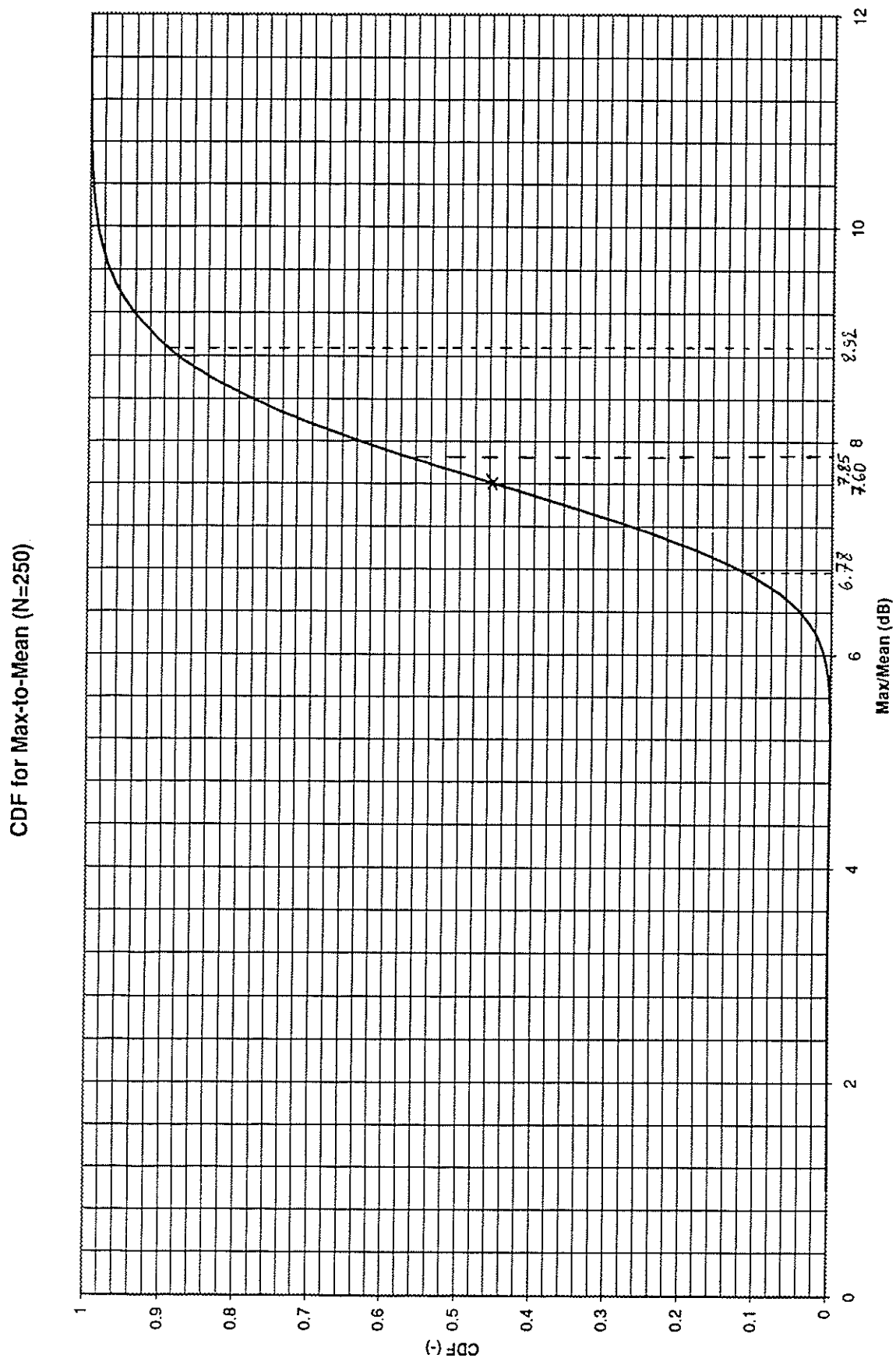


Fig 5.4

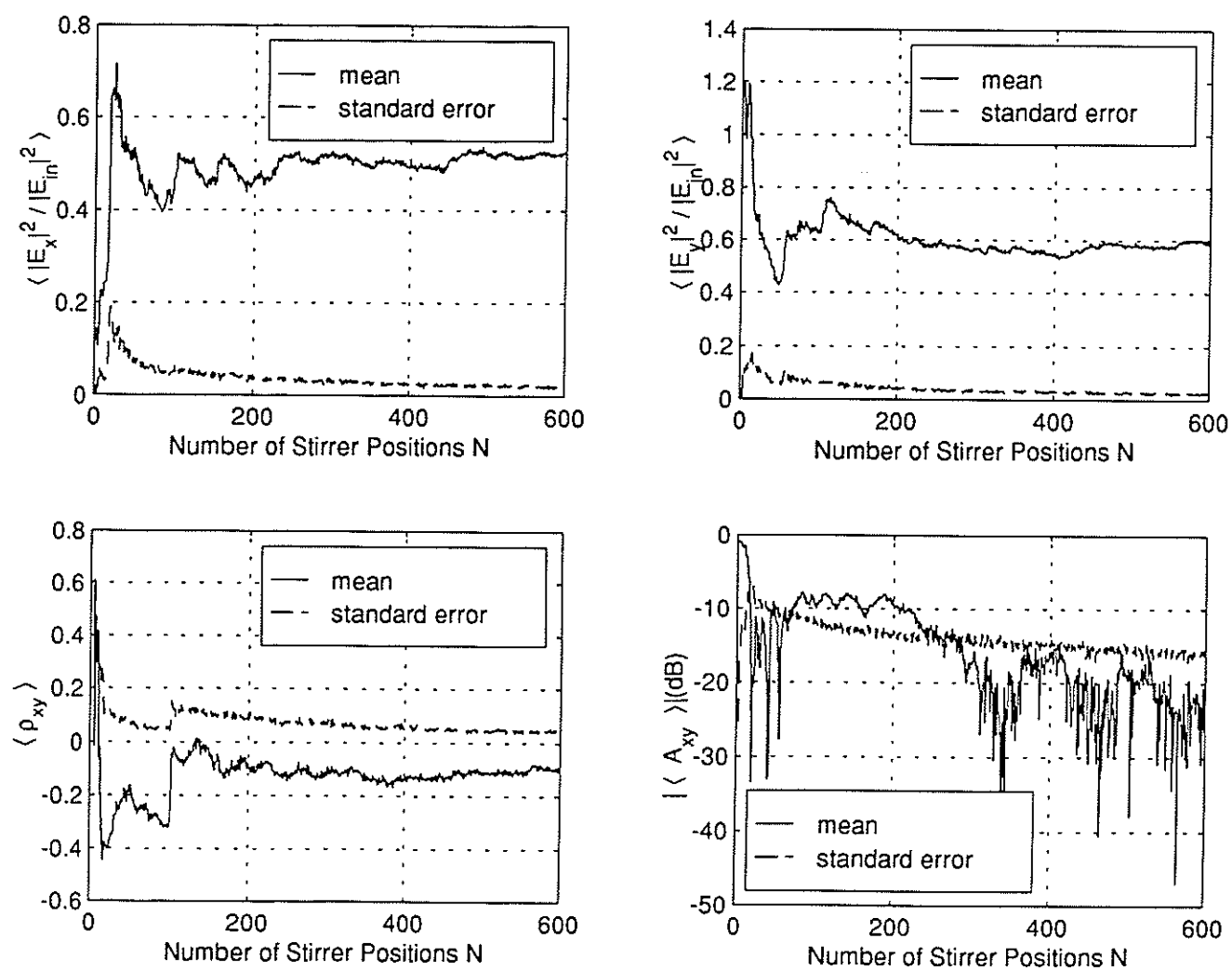


Fig 5.5

CDF of planar field anisotropy @ 8.2GHz

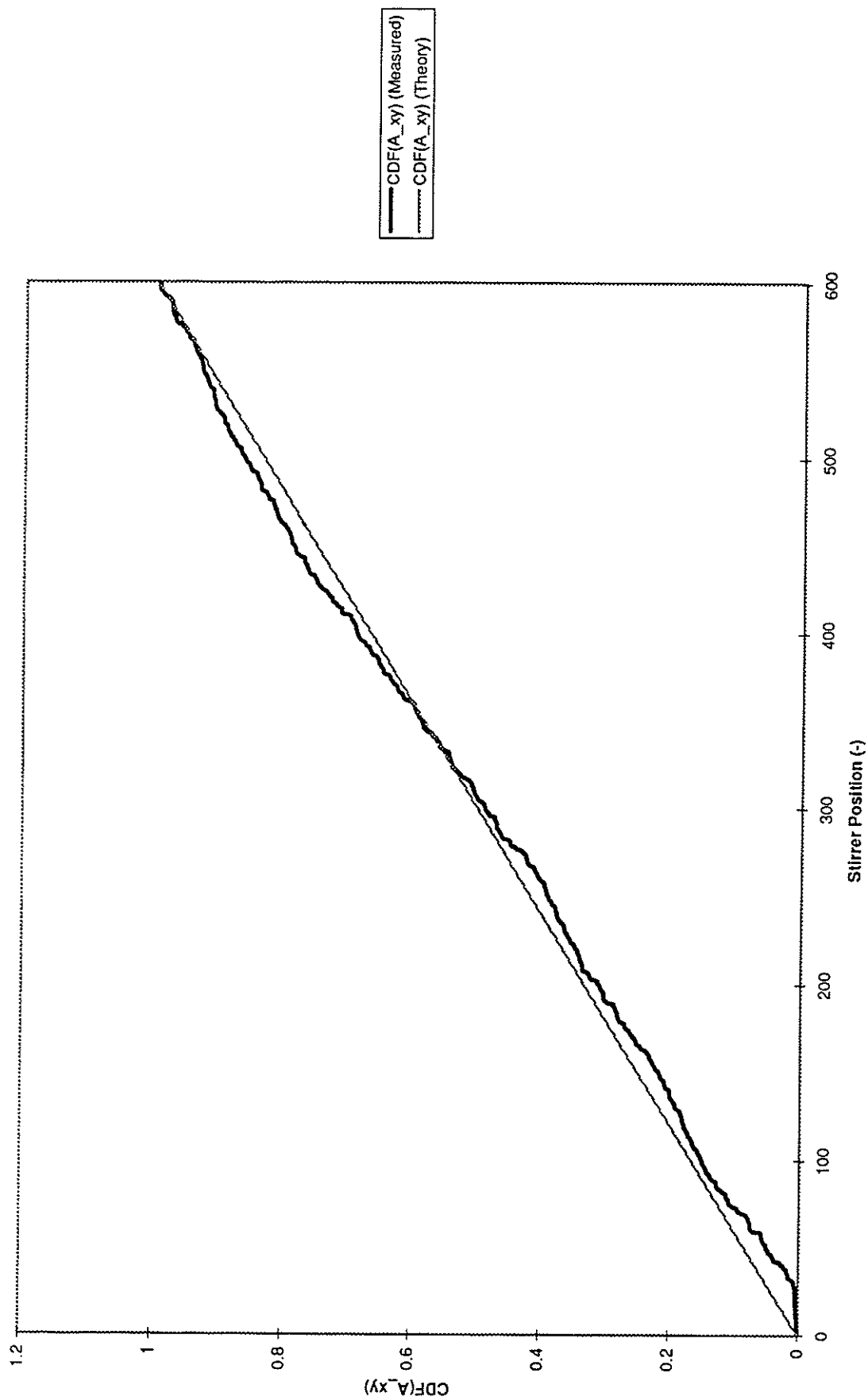


Fig 5.6

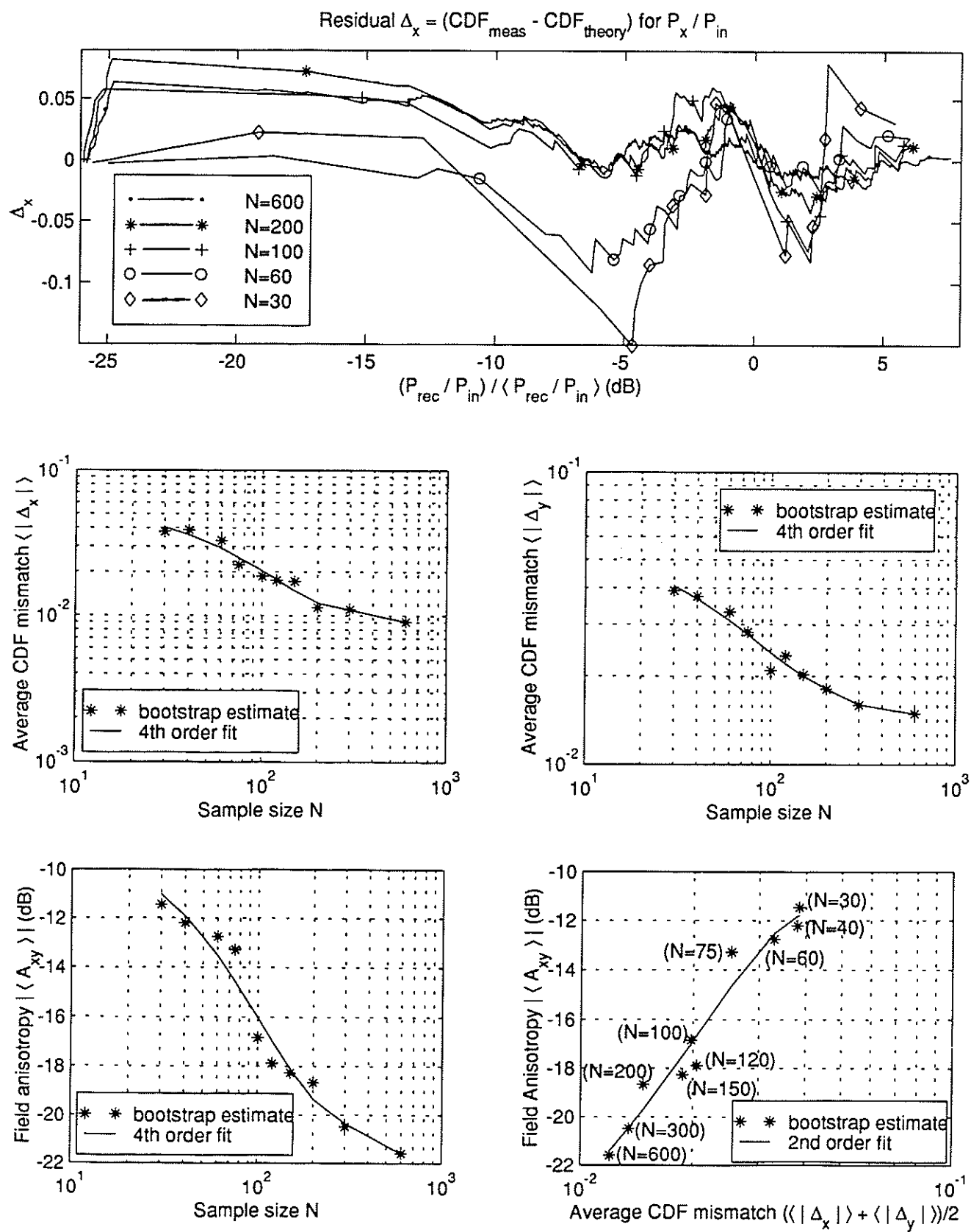


Fig 5.7

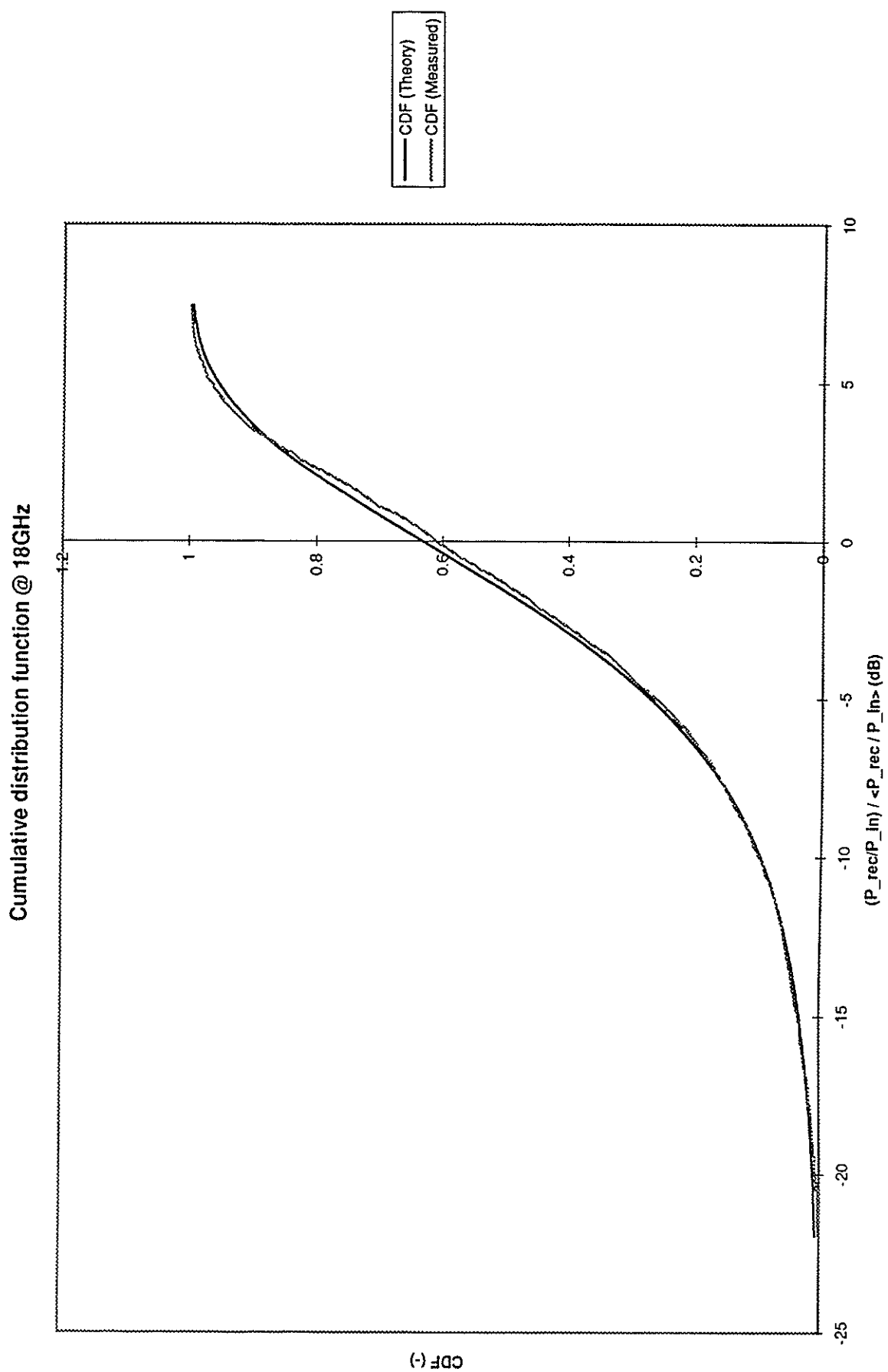


Fig 5.8

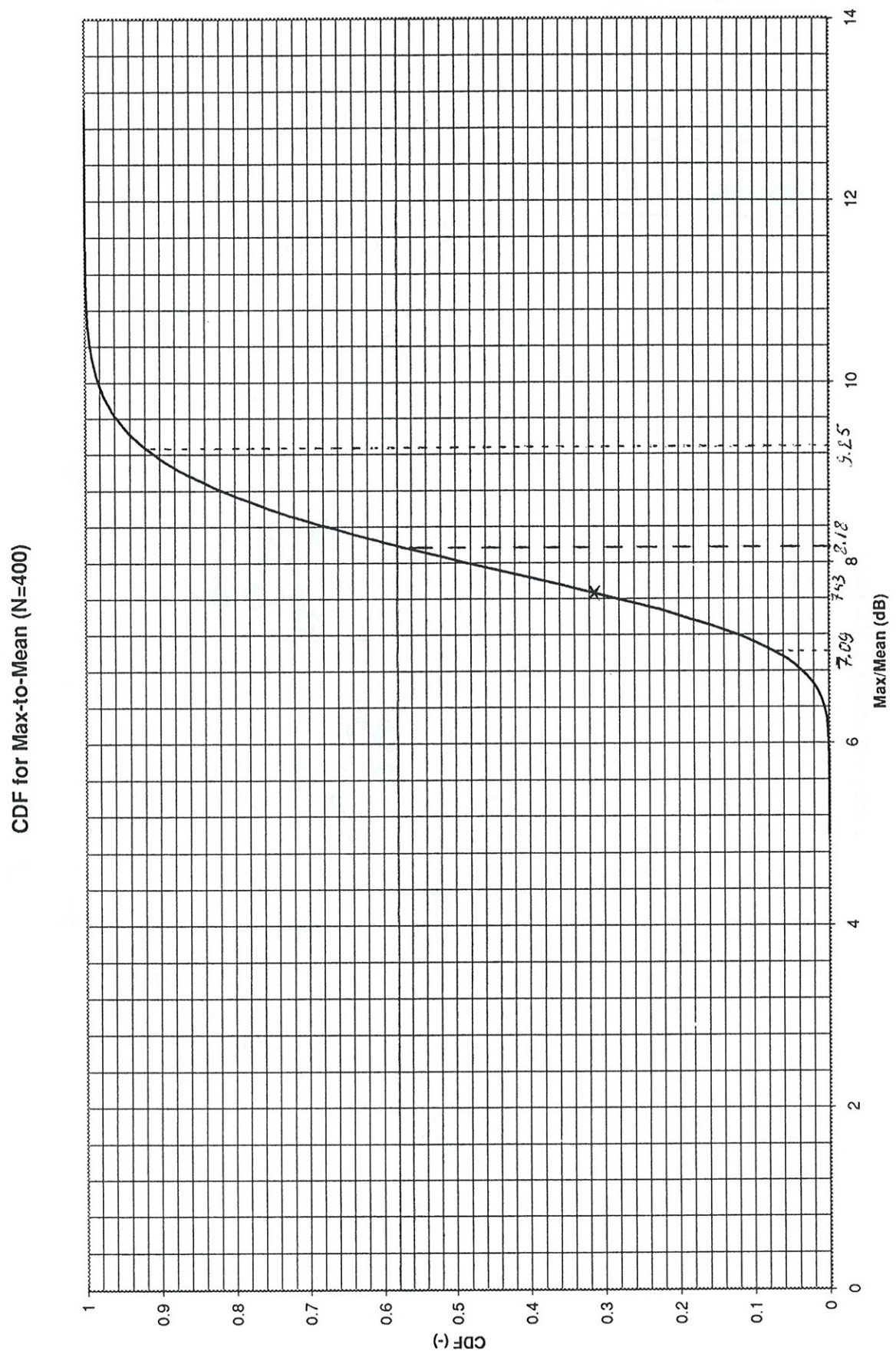


Fig 5.9

Cavity uniformity at 8.2 GHz

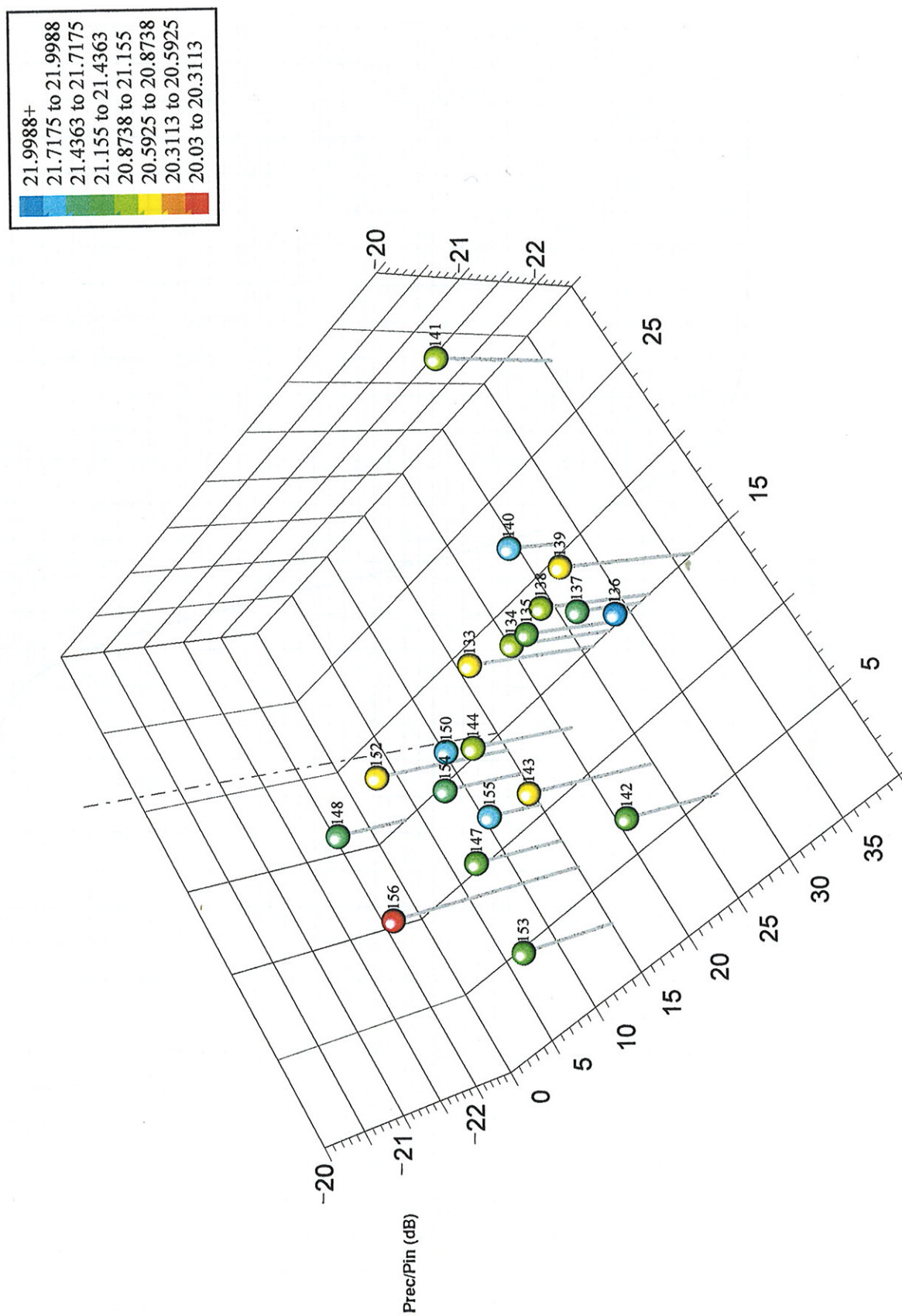


Fig 6.1

Spatial correlation function (Power CCF) for Files 133-139

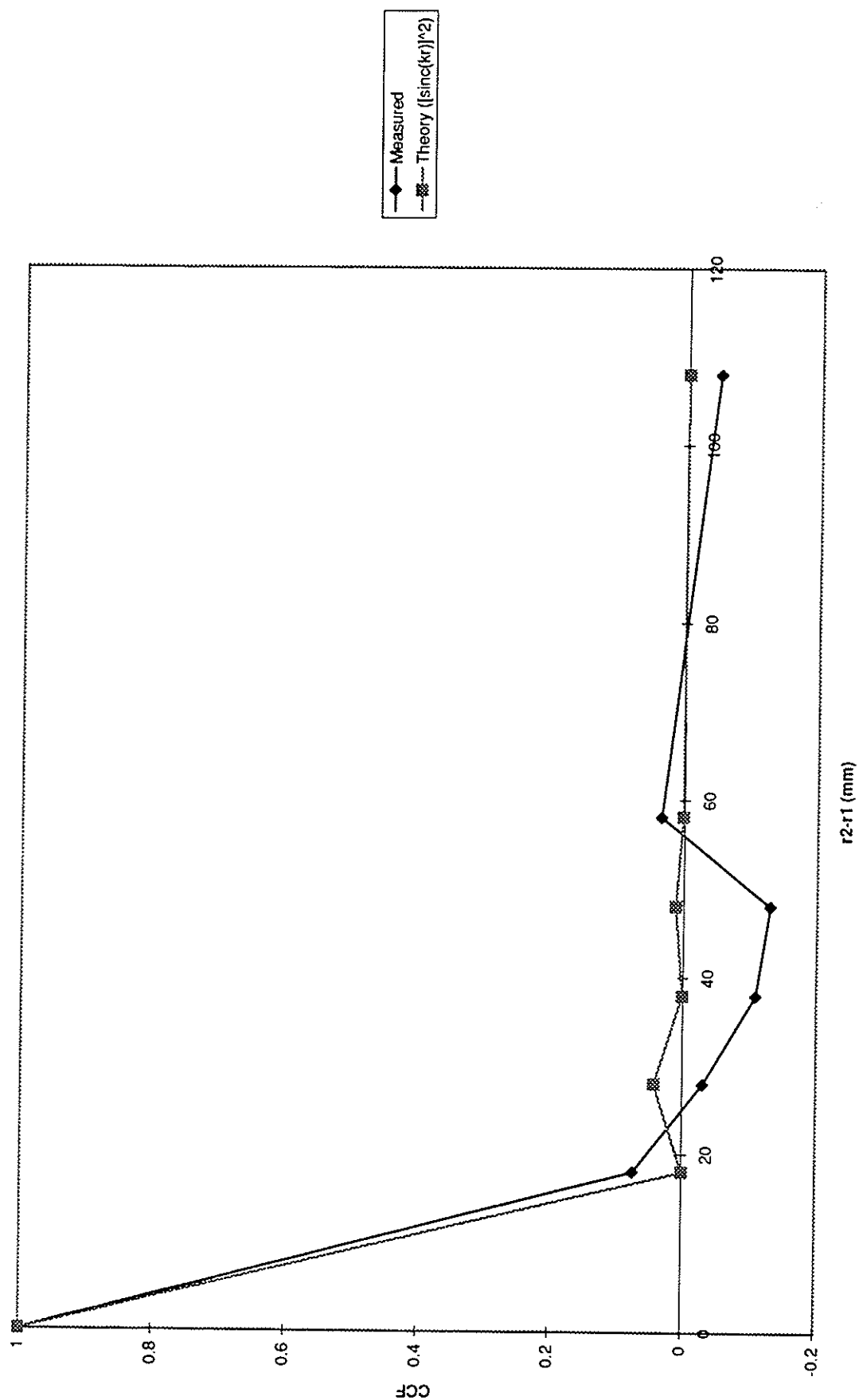


Fig 6.2

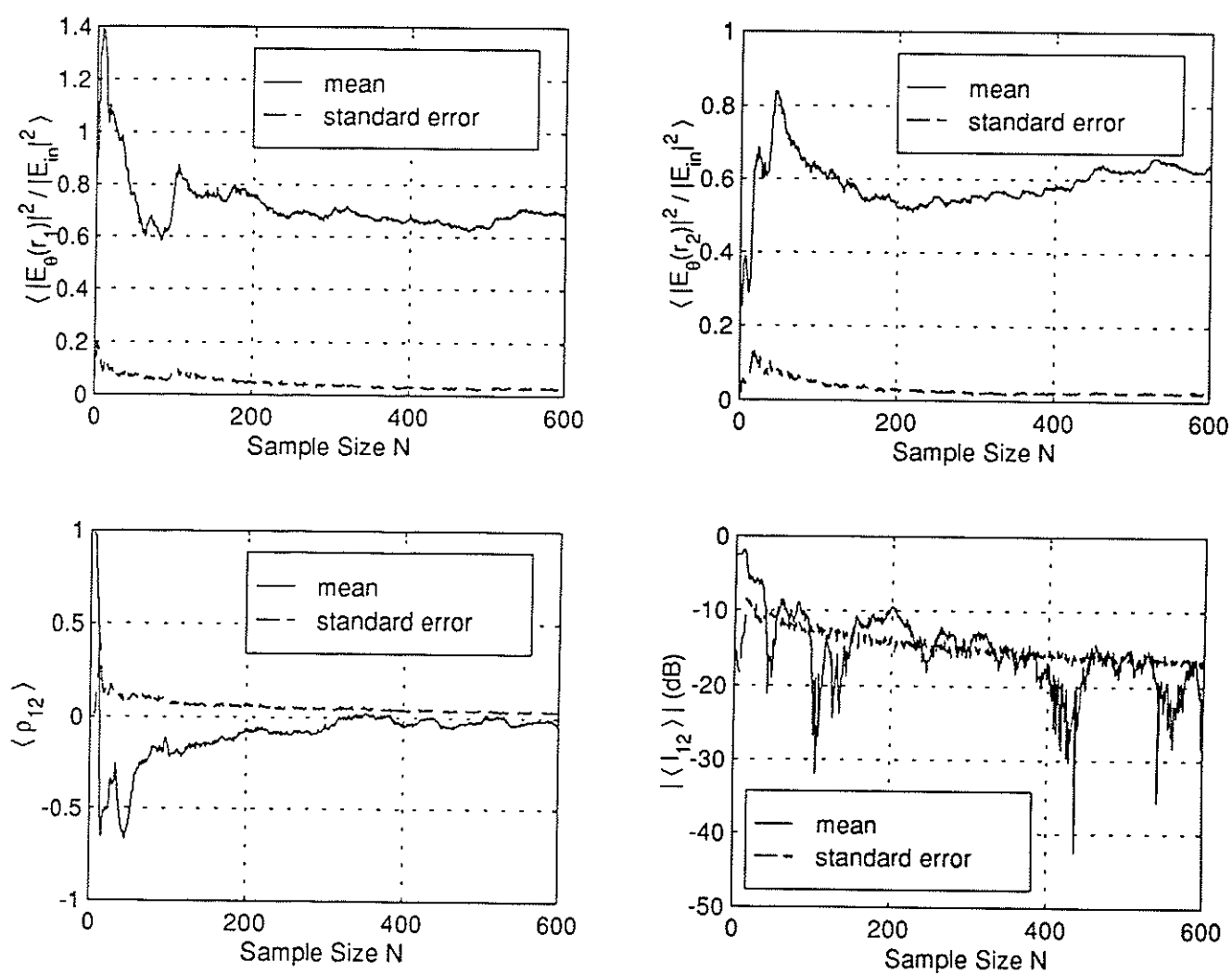


Fig 6.3

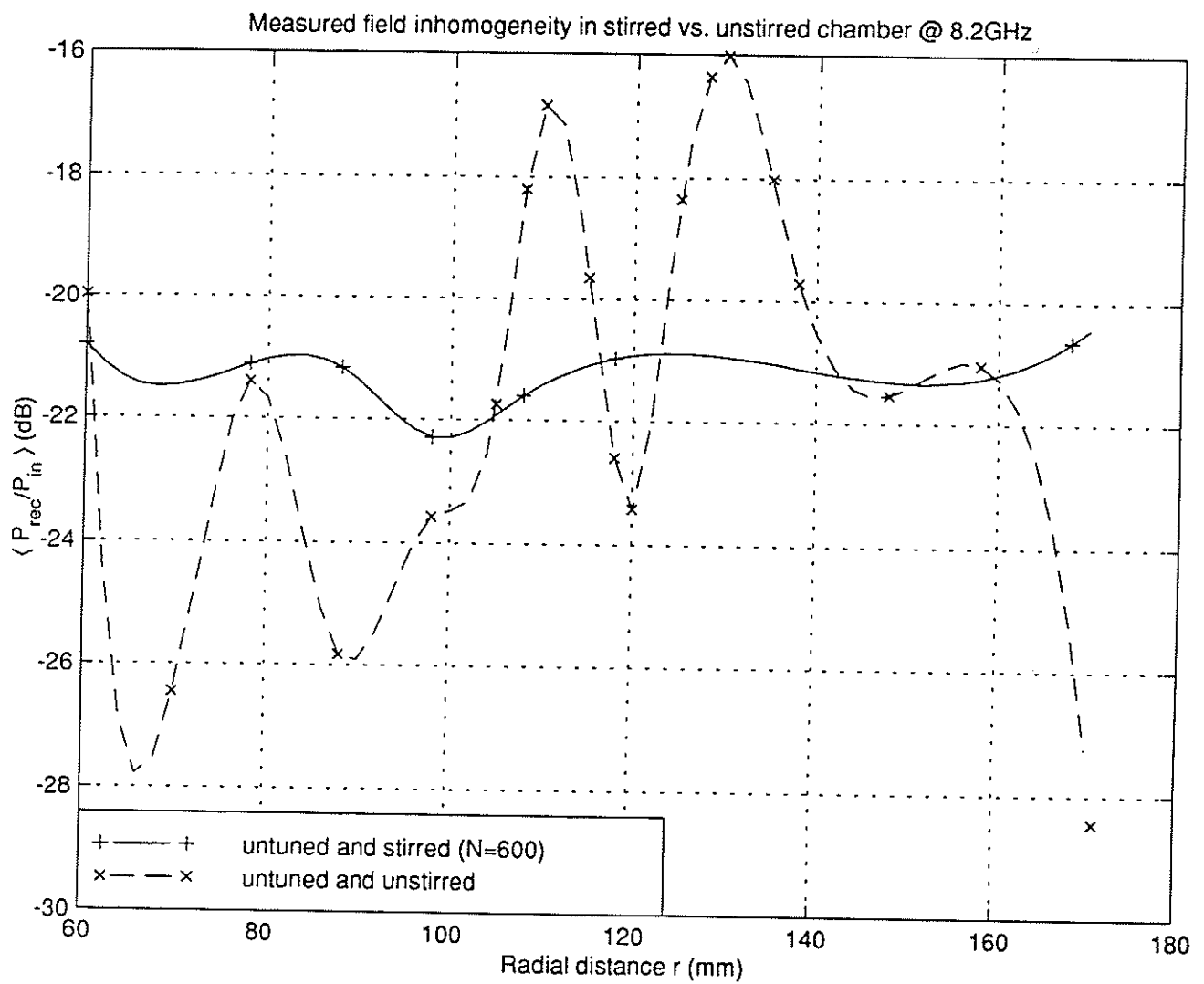


Fig 6.4

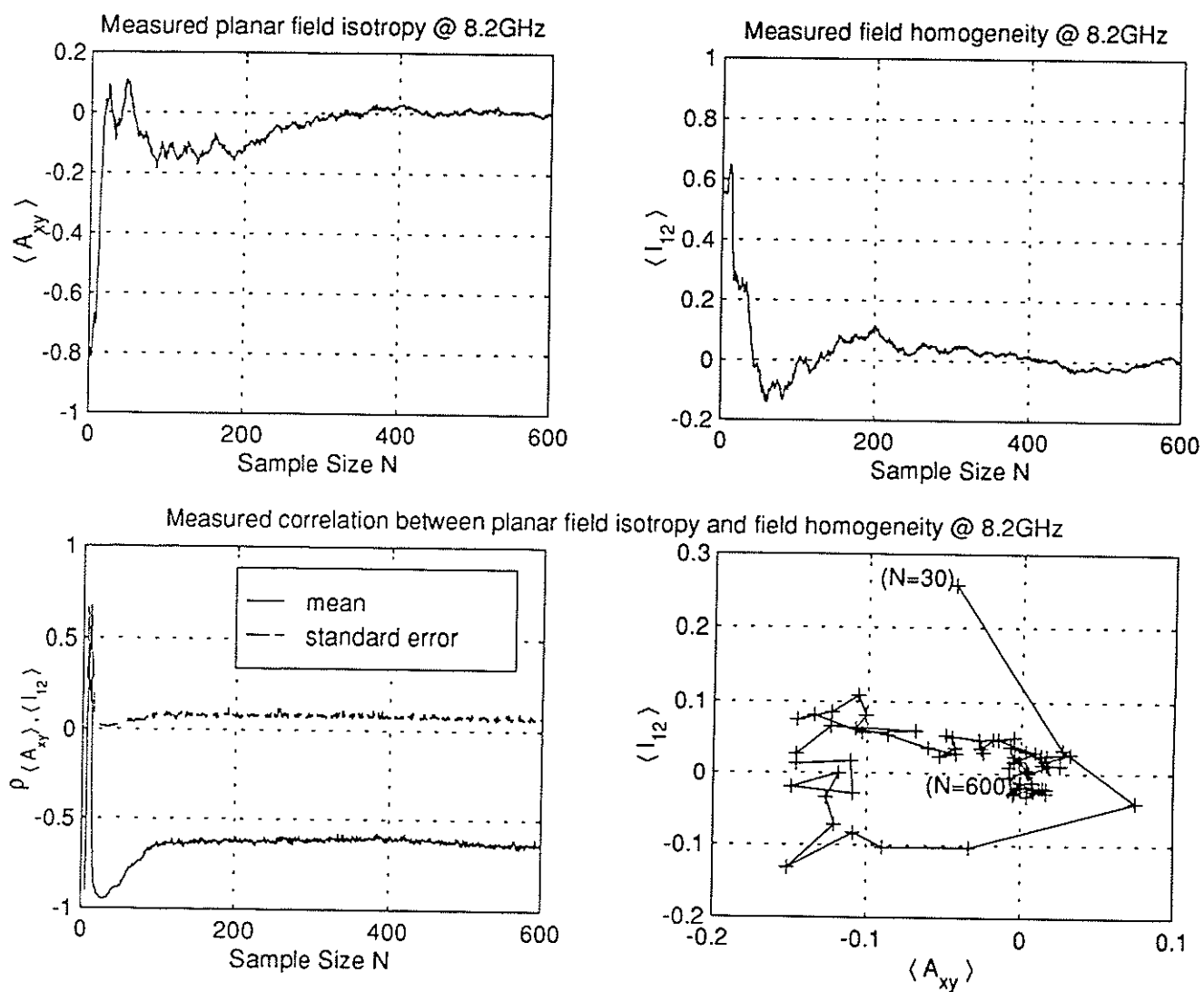


Fig 7.1

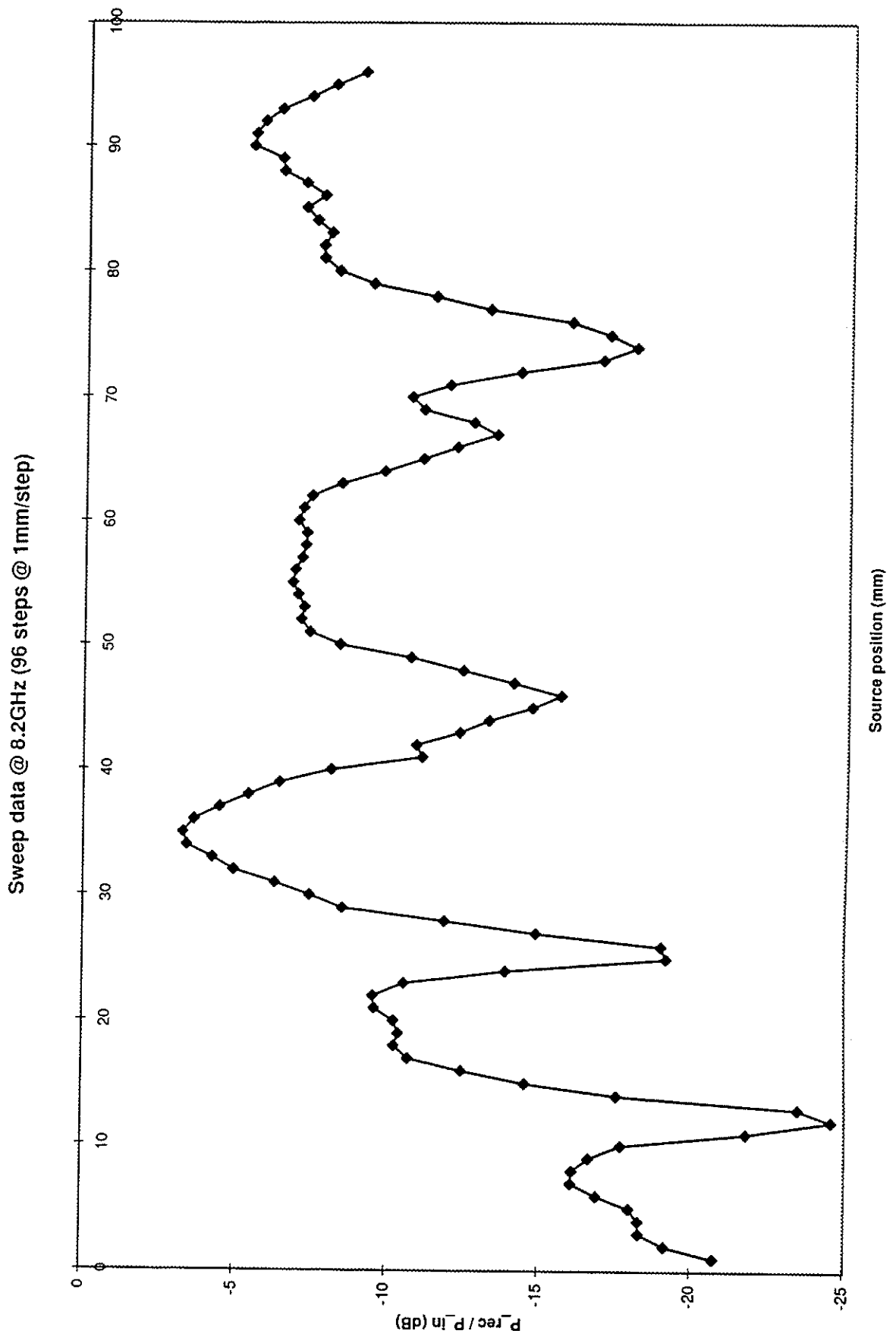


Fig 9.1

Cumulative distribution function @ 8.2GHz (96 source steps @ 1mm/step)

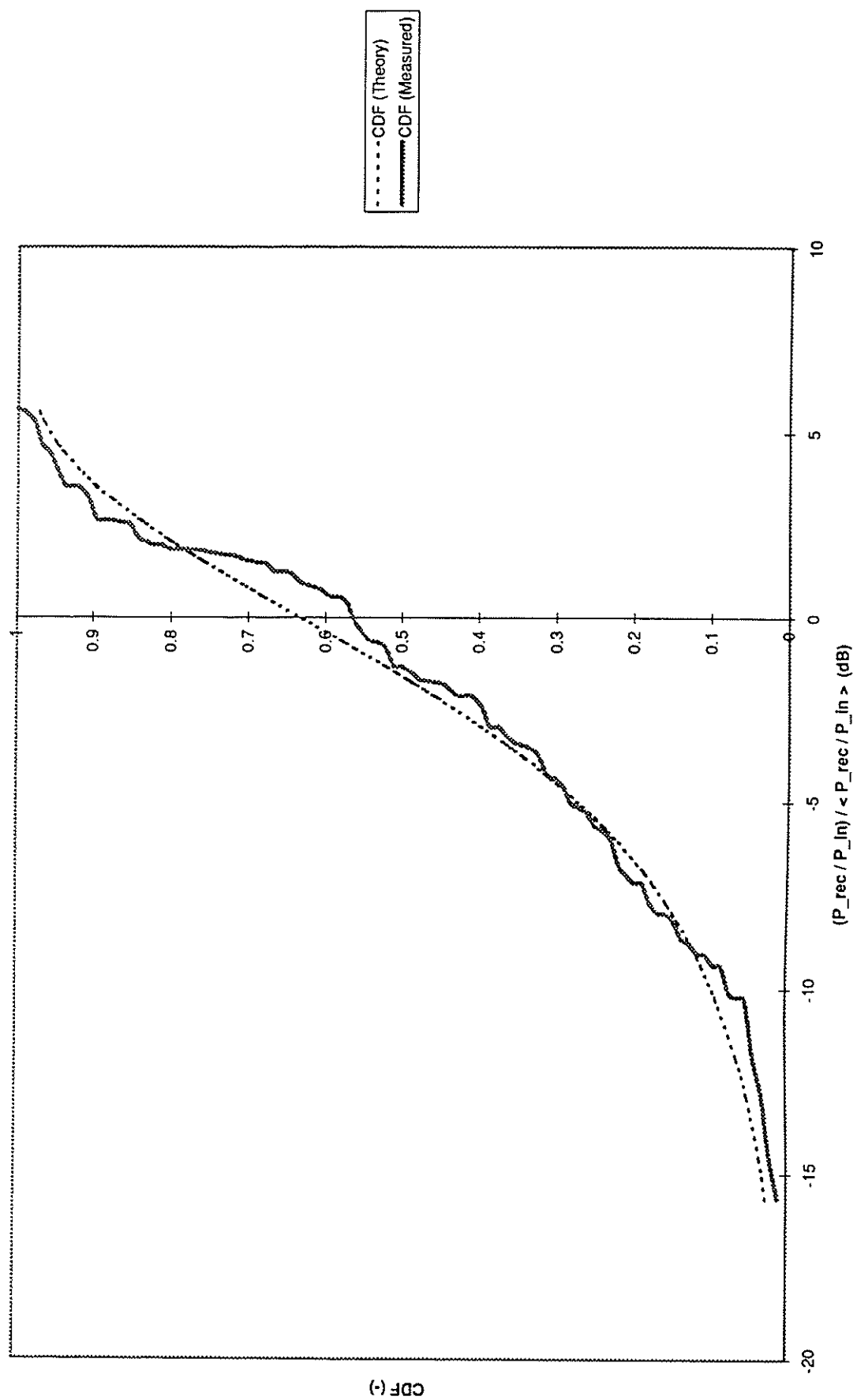


Fig 9.2

Autocorrelation function @ 8.2GHz (96 source steps @ 1mm/step)

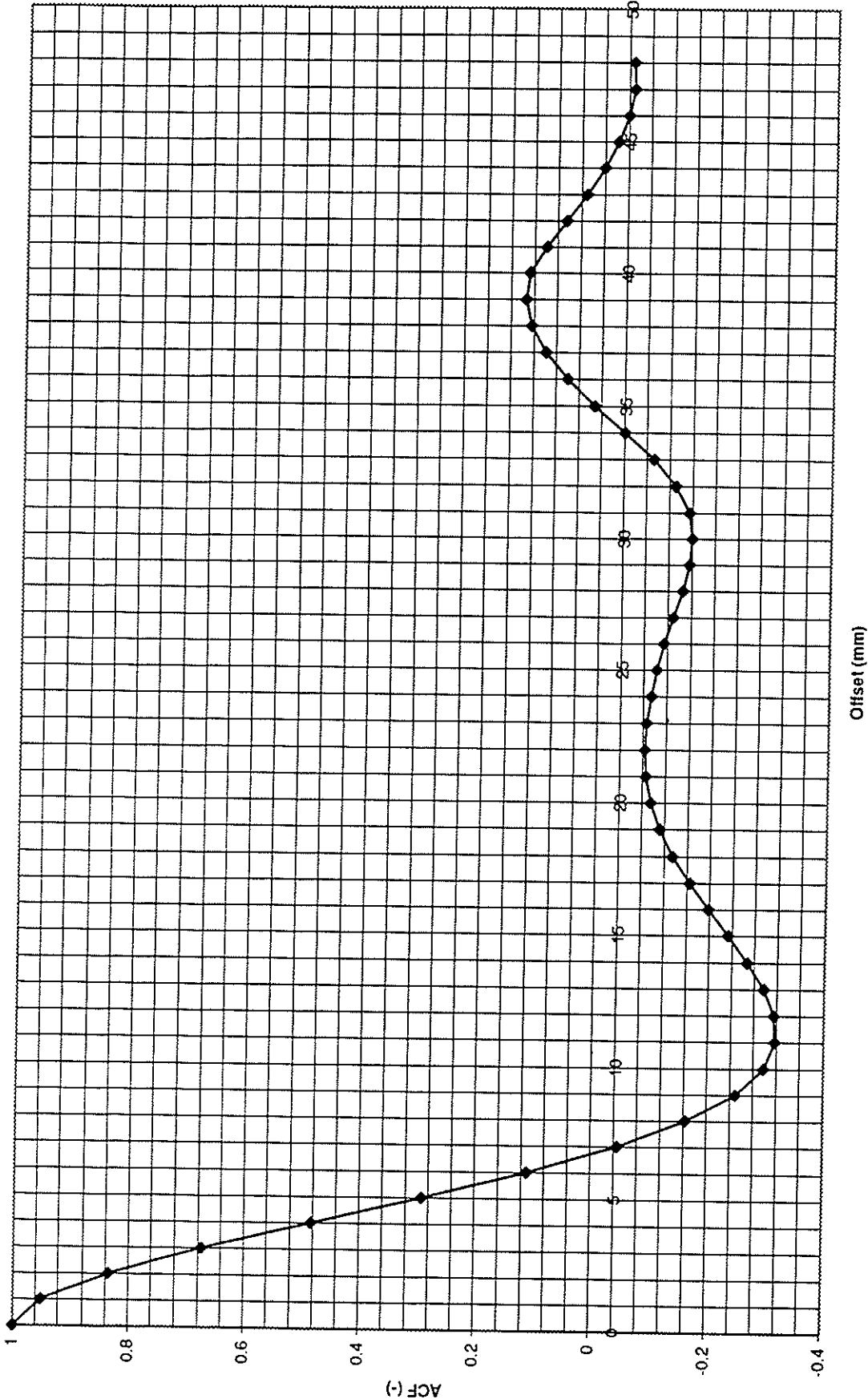


Fig 9.3

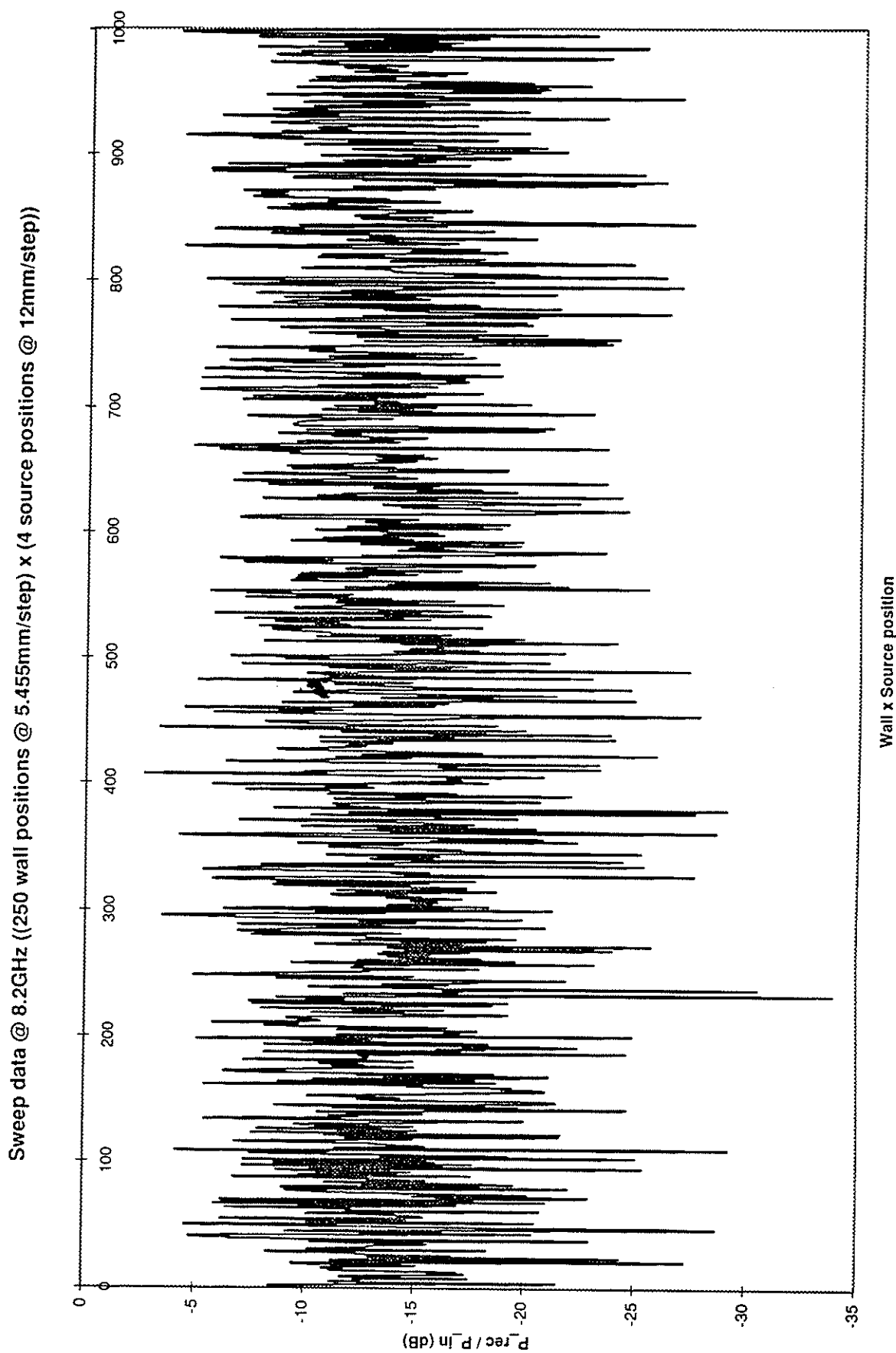


Fig 9.4

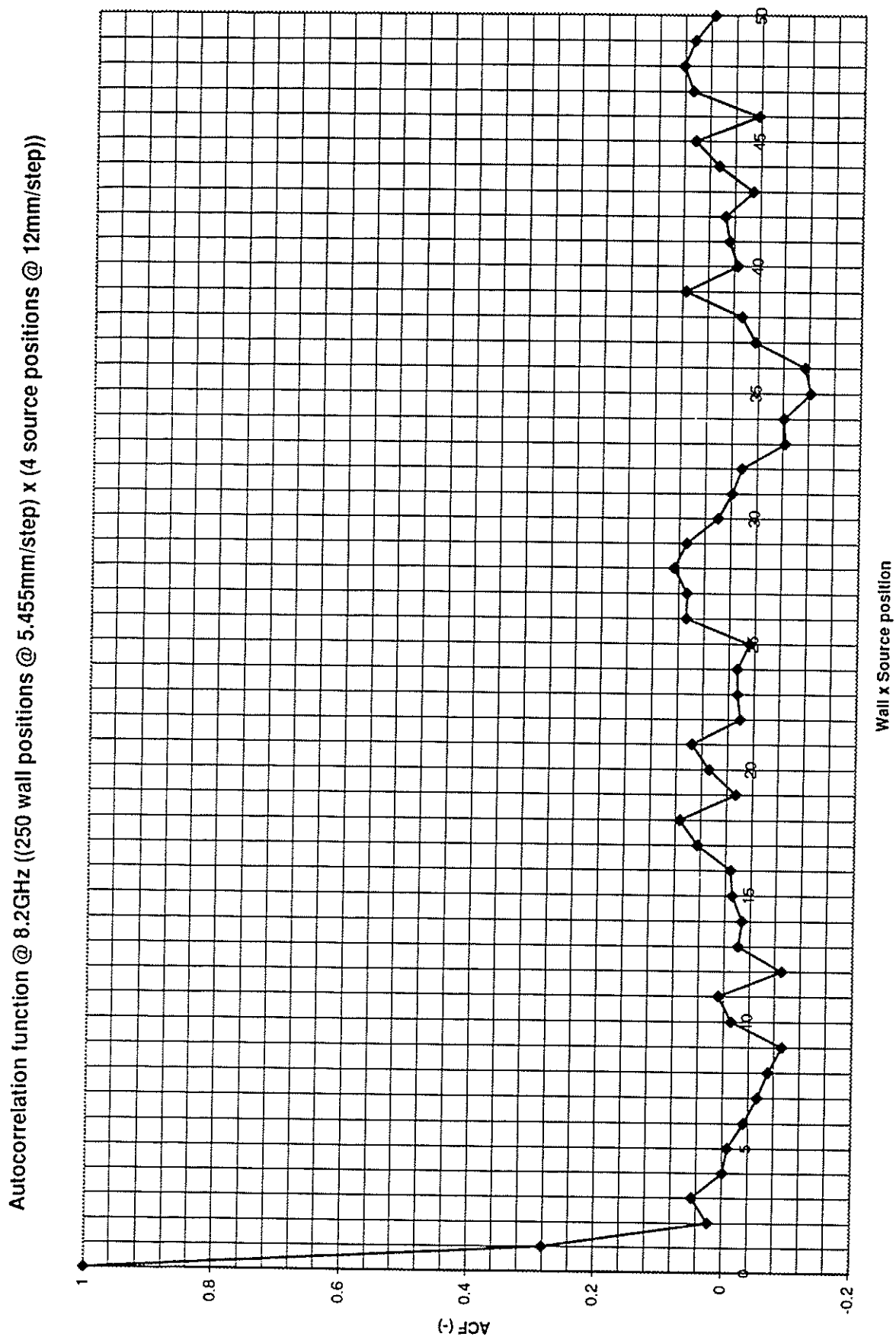


Fig 9.5

Cumulative distribution function @ 8.2GHz

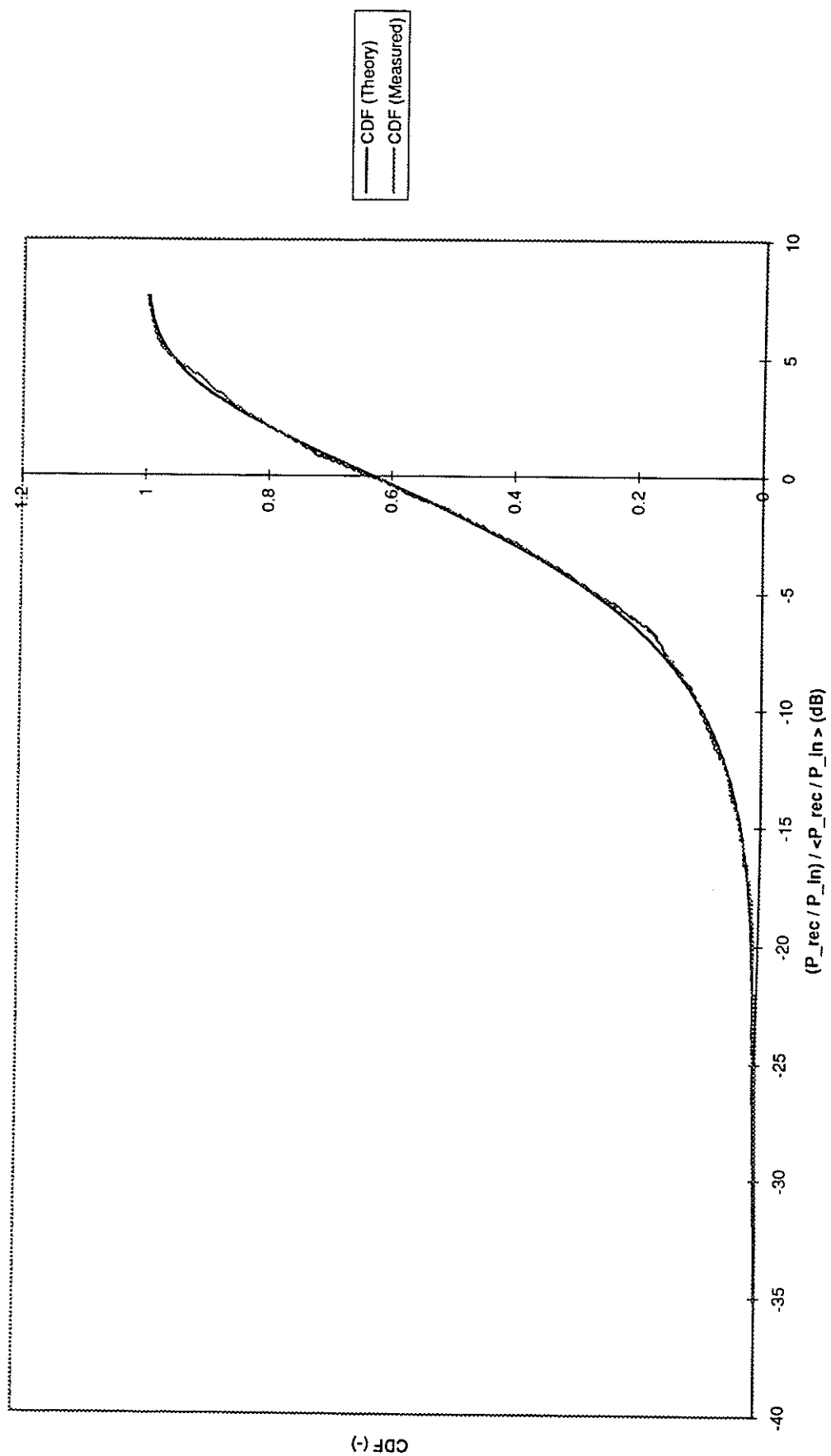


Fig 9.6

Cumulative distribution function @ 8.2GHz (50 wall positions @ 27.27mm/step) x (4 source positions @
r=26mm, 49mm, 76mm, 83mm)

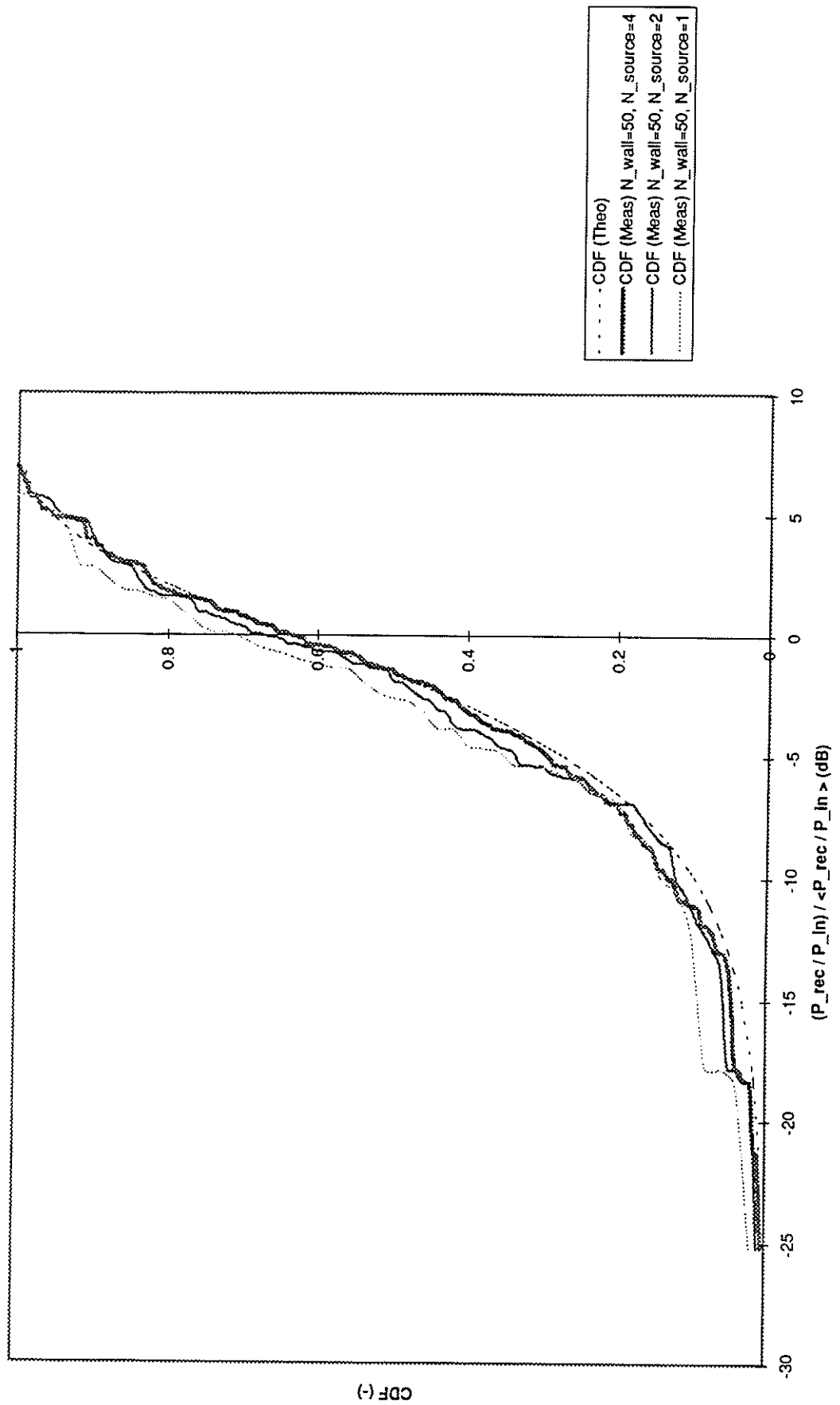


Fig 9.7

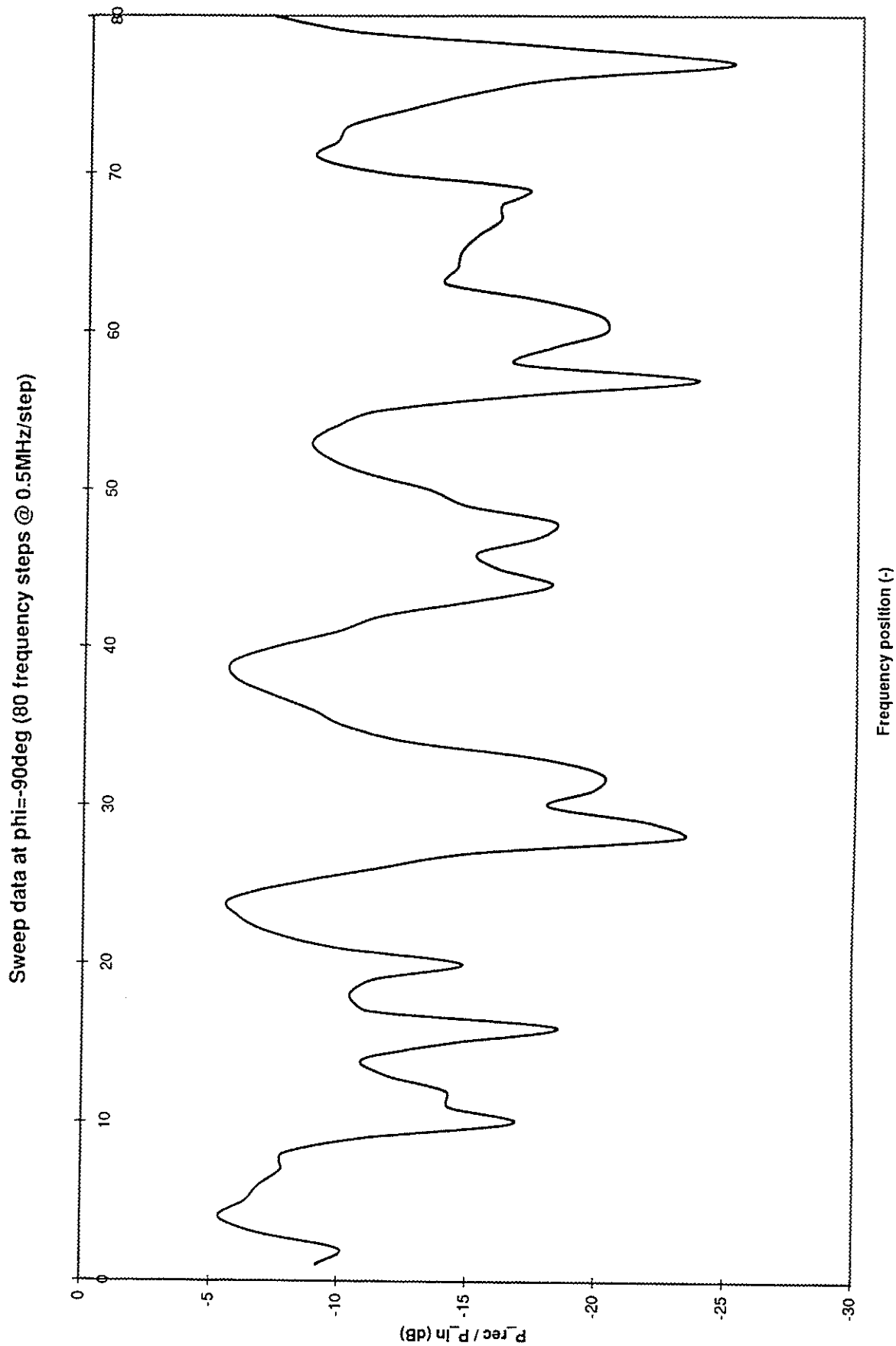


Fig 9.8

Autocorrelation function at $\phi = -90^\circ$ (80 frequency steps @ 0.5MHz/step)

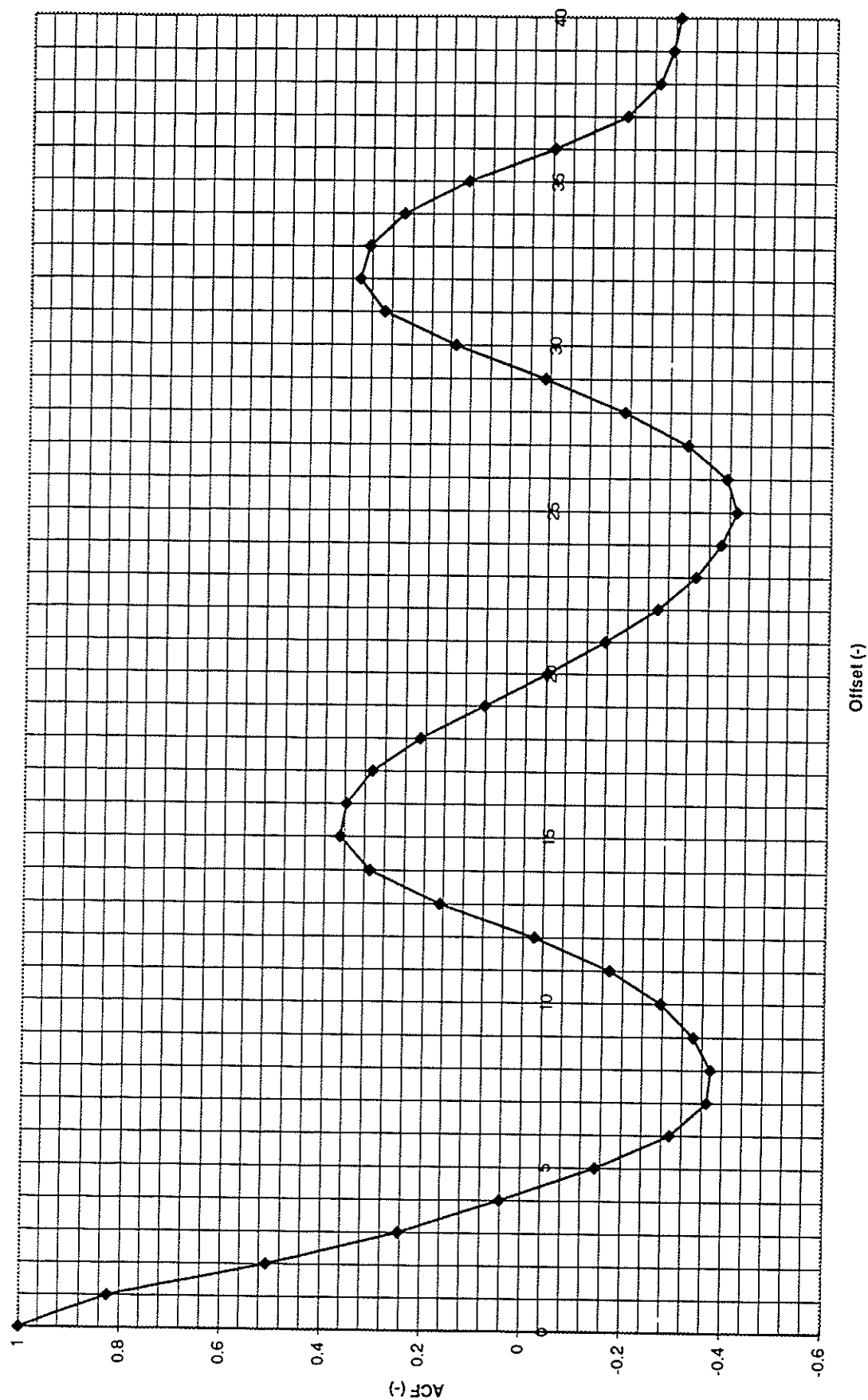


Fig 9.9

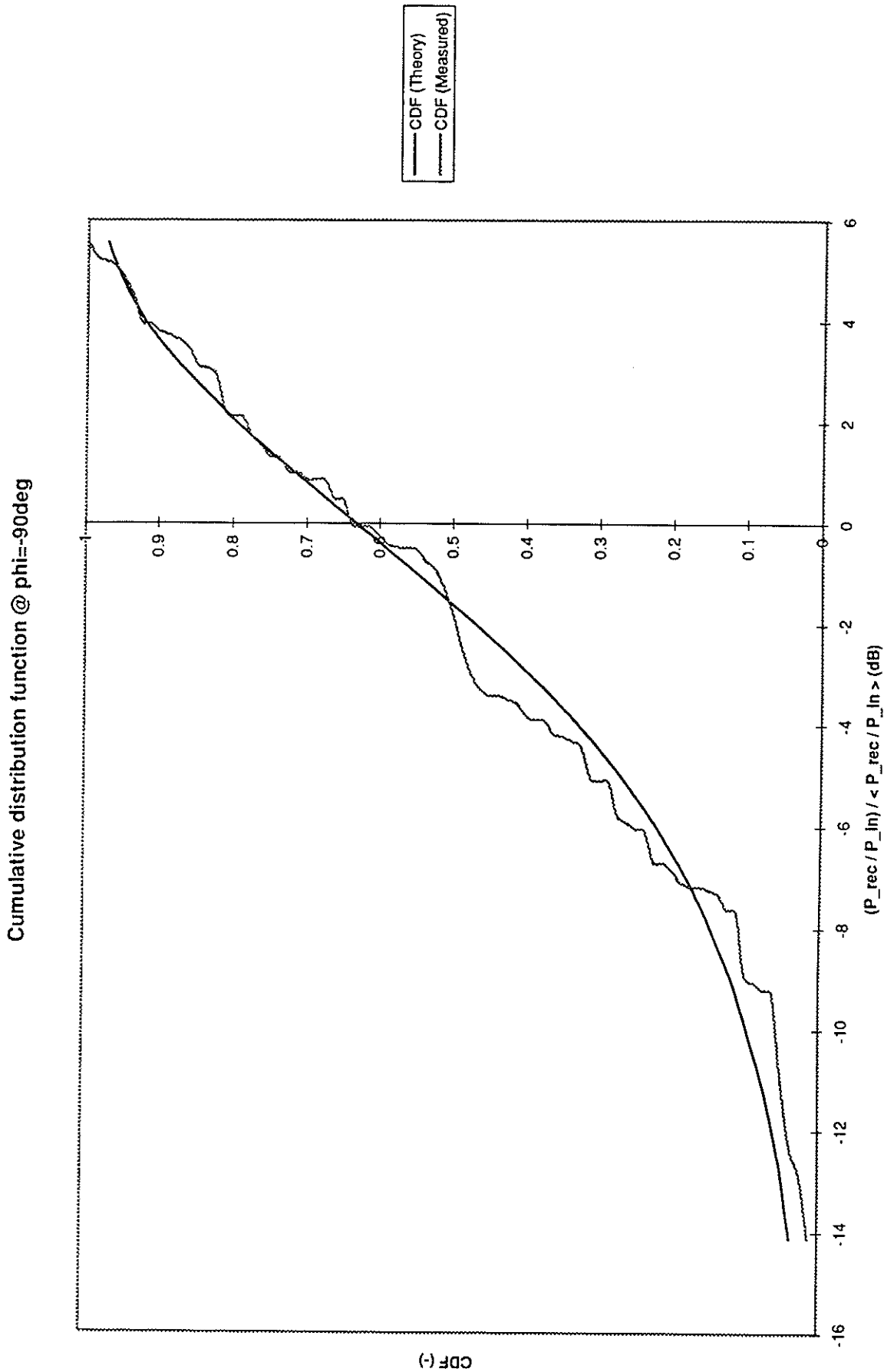


Fig 9.10

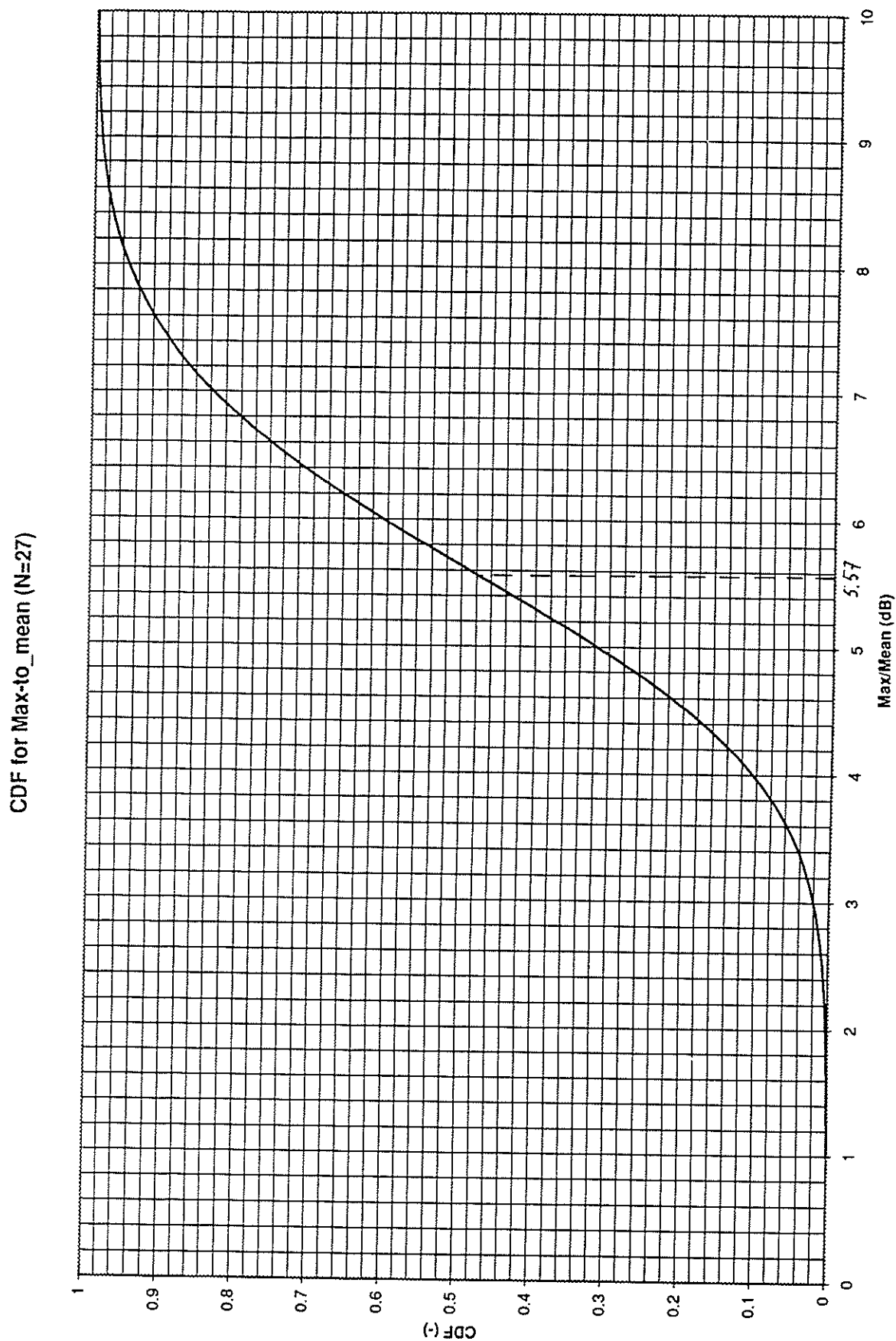


Fig 9.11

Theoretical Qc, Qwall, Qant, Qtot for spherical aluminium chamber (R=350mm)

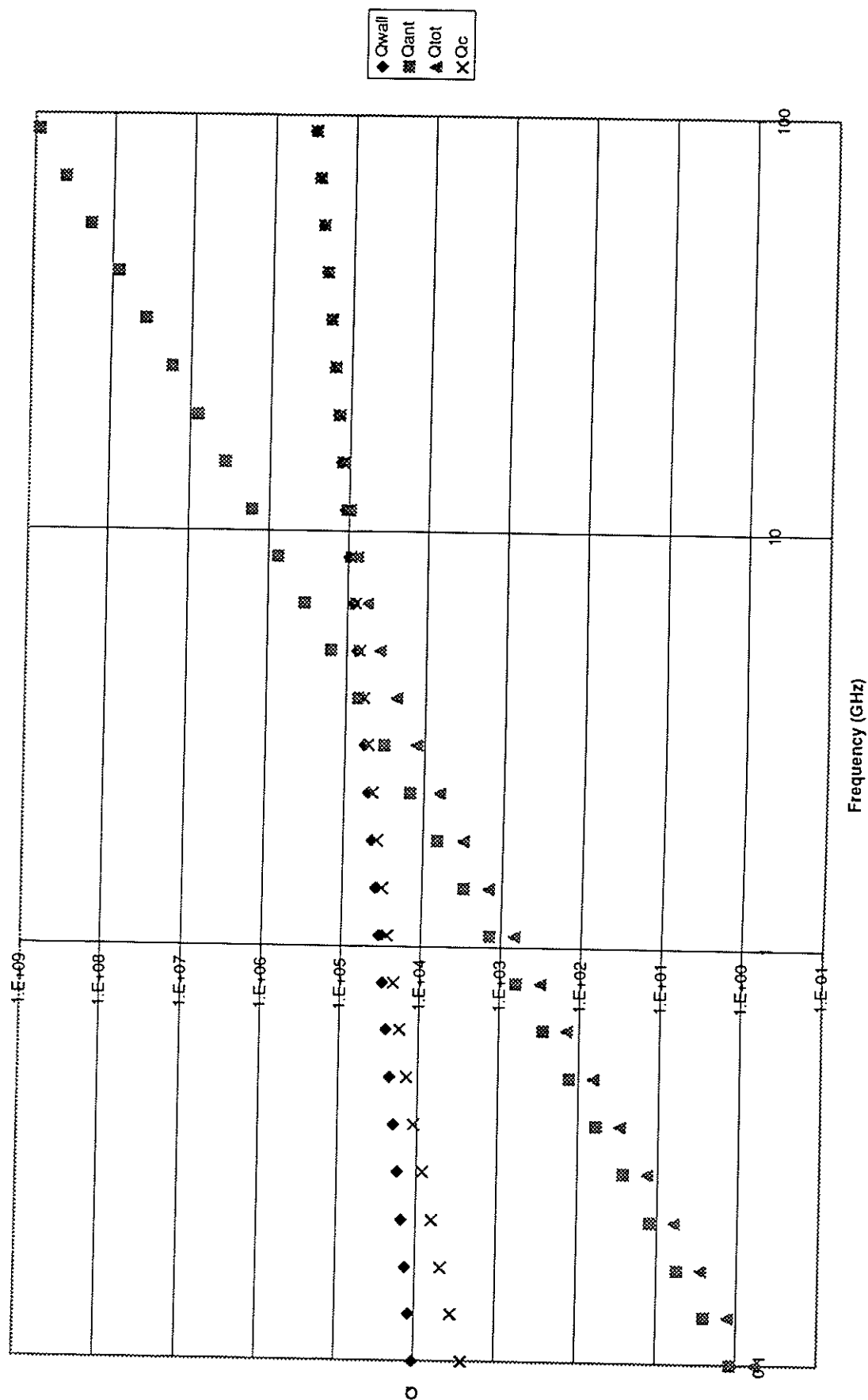


Fig 10.1

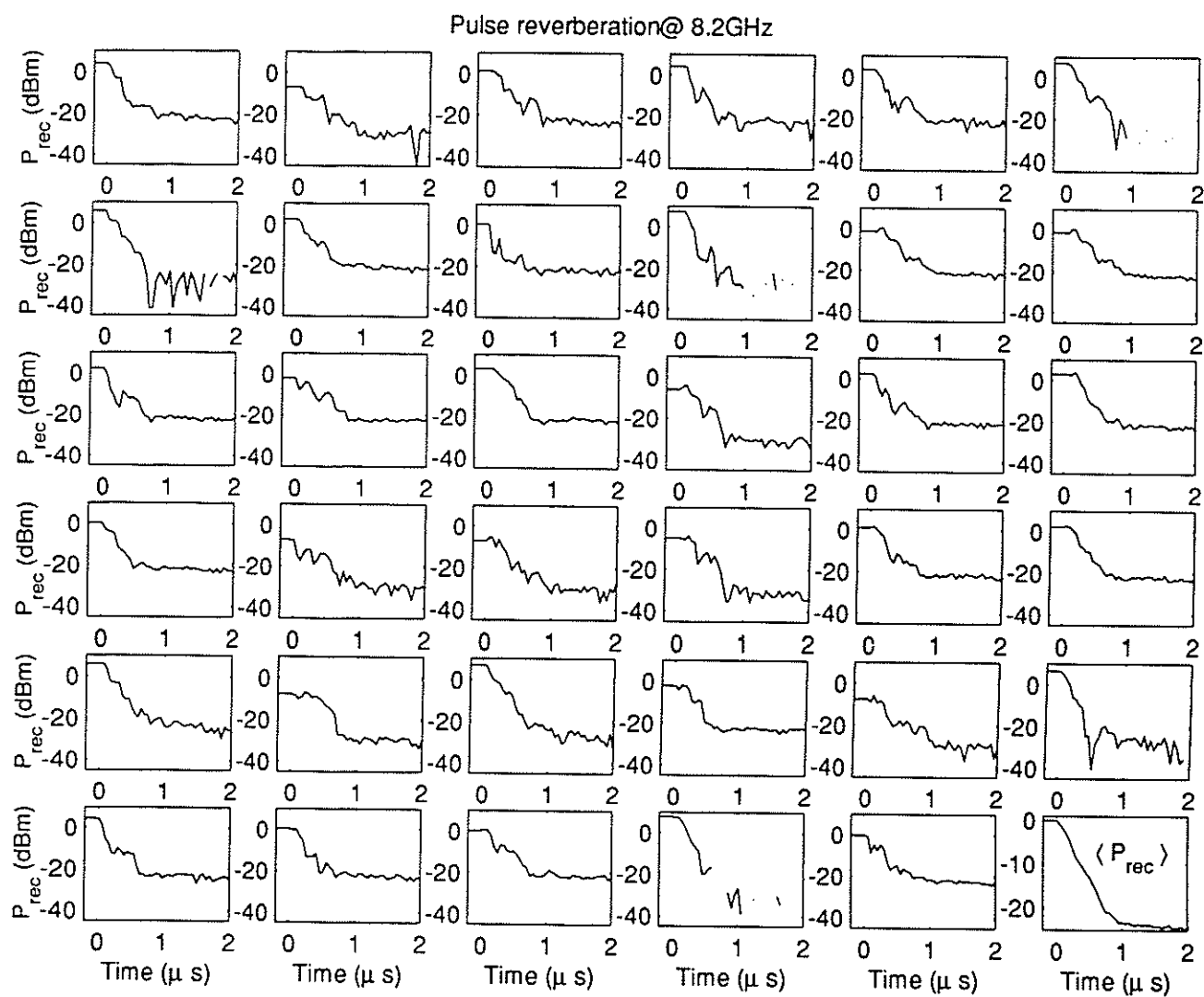


Fig 10.2

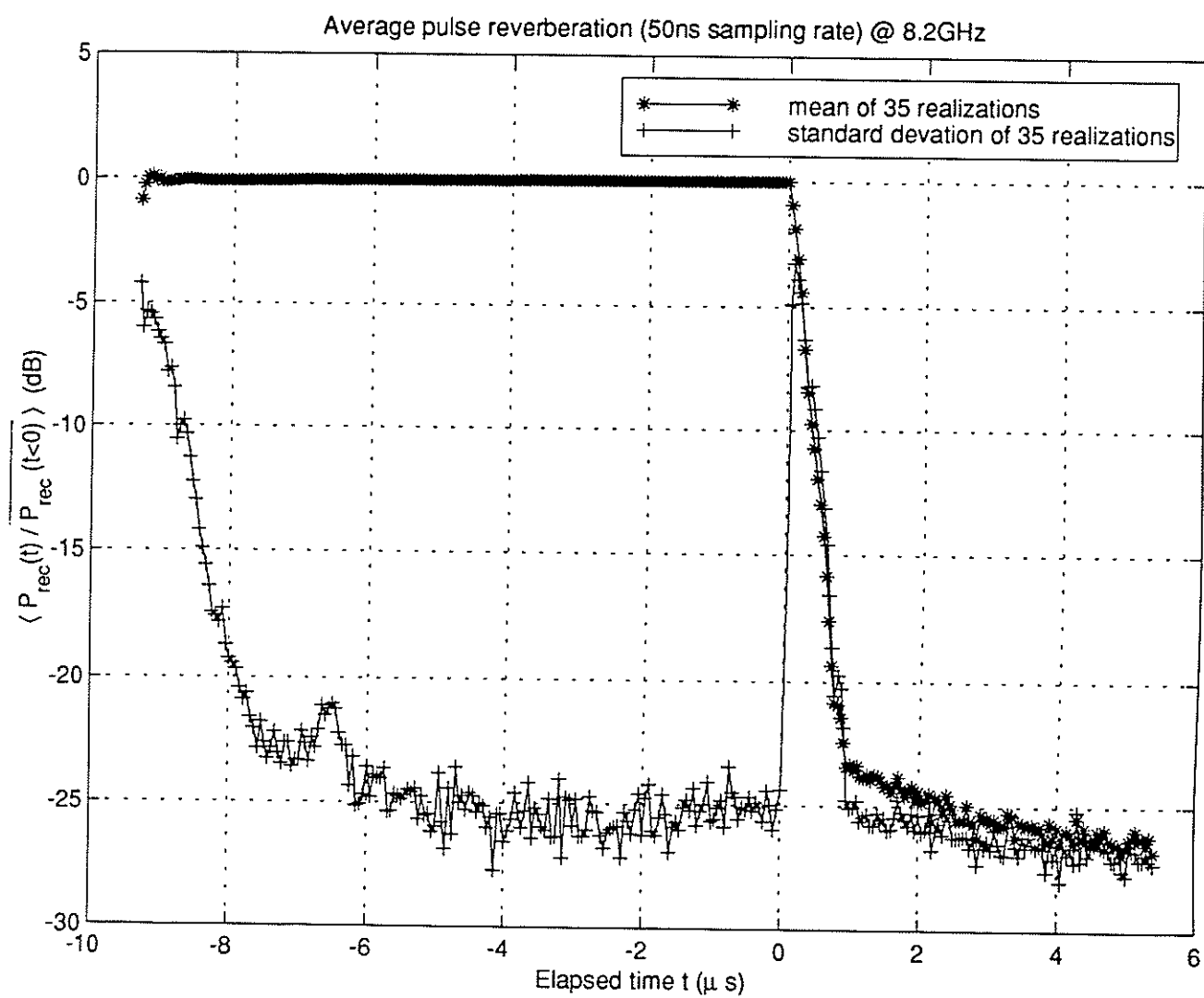


Fig 10.3

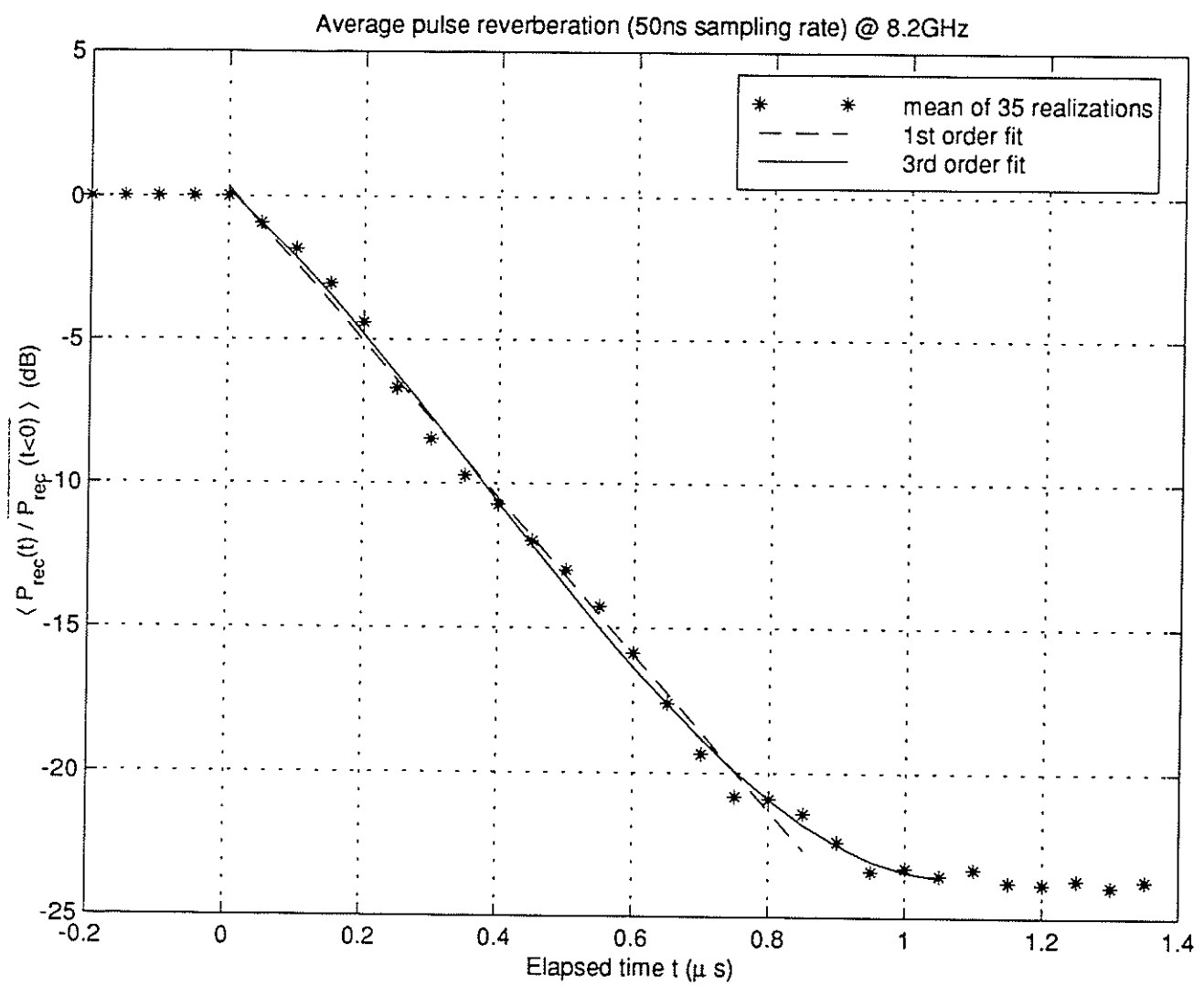
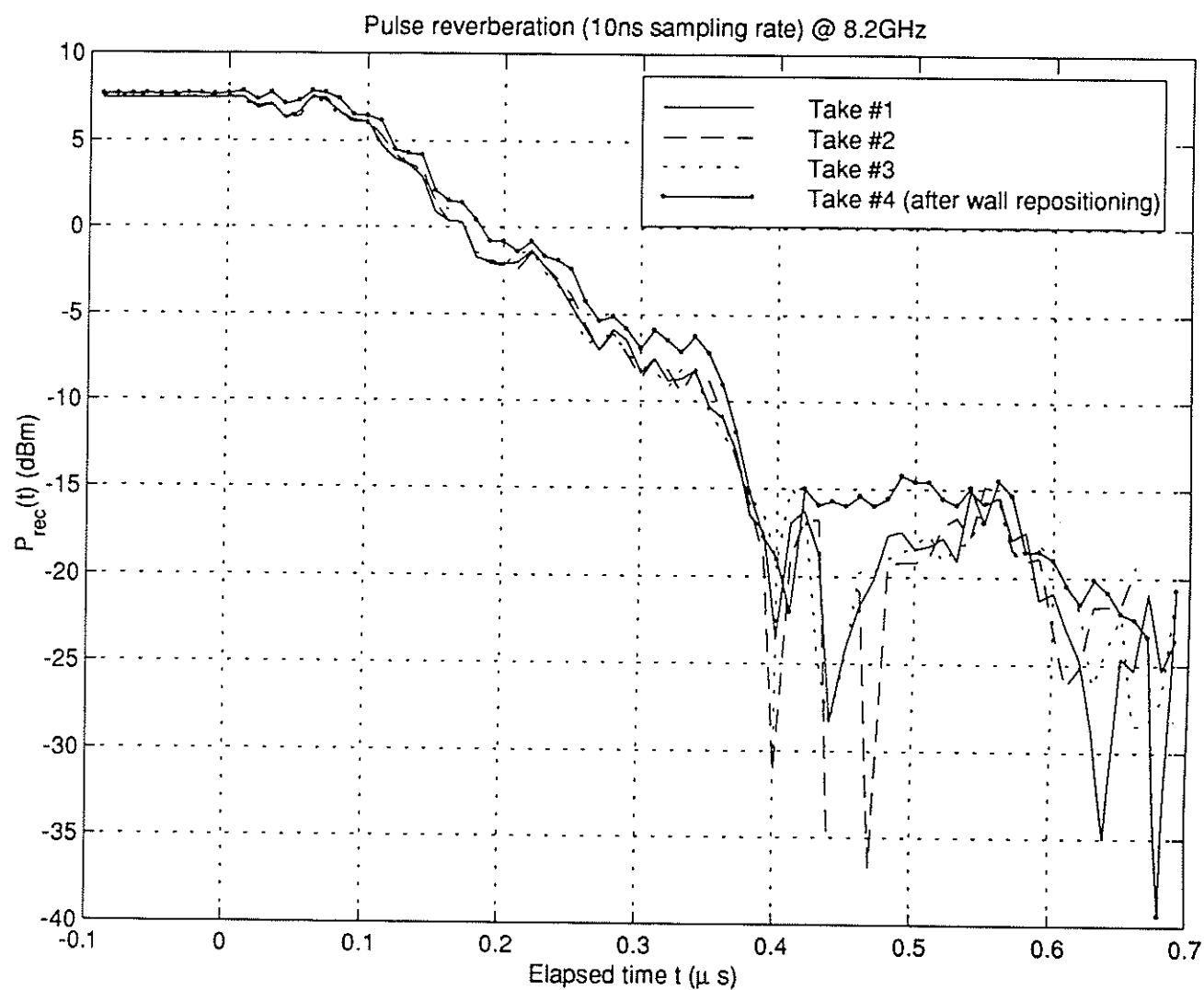
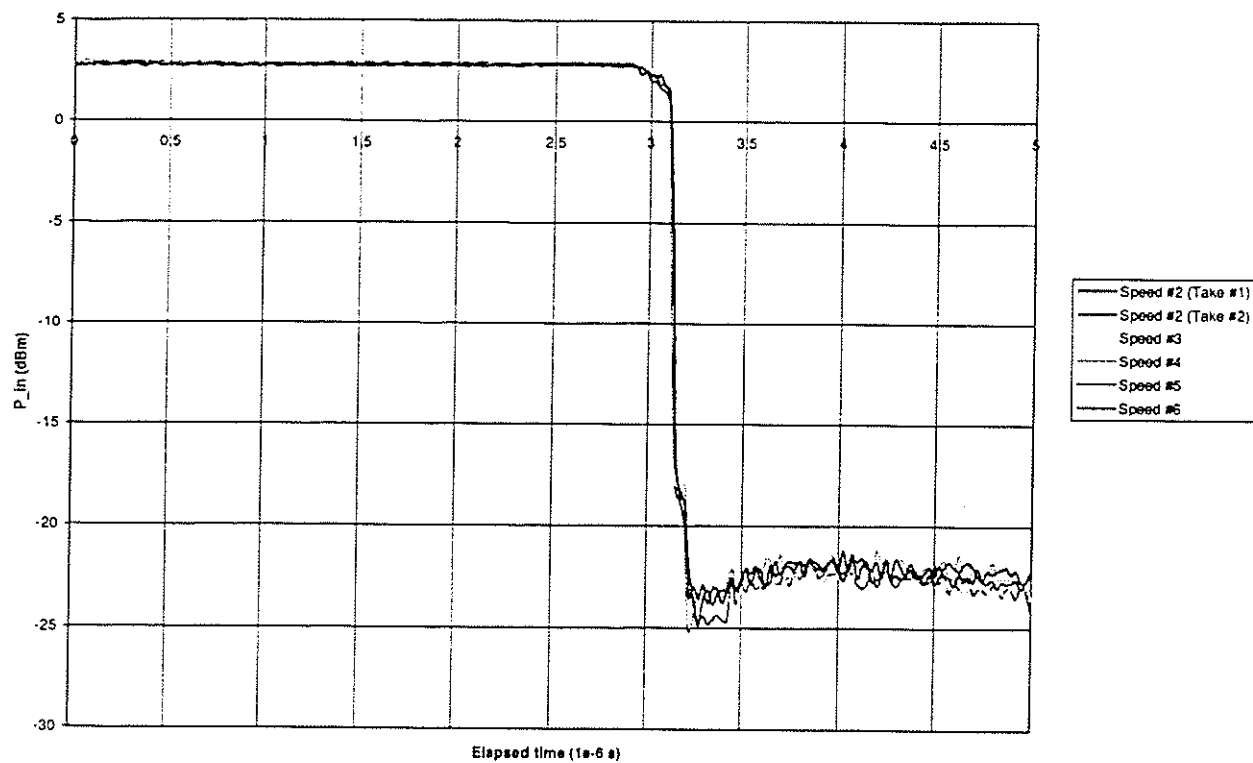


Fig 10.4

**Fig 10.5**

Input pulses for mode stirring @ 8.2GHz



Pulse reverberation for mode stirring @ 8.2GHz

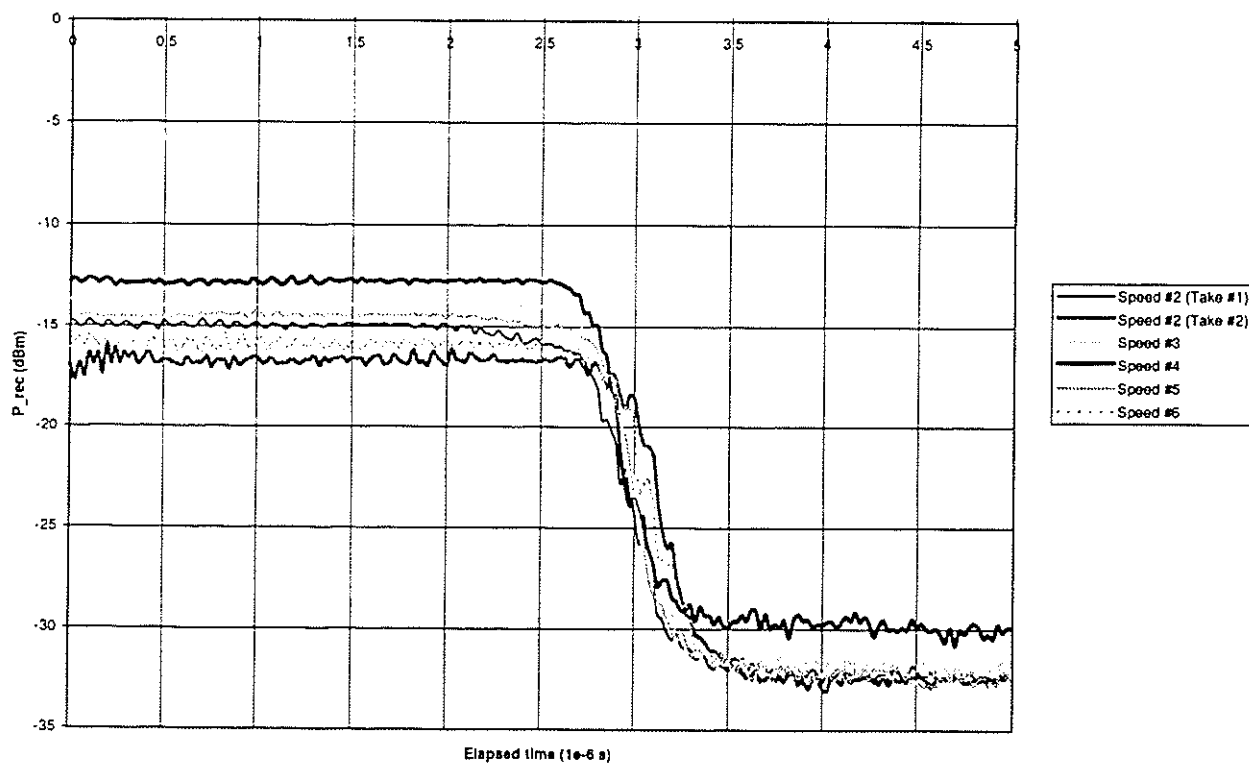


Fig 10.6

Speed averaged pulse reverberation for mode stirring @ 8.2GHz

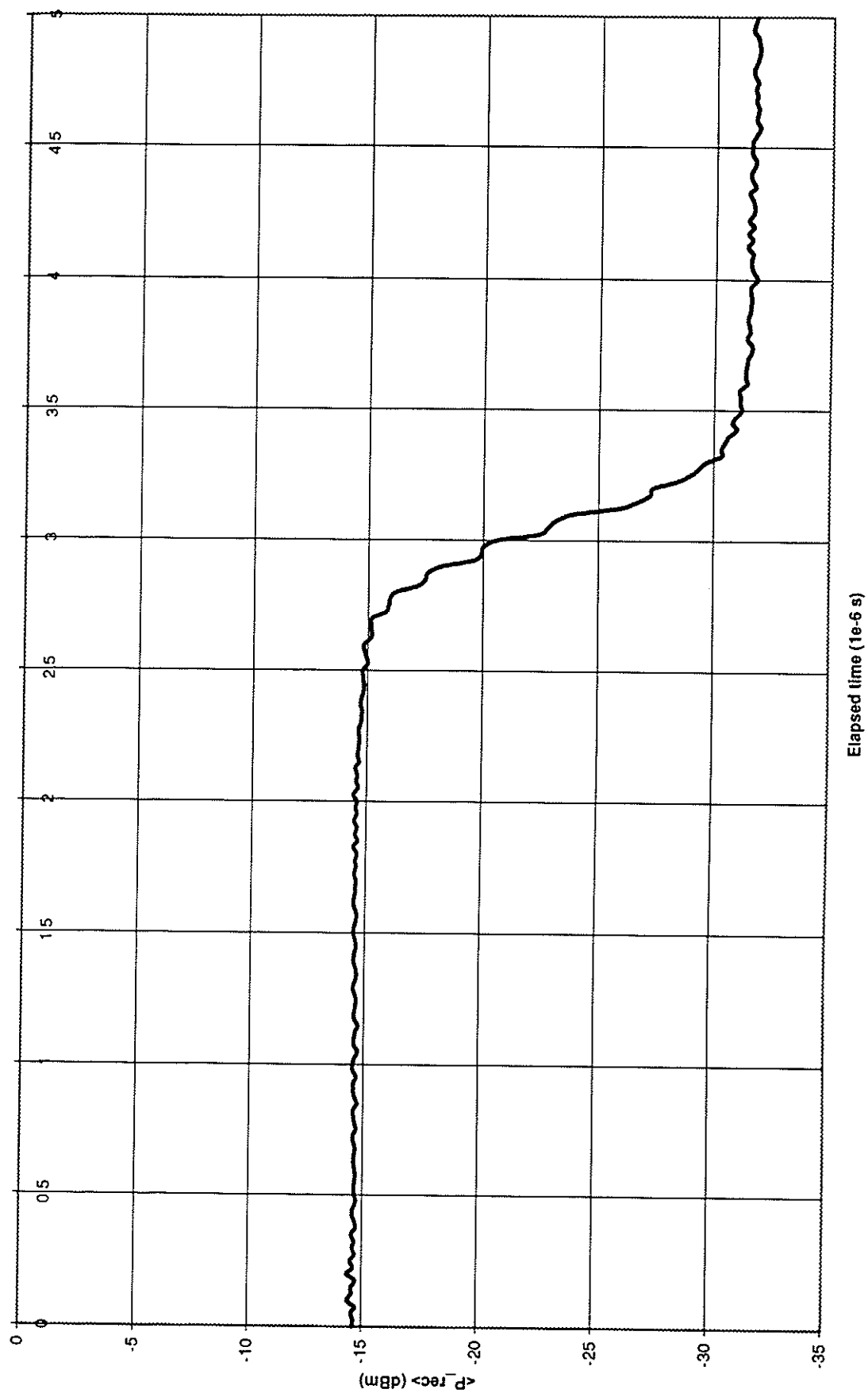


Fig 10.7

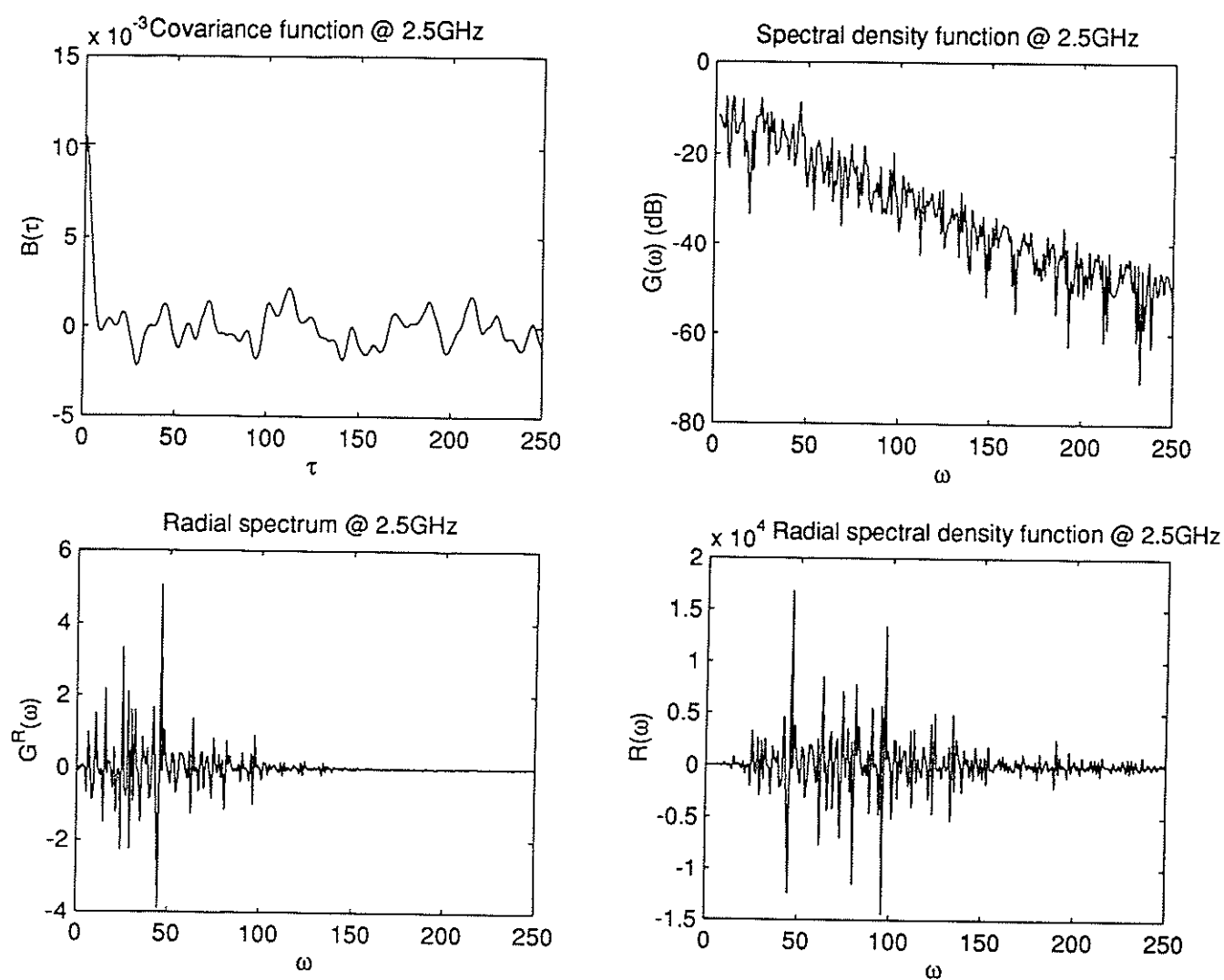
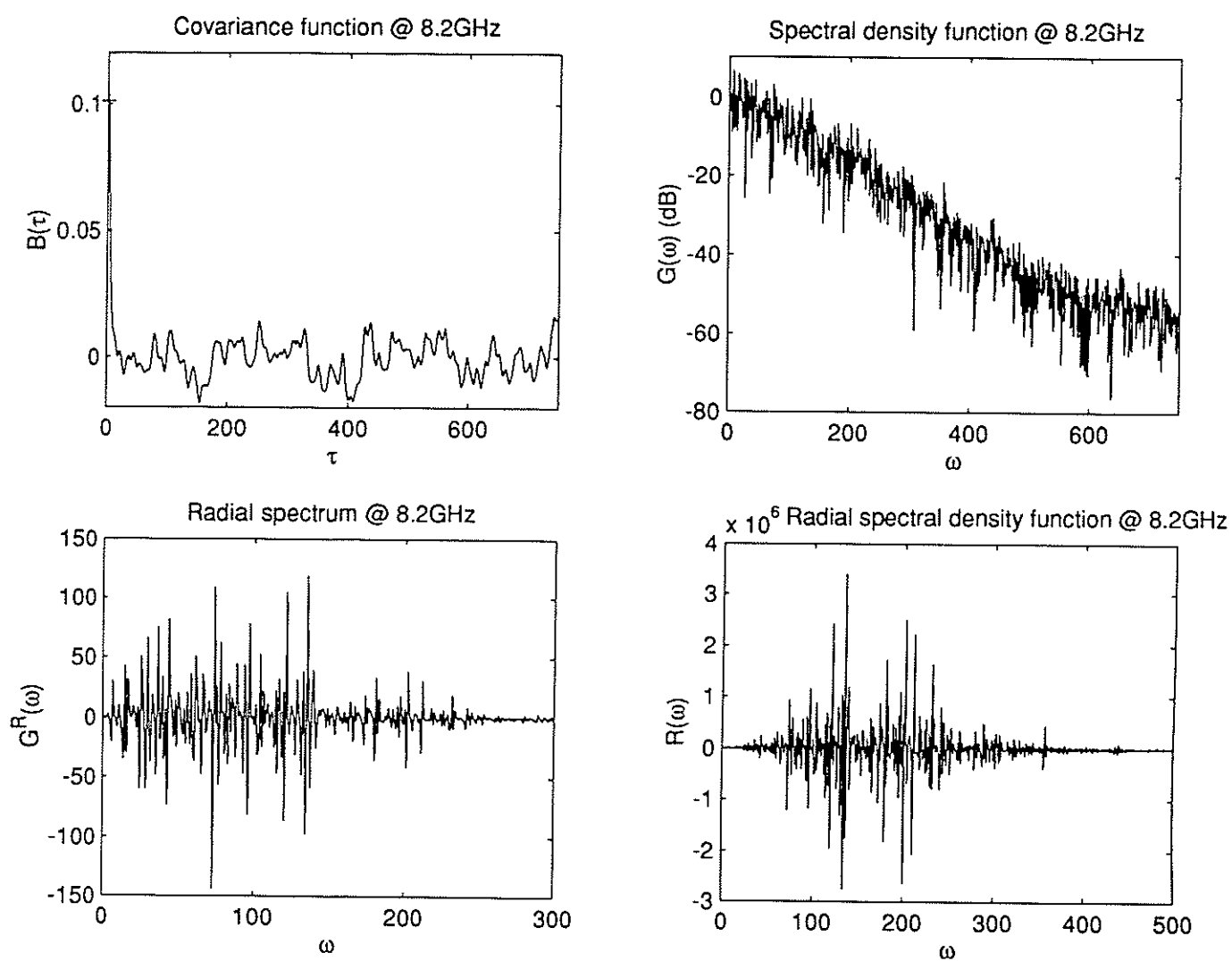


Fig 12.1

**Fig 12.2**

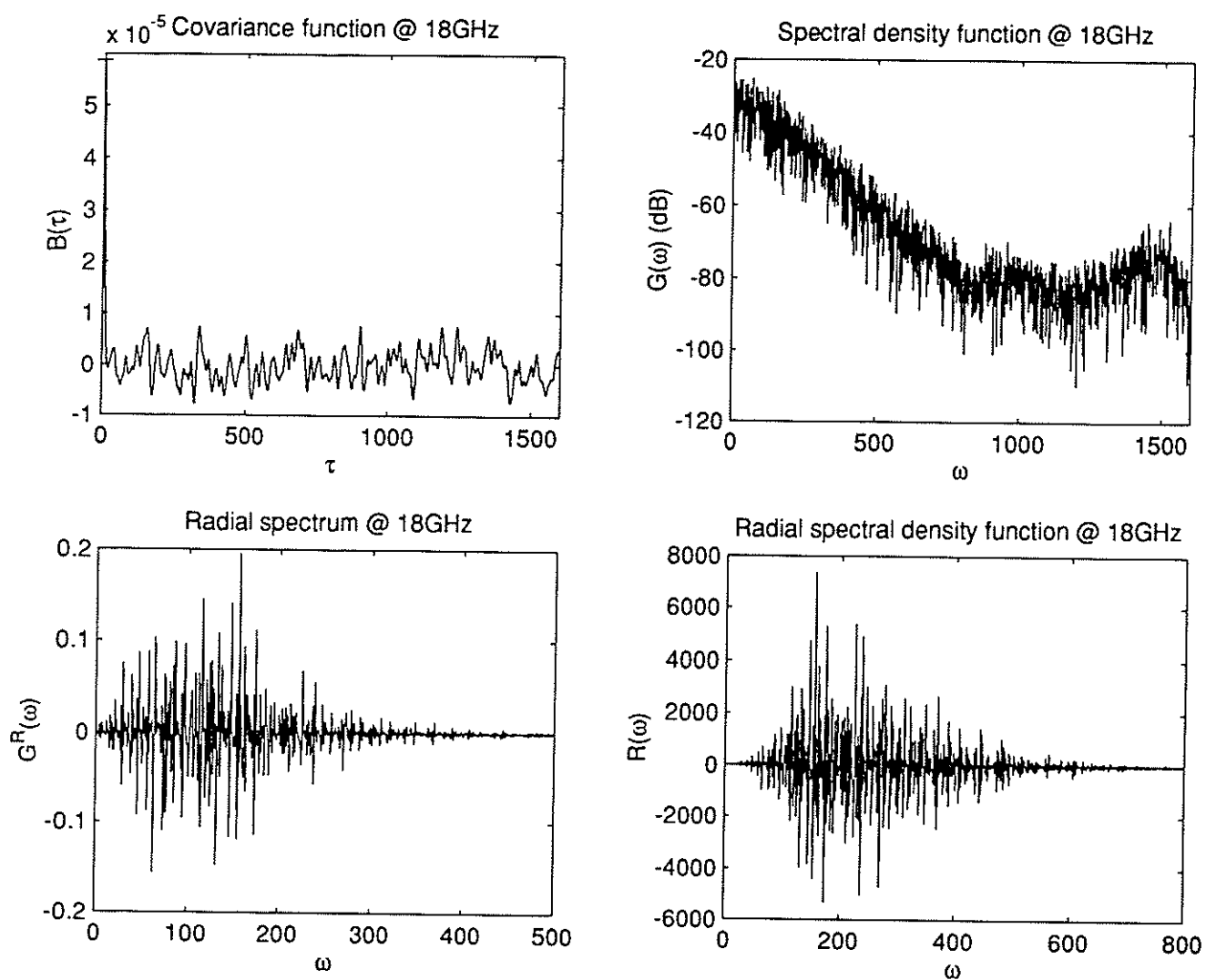


Fig 12.3

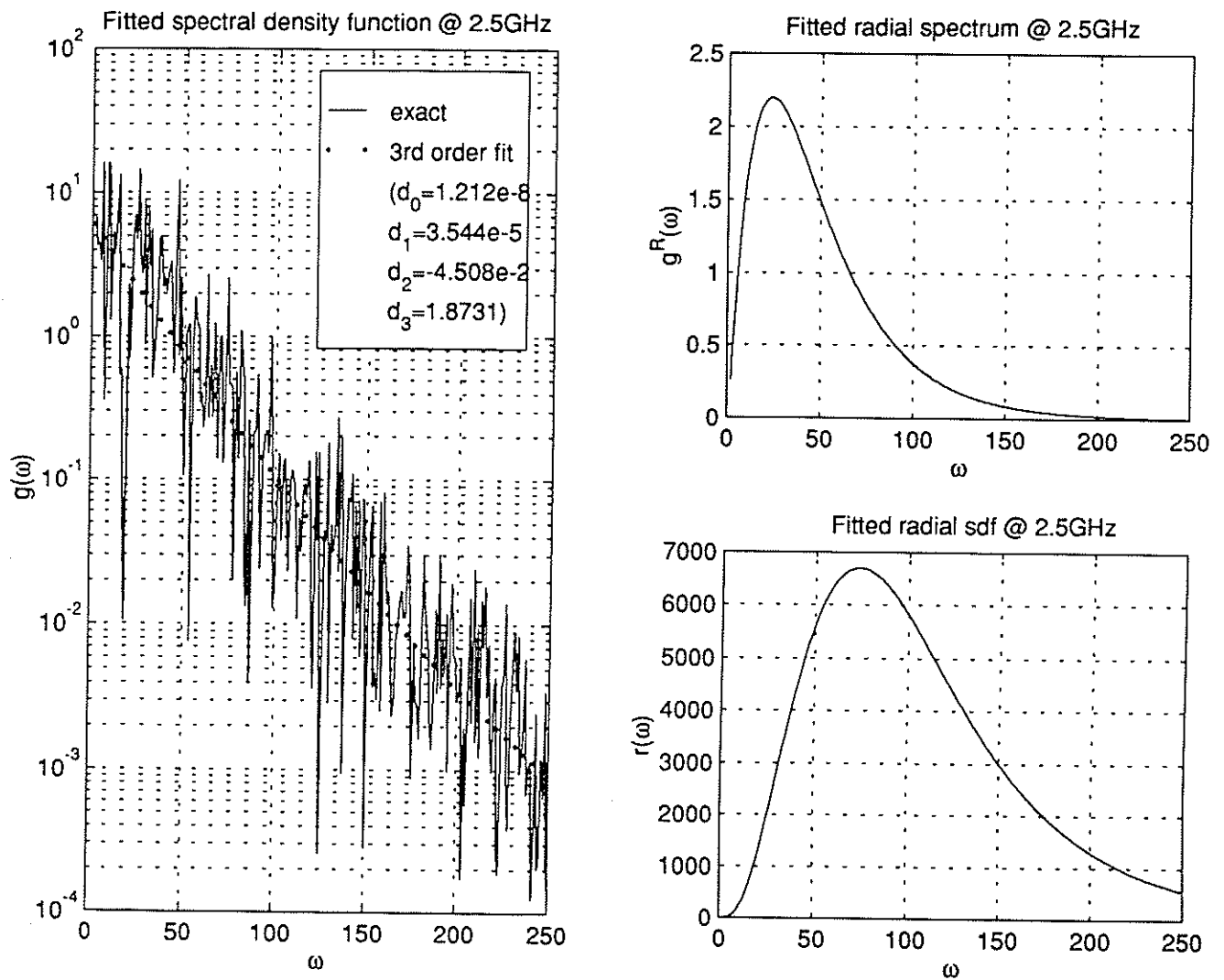


Fig 12.4

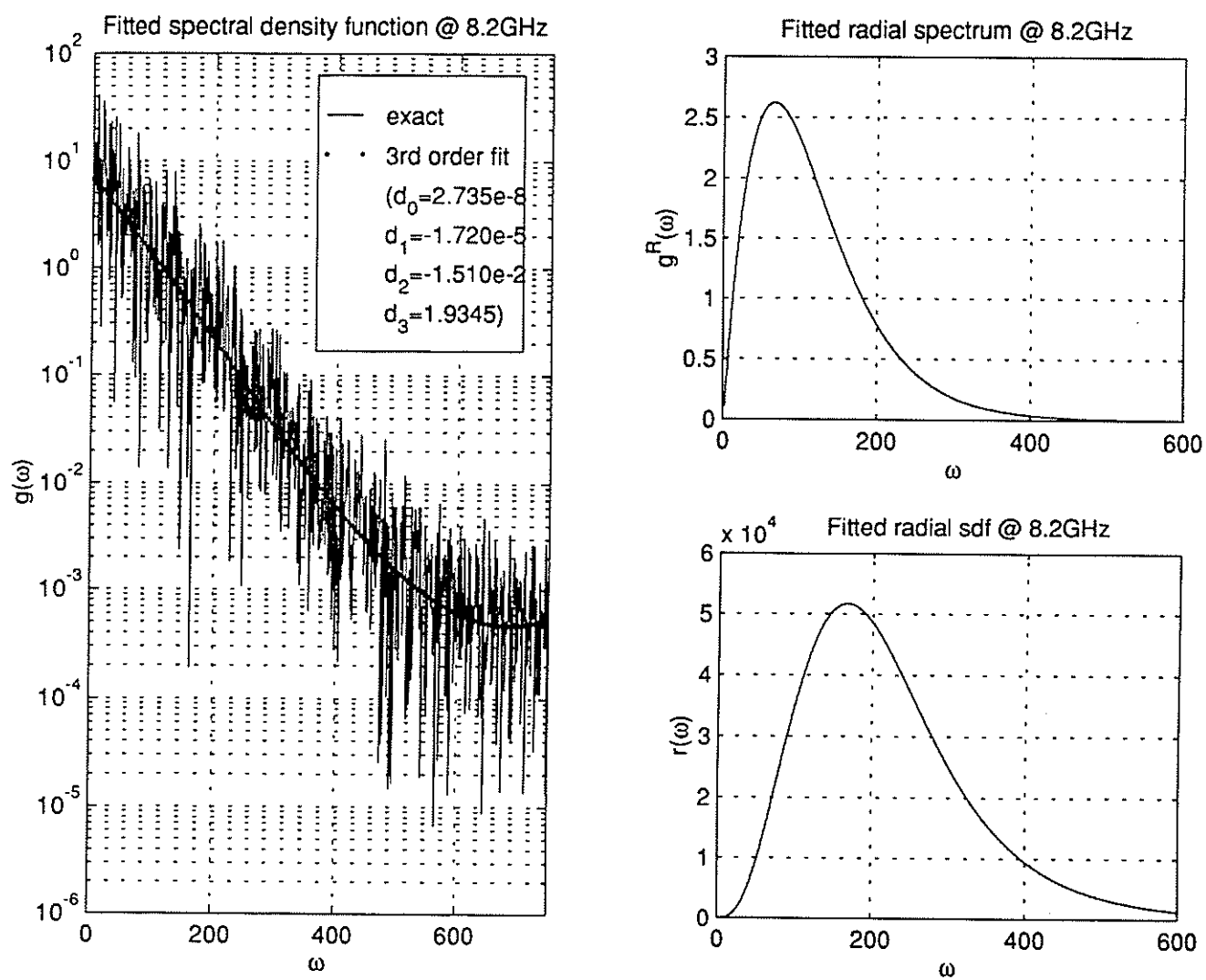


Fig 12.5

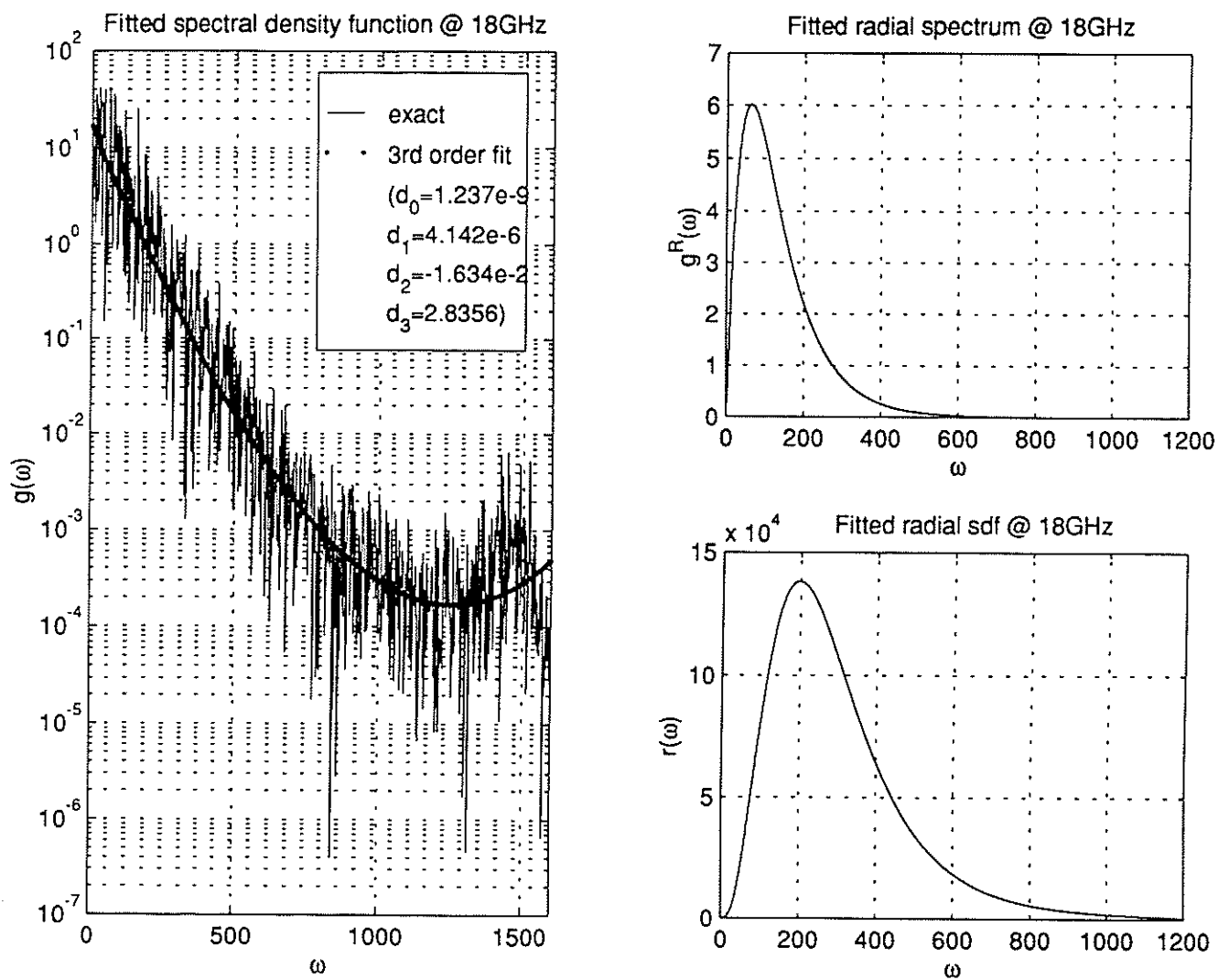


Fig 12.6

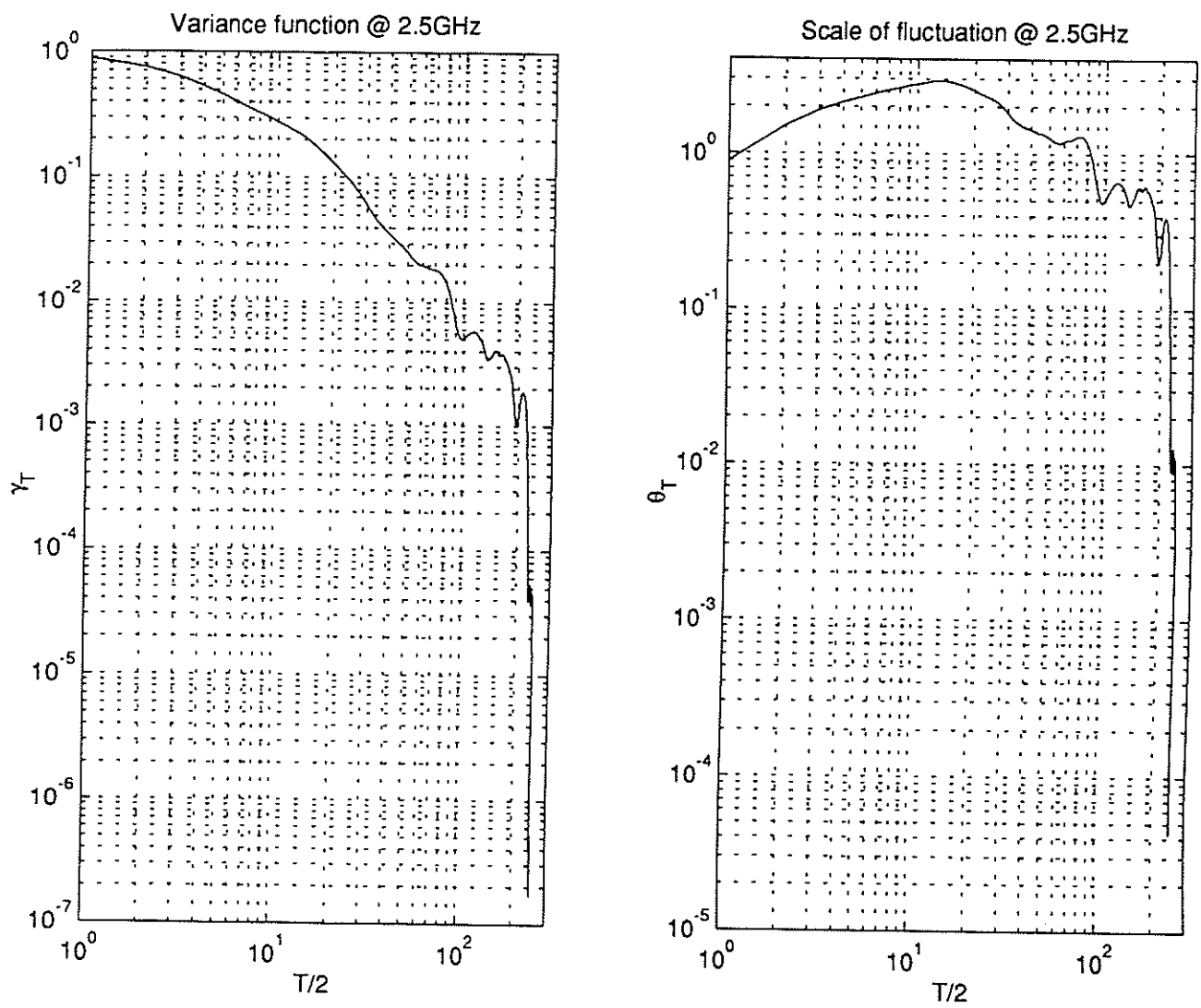


Fig 12.7

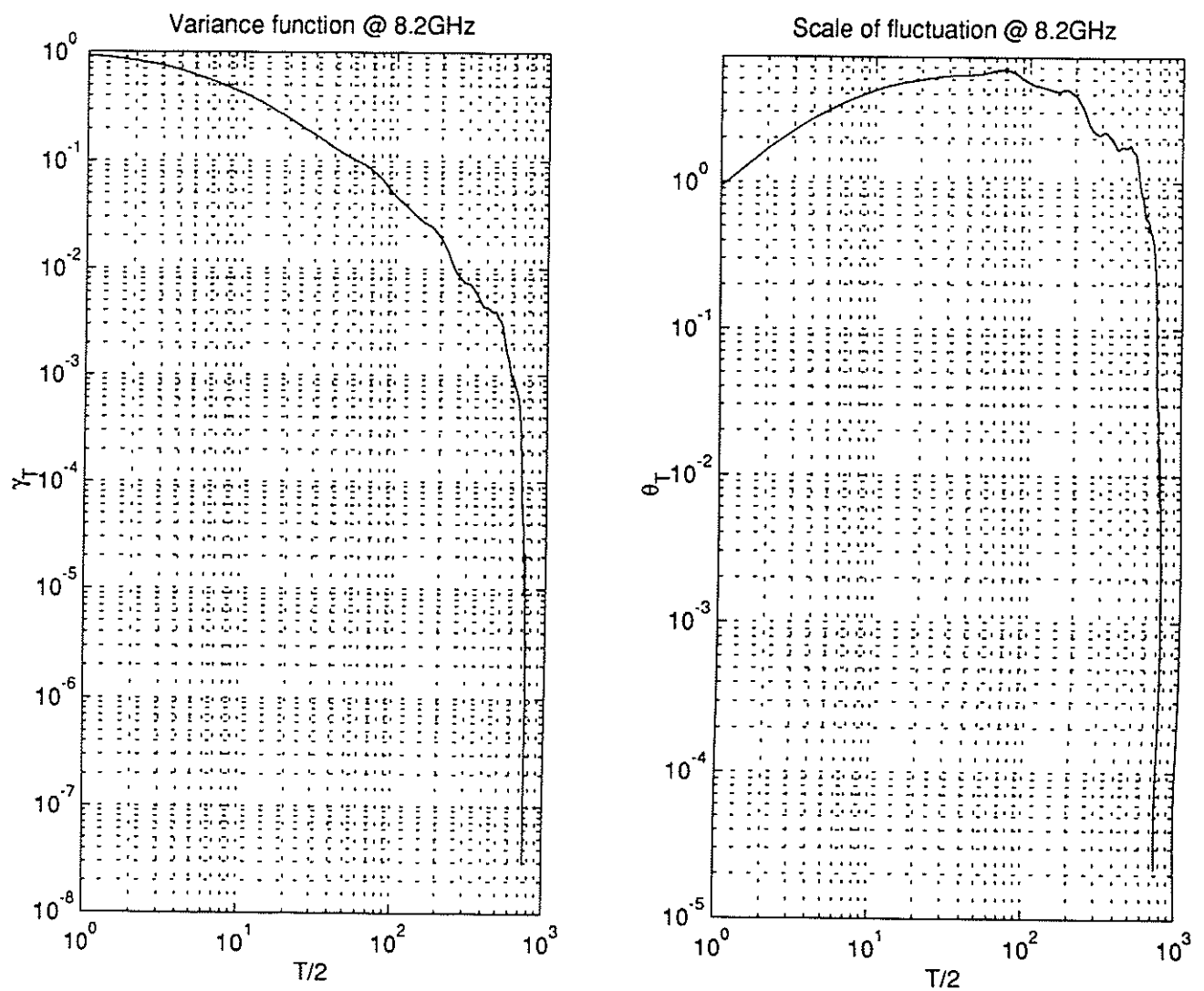


Fig 12.8

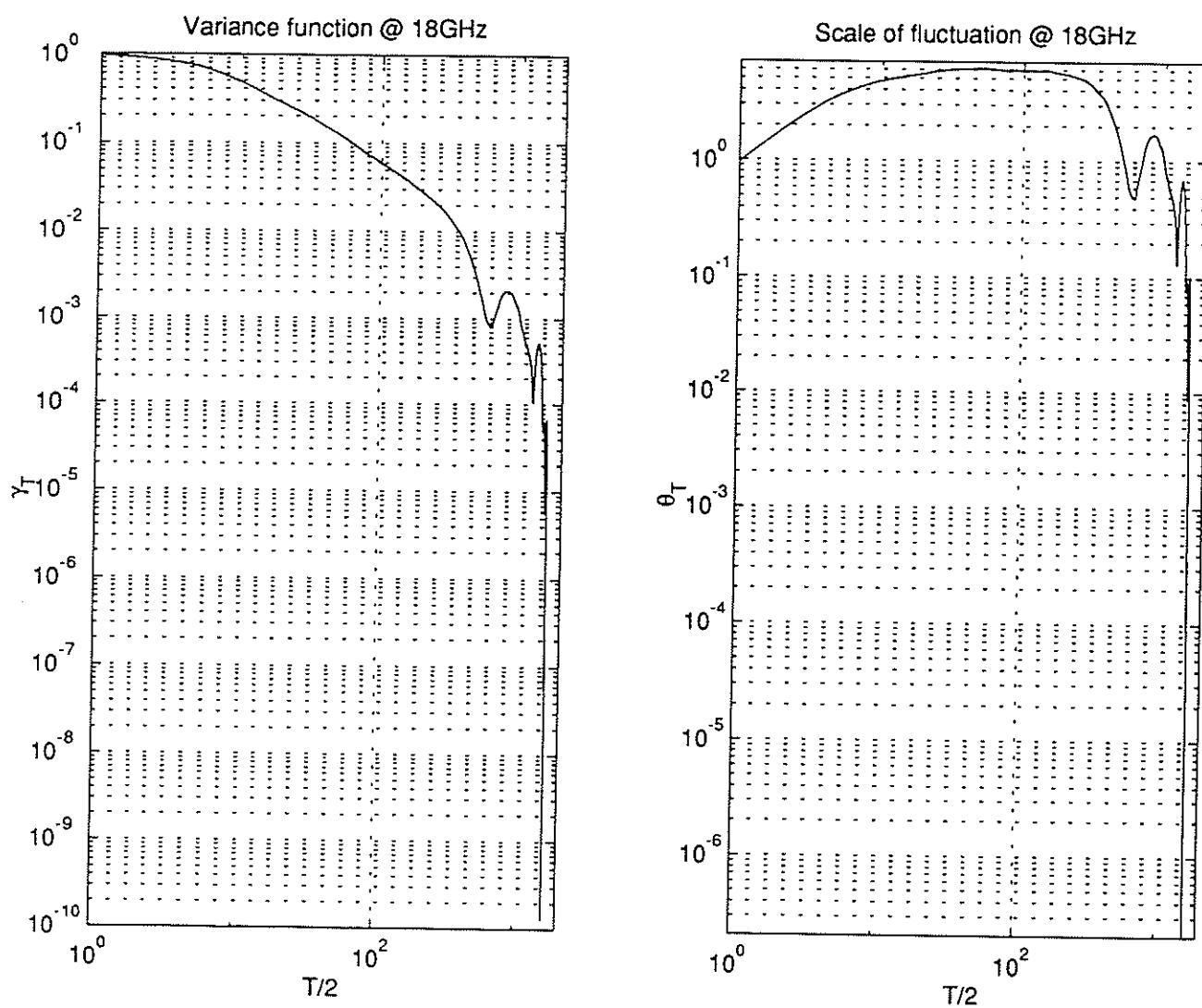
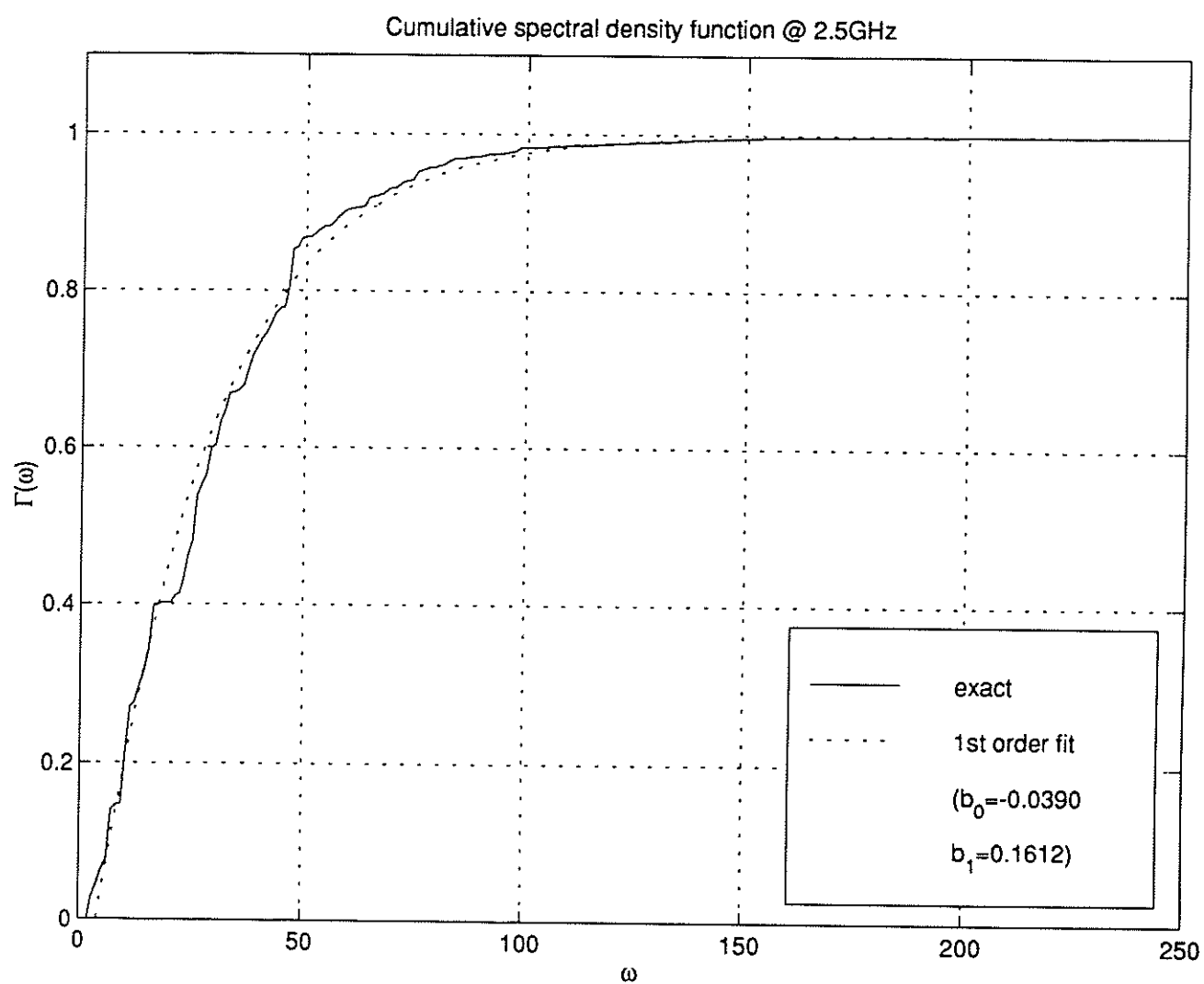


Fig 12.9

**Fig 12.10**

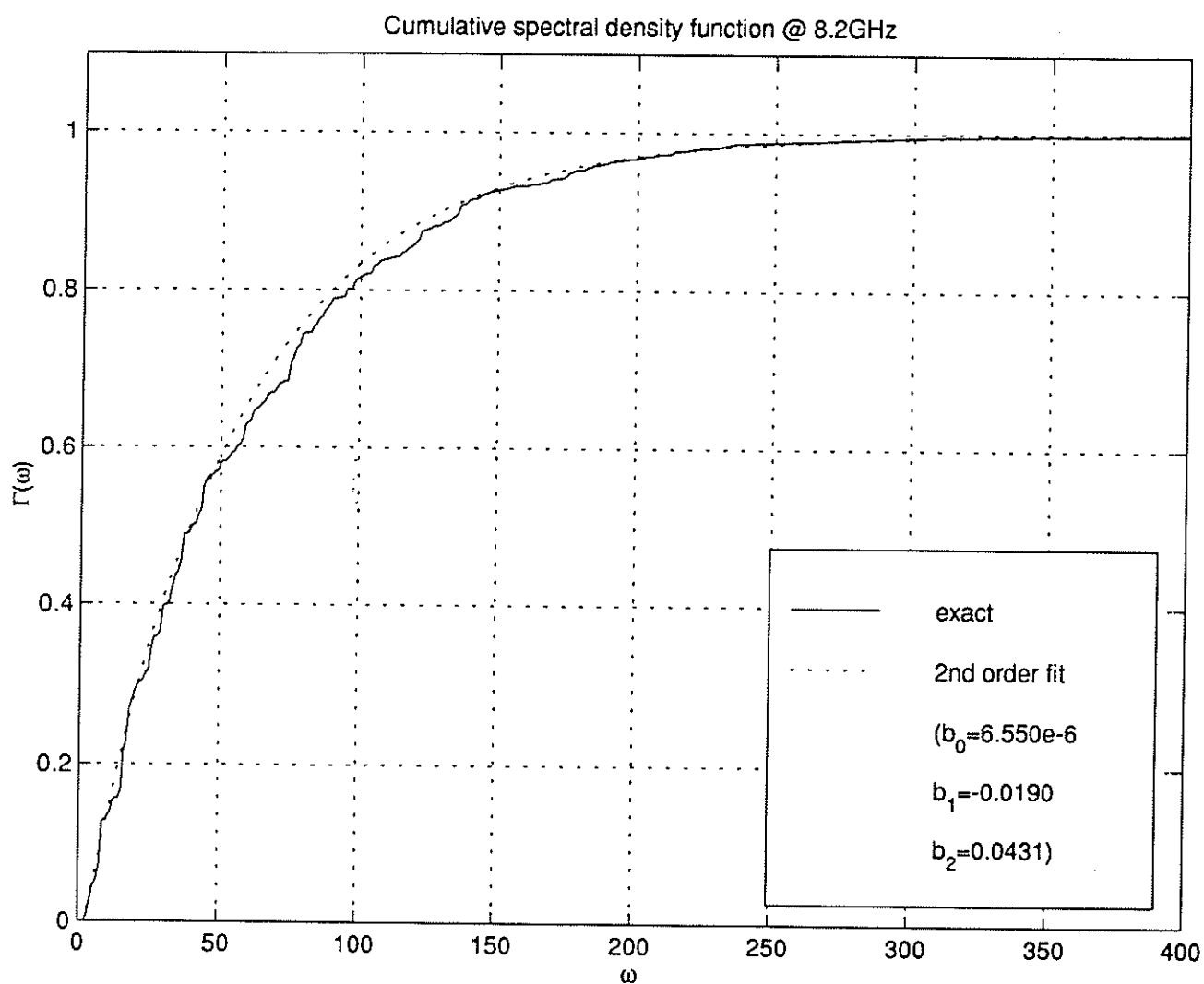


Fig 12.11

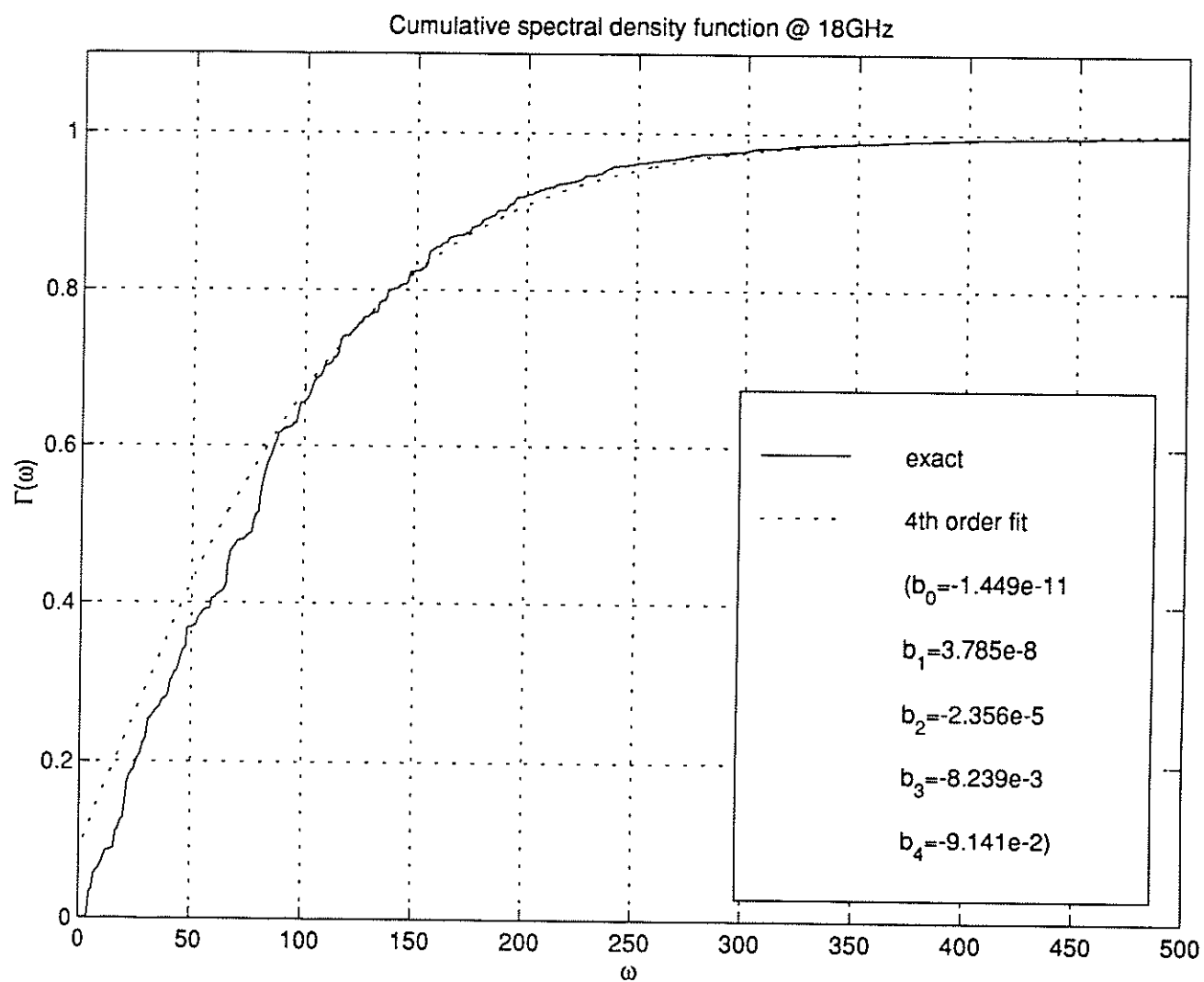


Fig 12.12

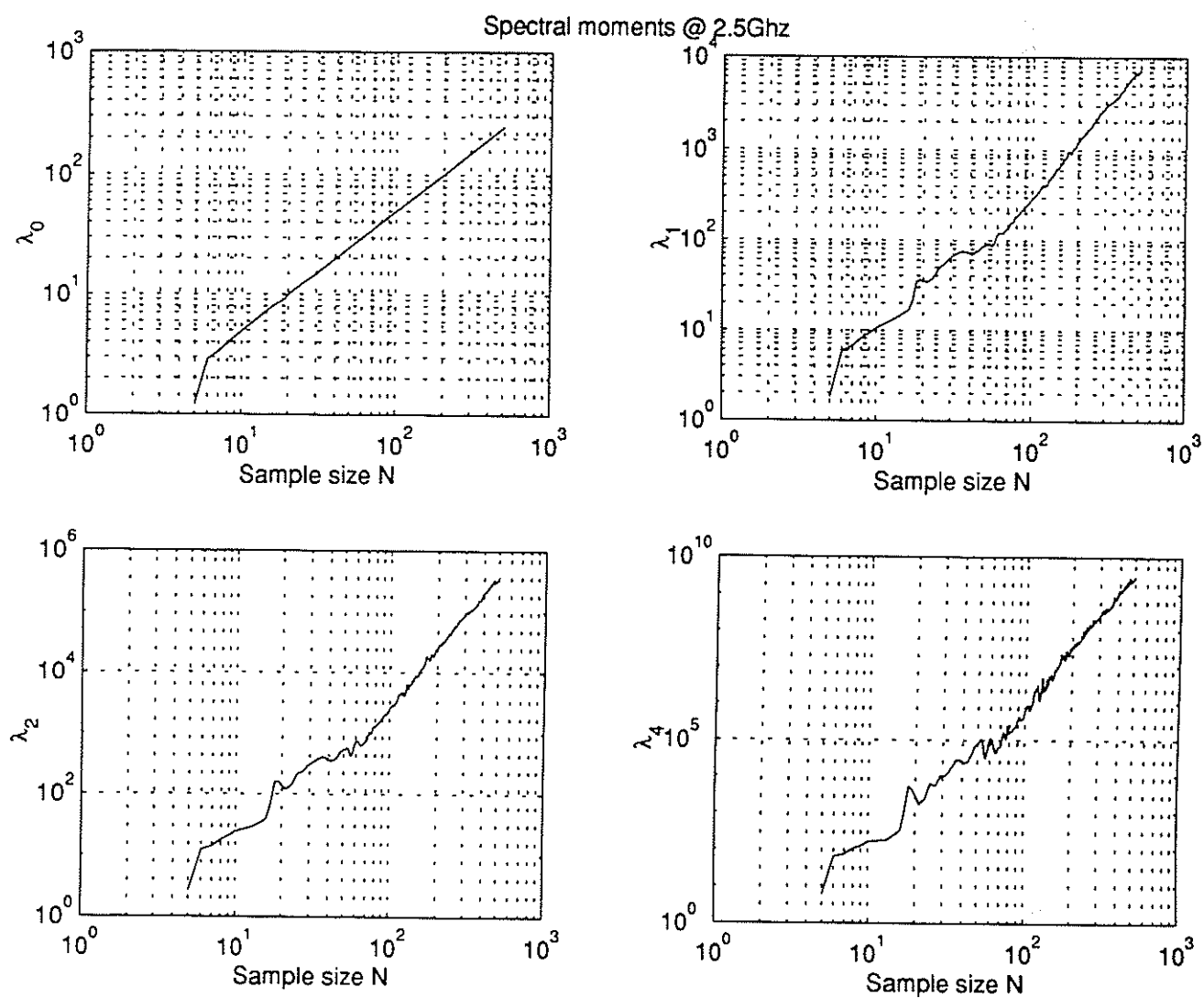


Fig 12.13

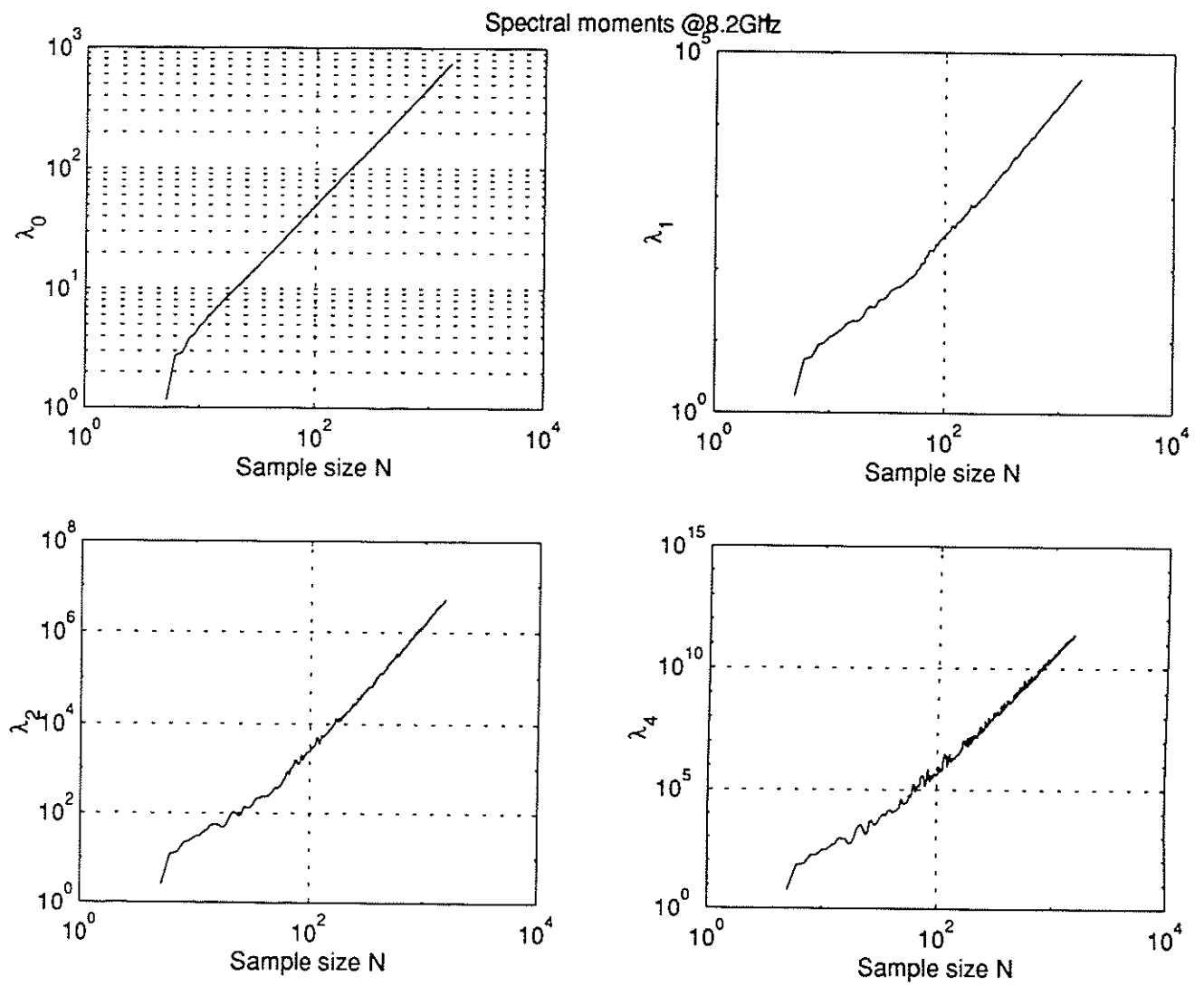


Fig 12.14

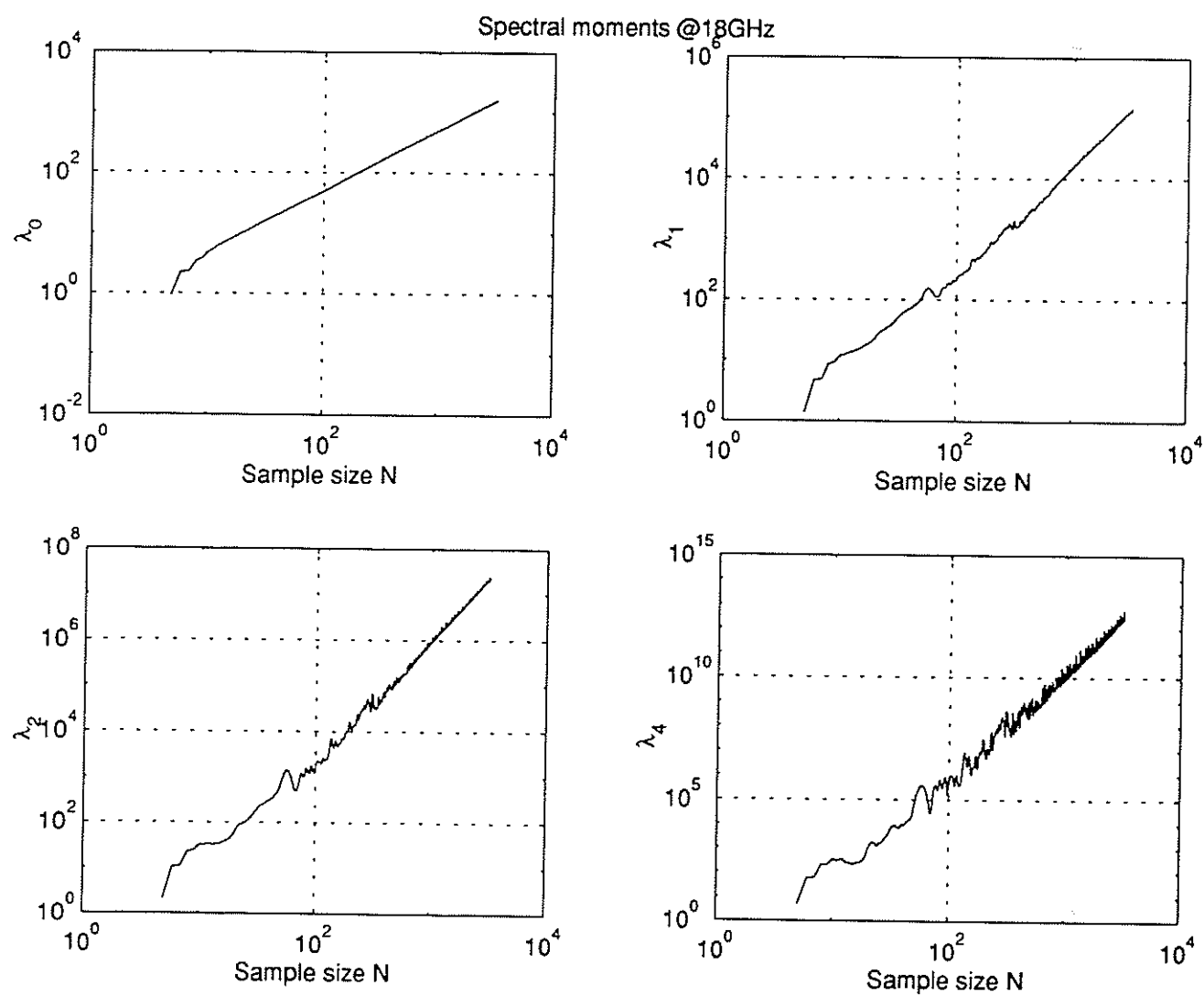


Fig 12.15

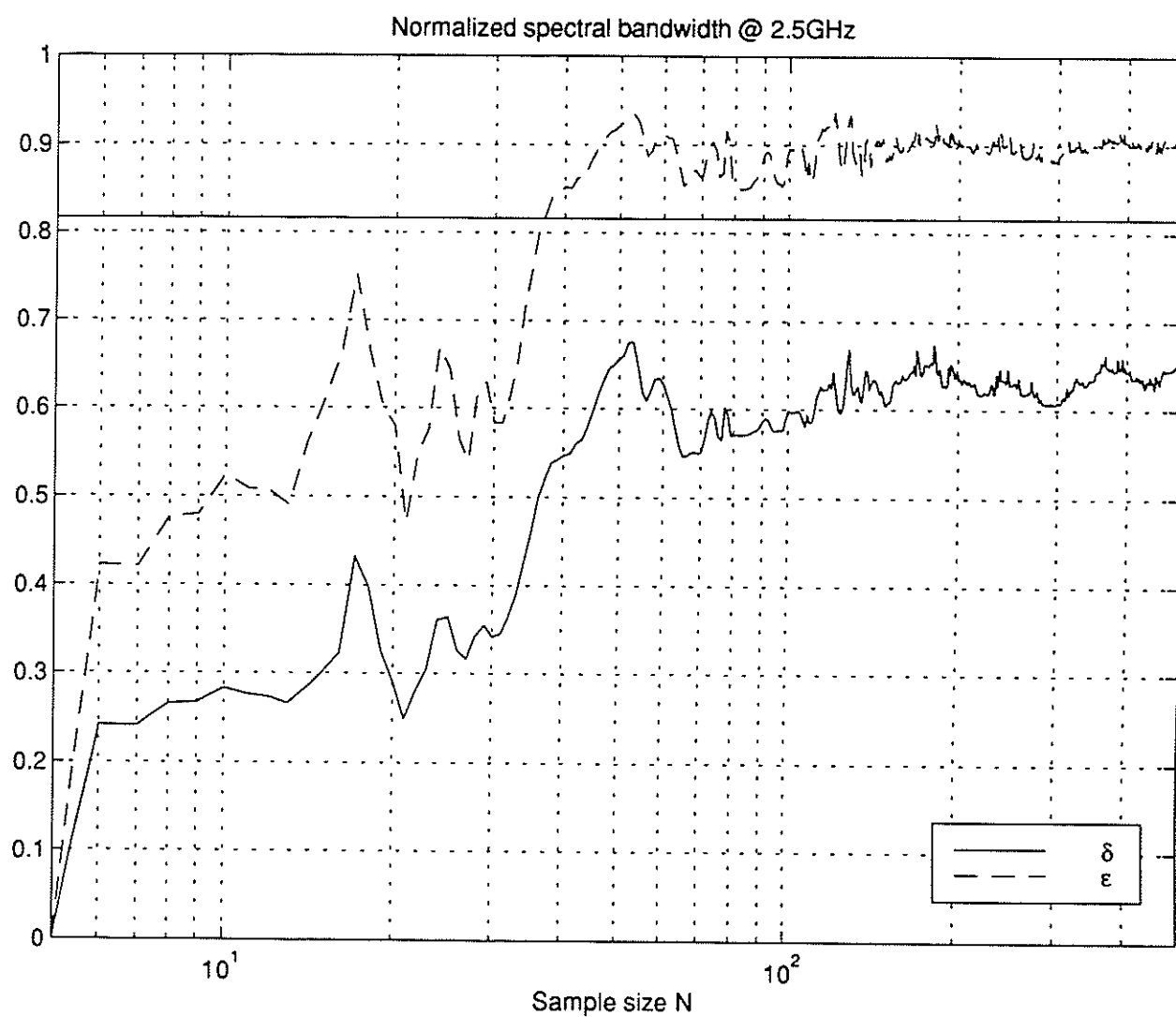


Fig 12.16

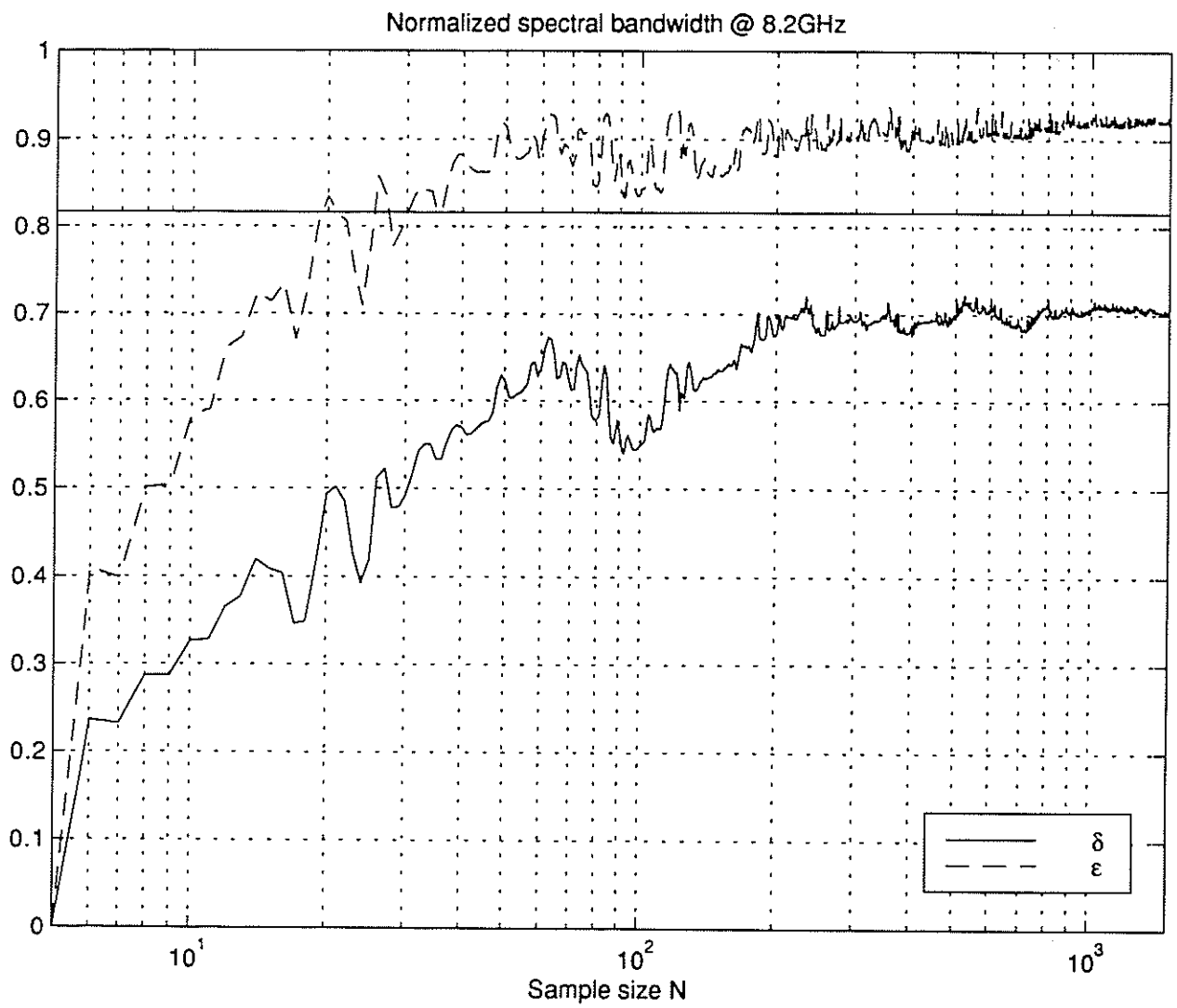
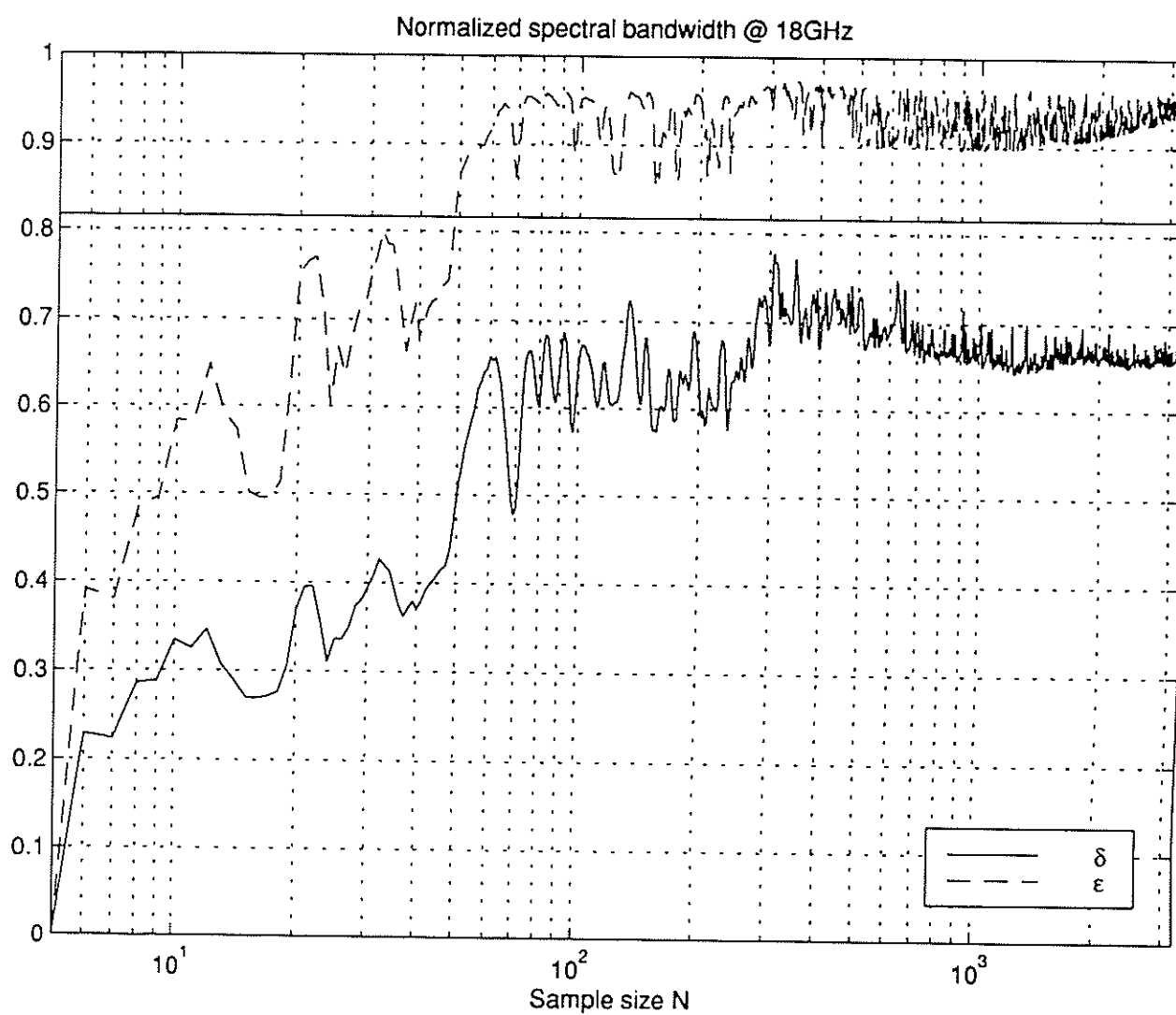


Fig 12.17

**Fig 12.18**

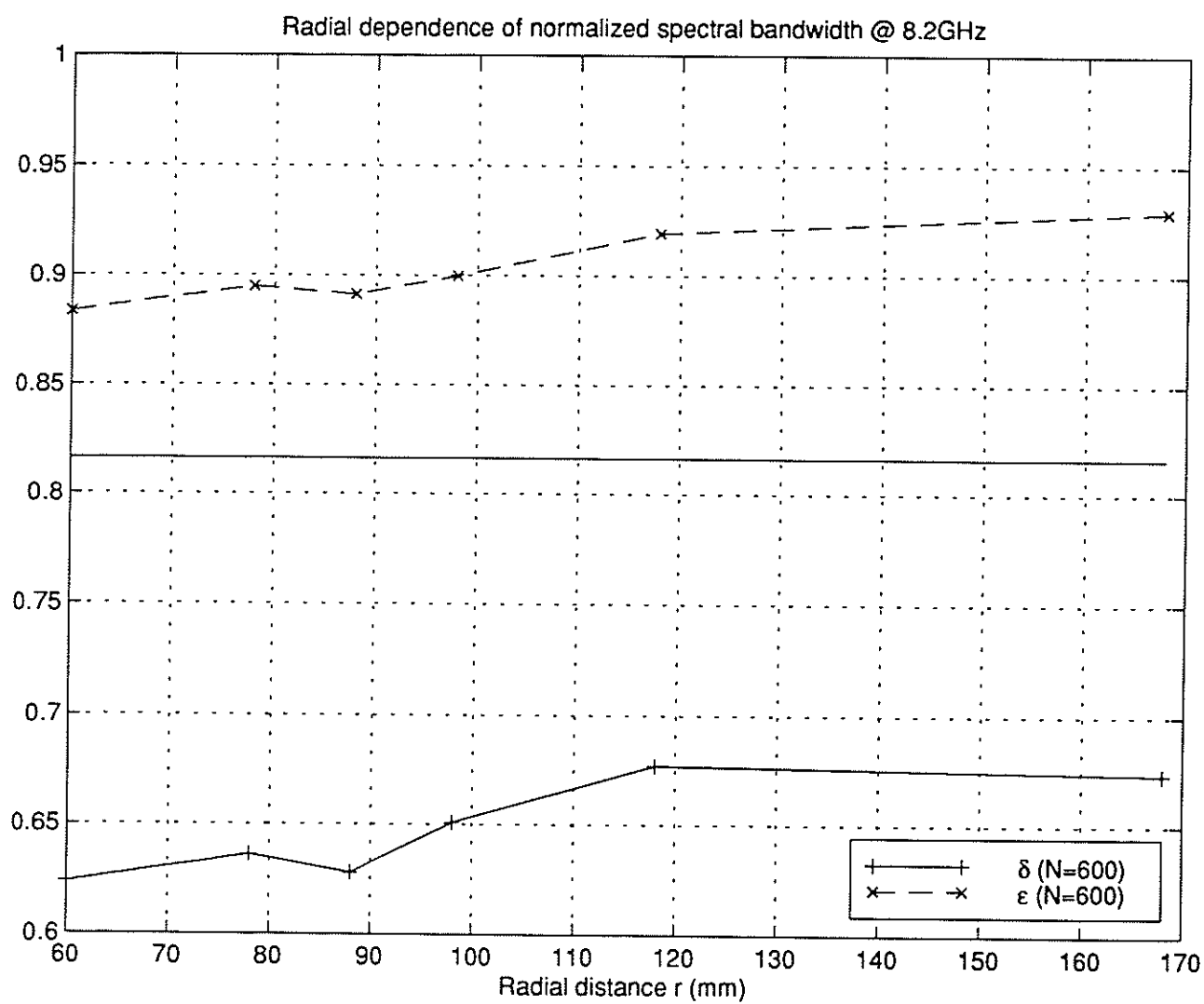


Fig 12.19

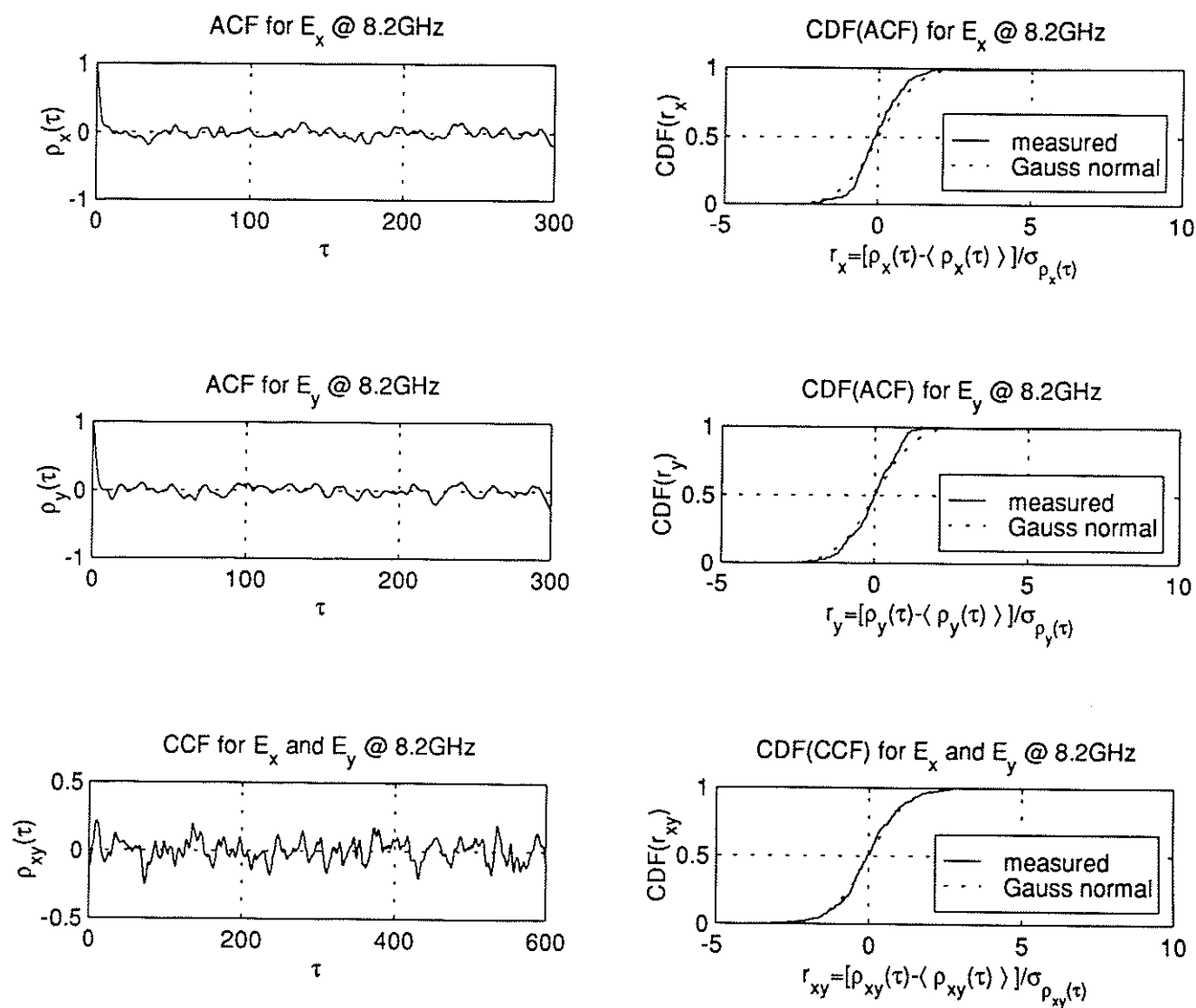


Fig 12.20

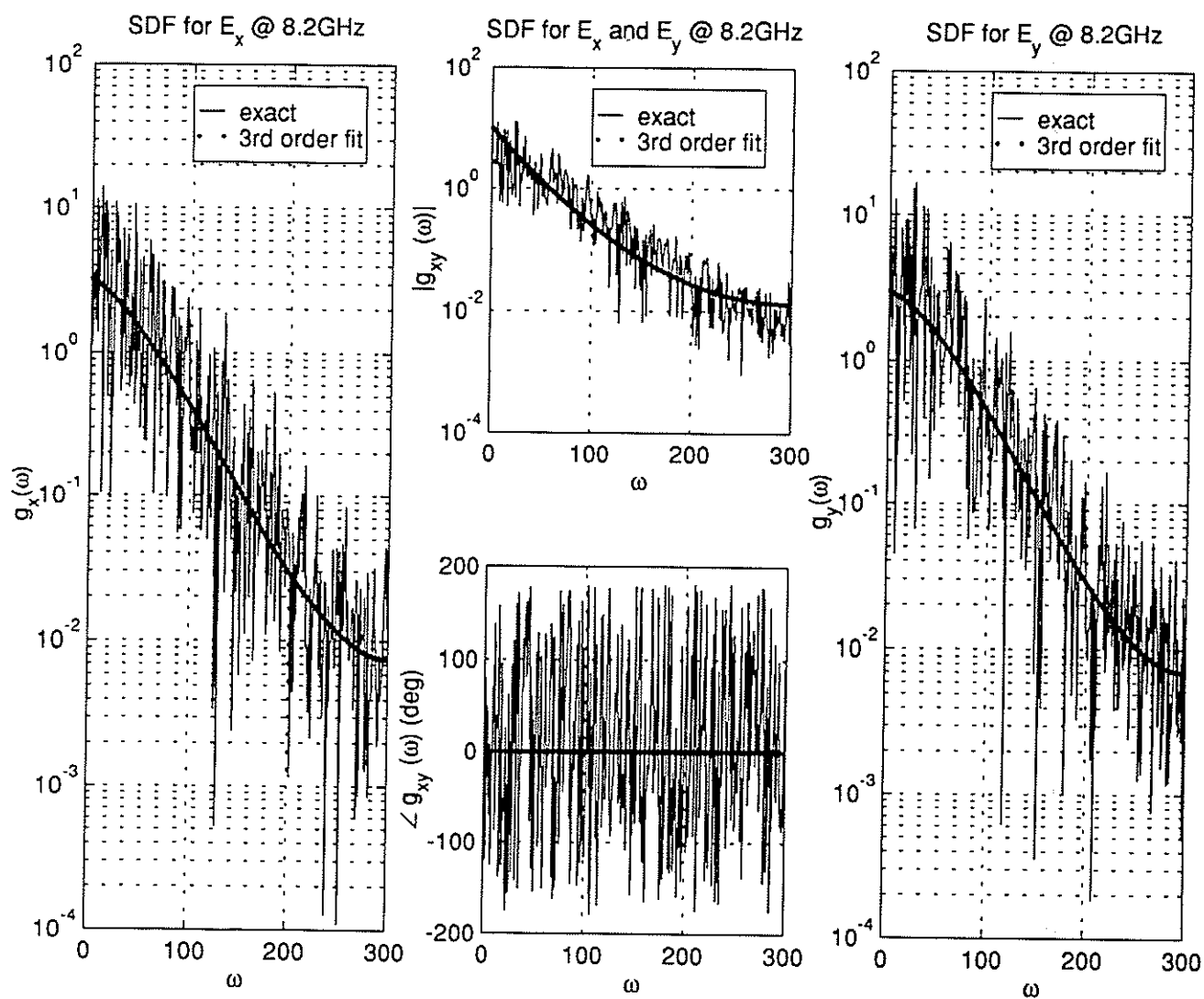


Fig 12.21

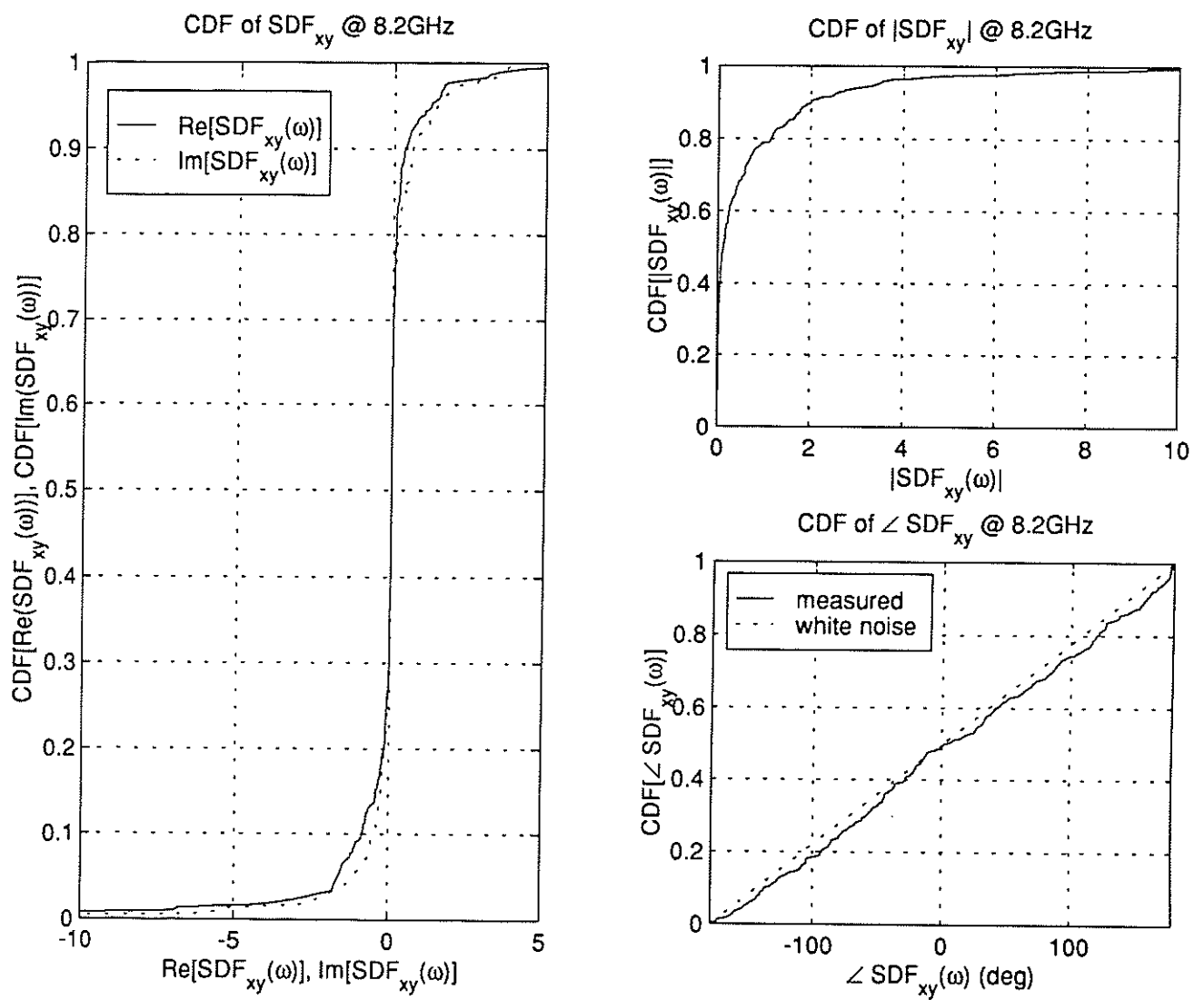


Fig 12.22

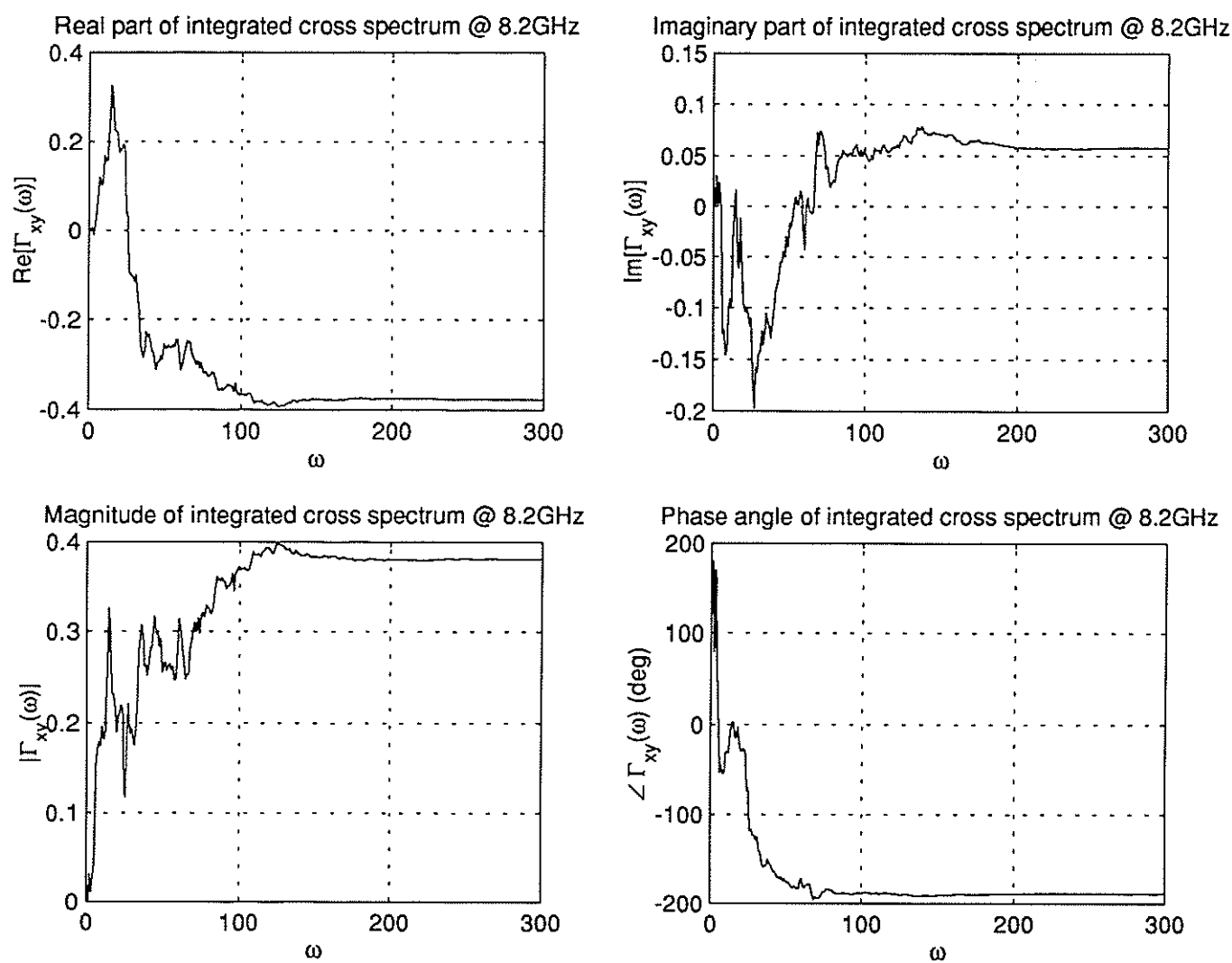


Fig 12.23

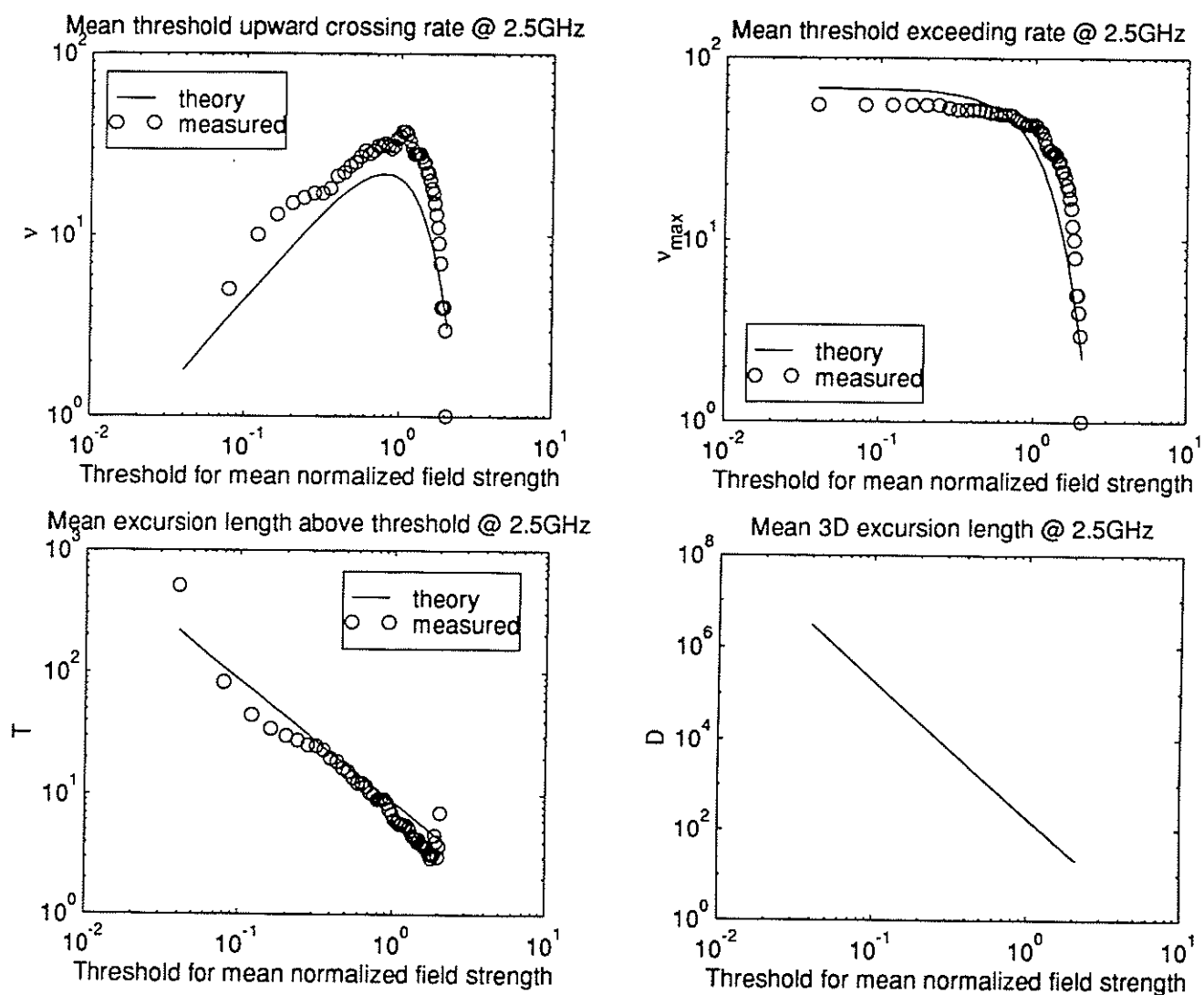


Fig 12.24

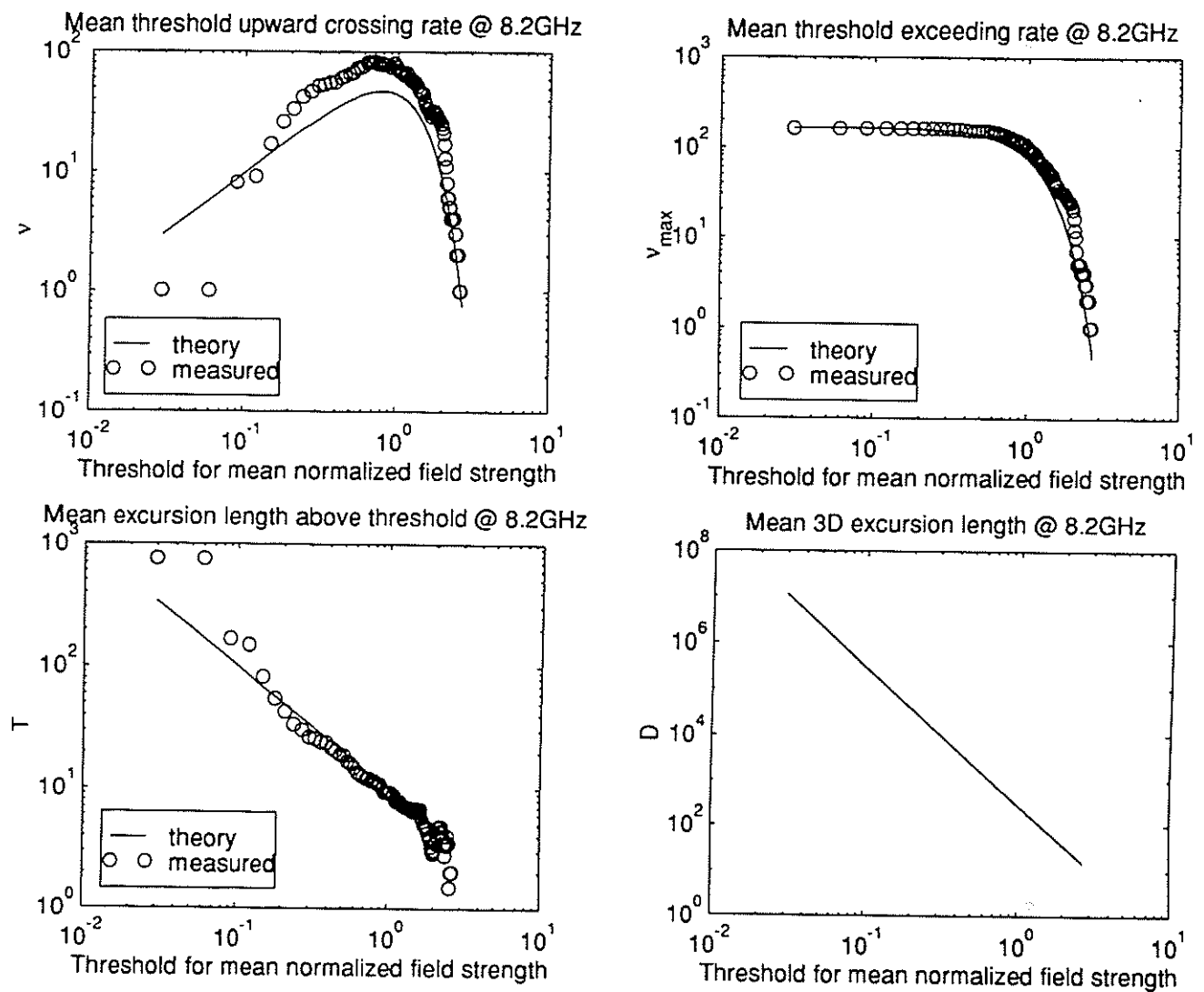


Fig 12.25

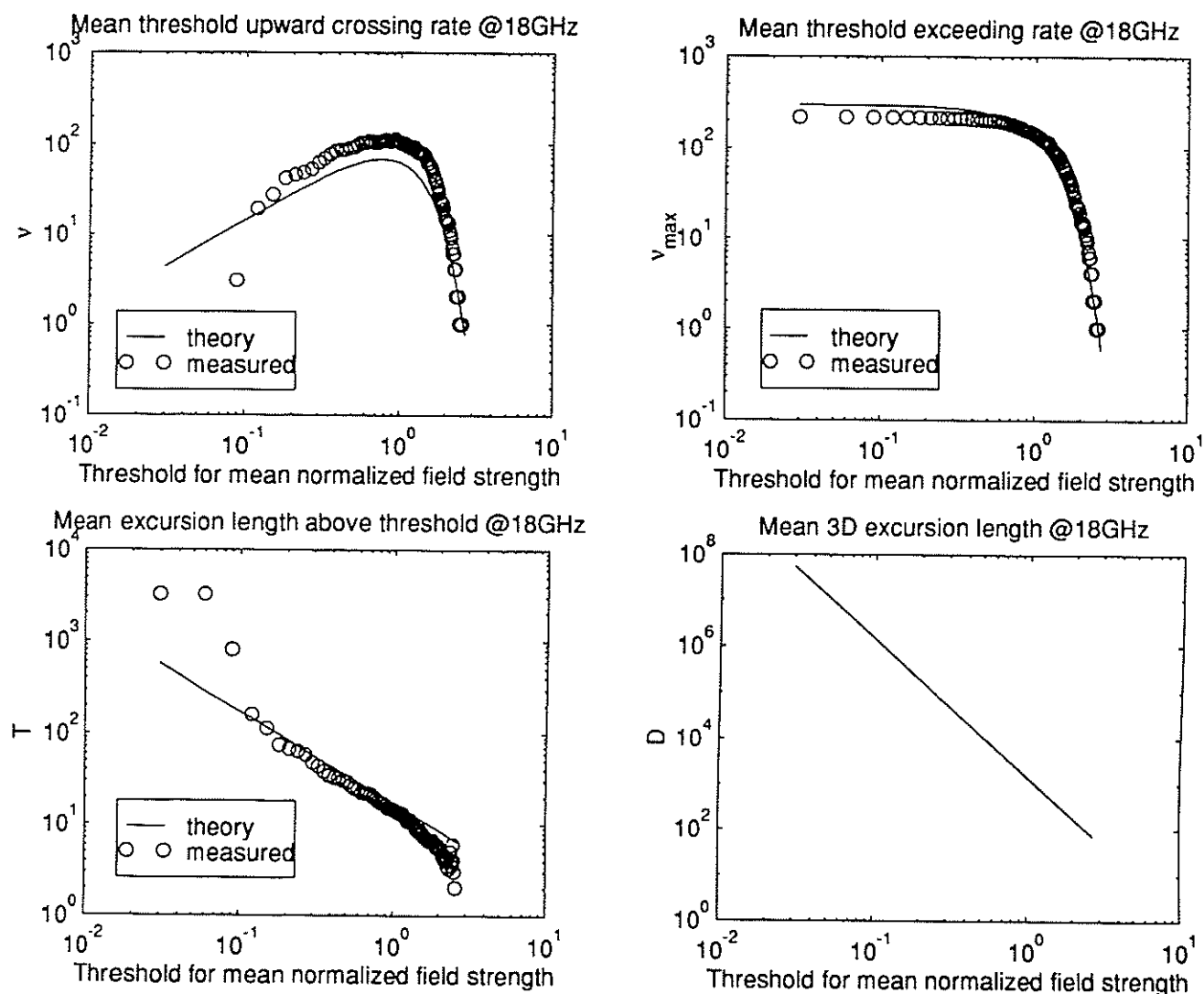


Fig 12.26

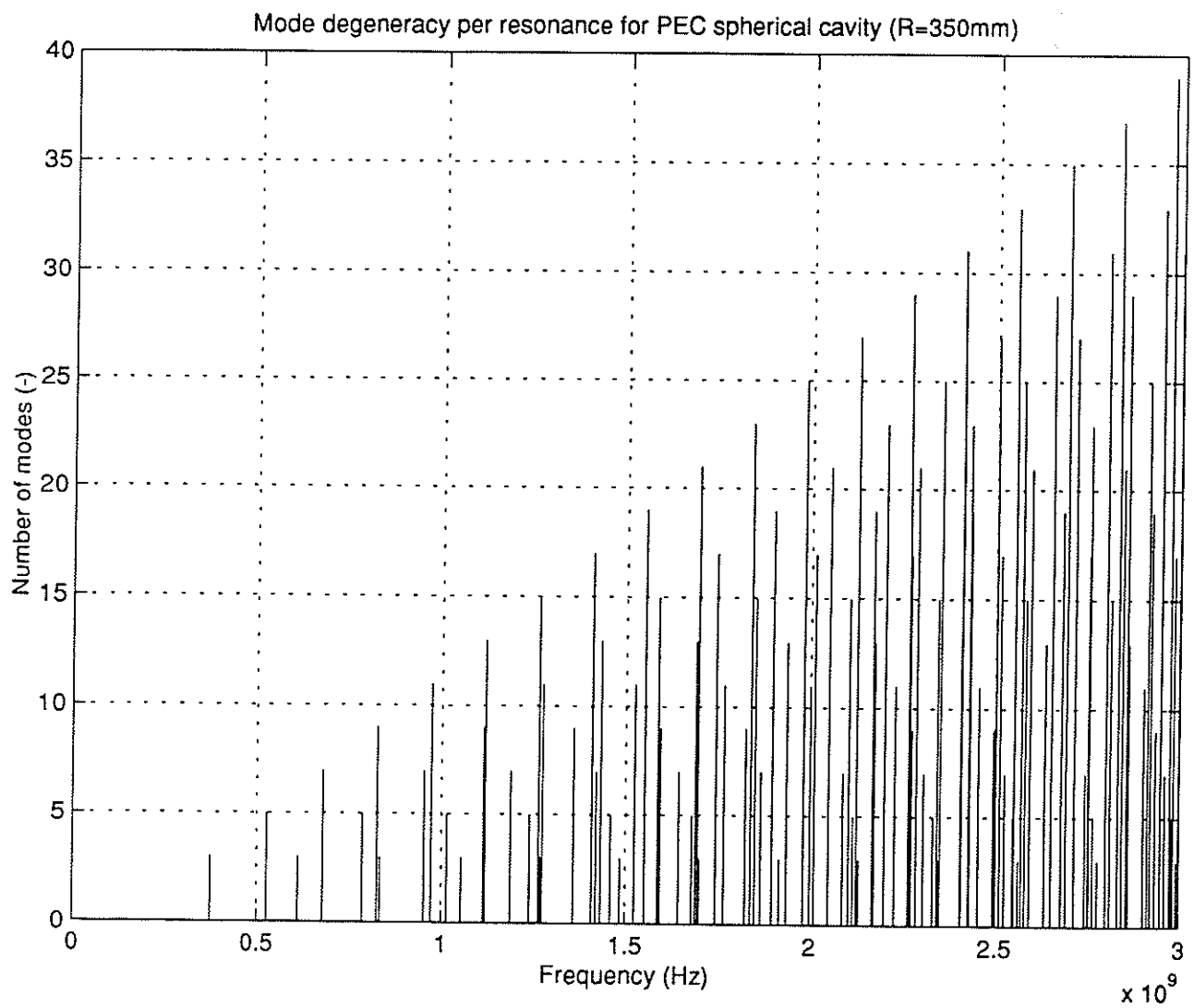


Fig A.1

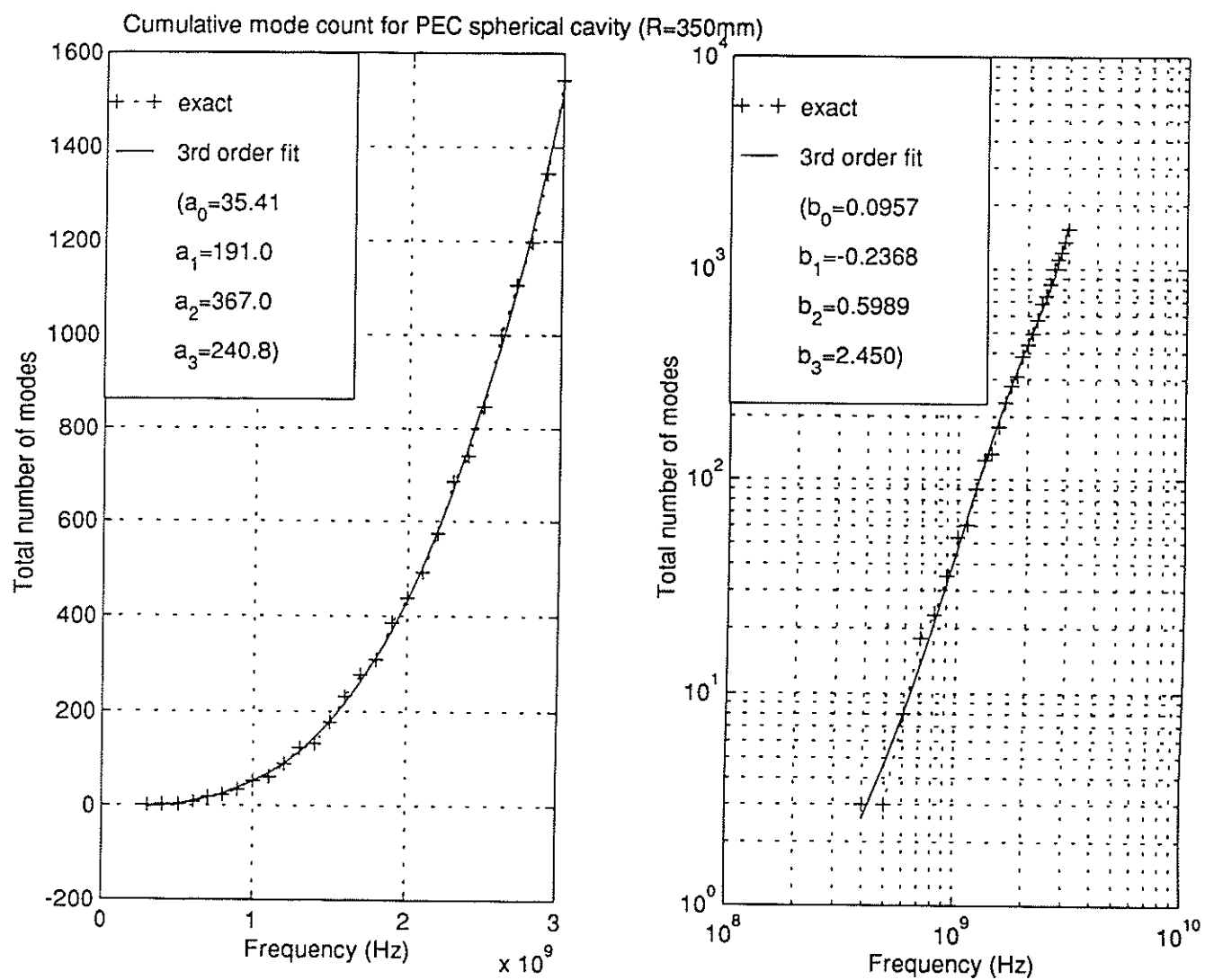


Fig A.2

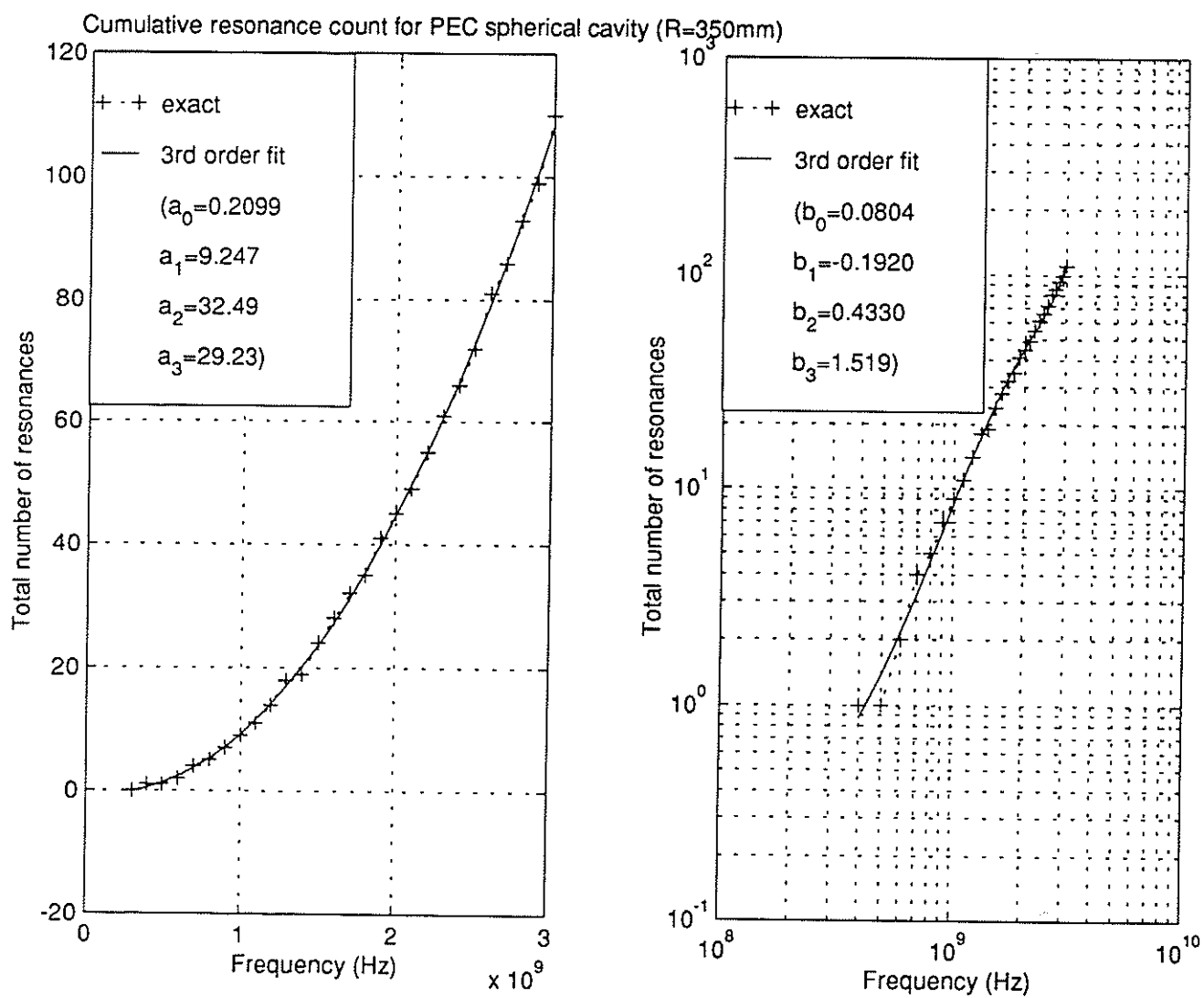


Fig A.3

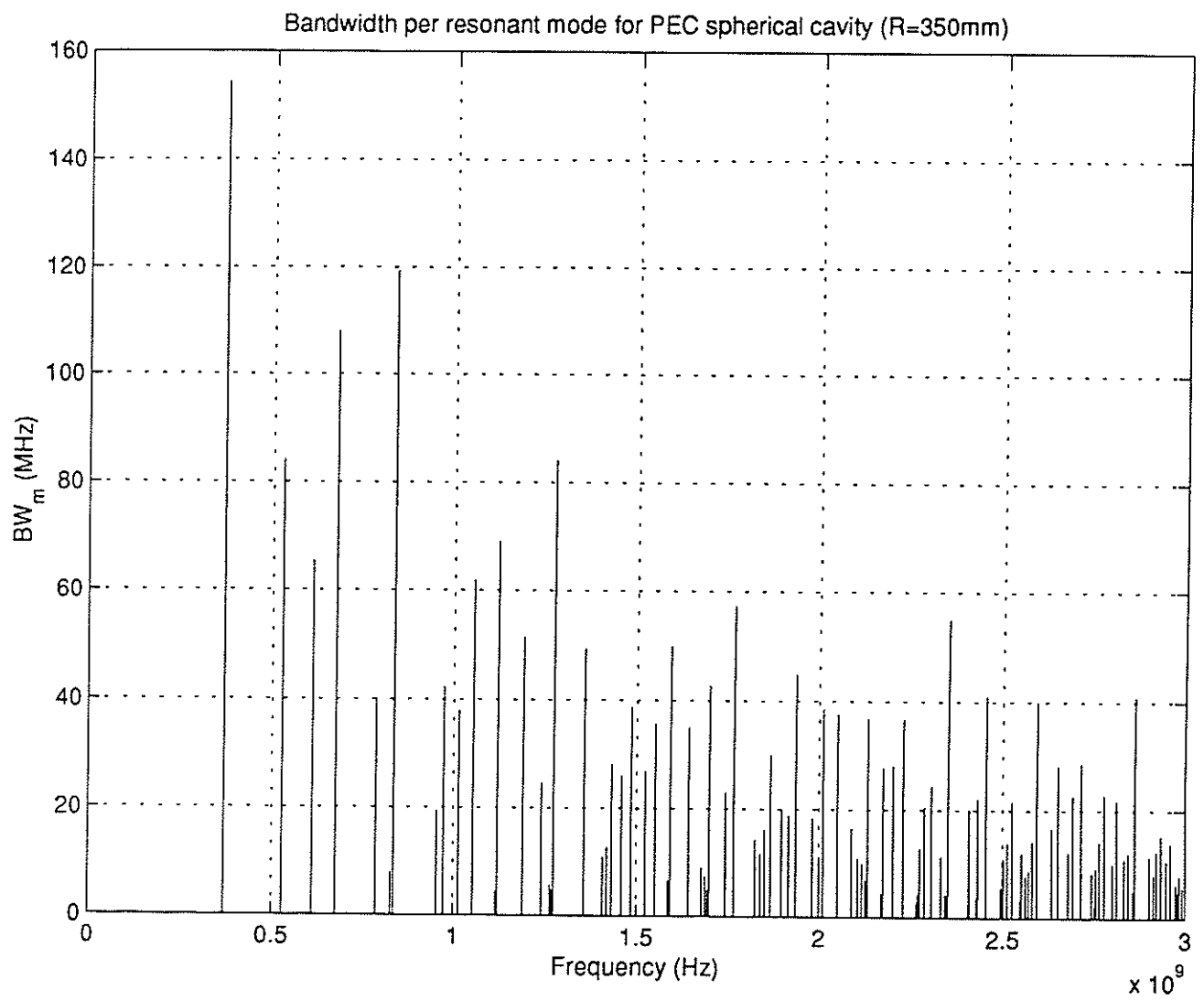


Fig A.4

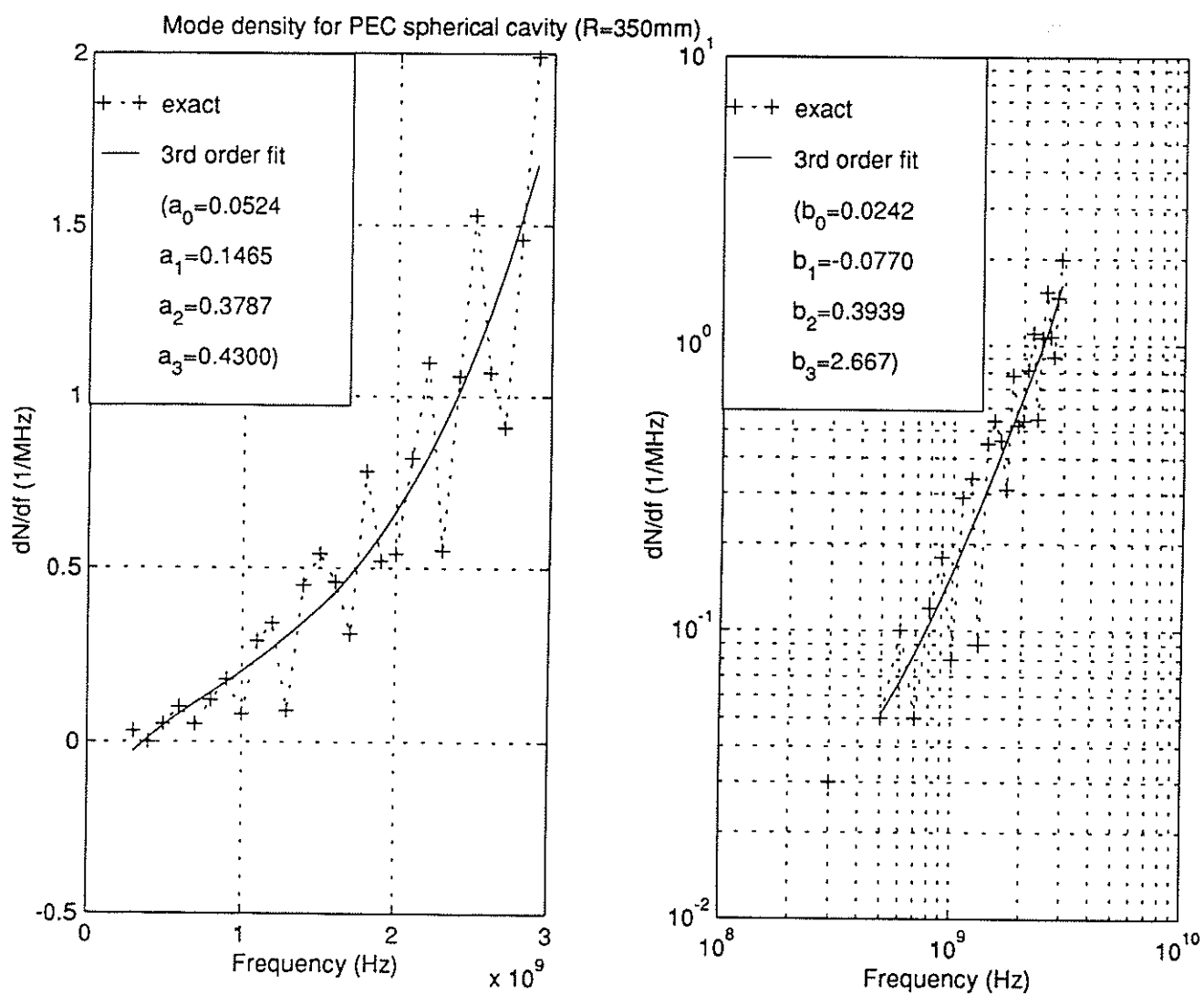


Fig A.5

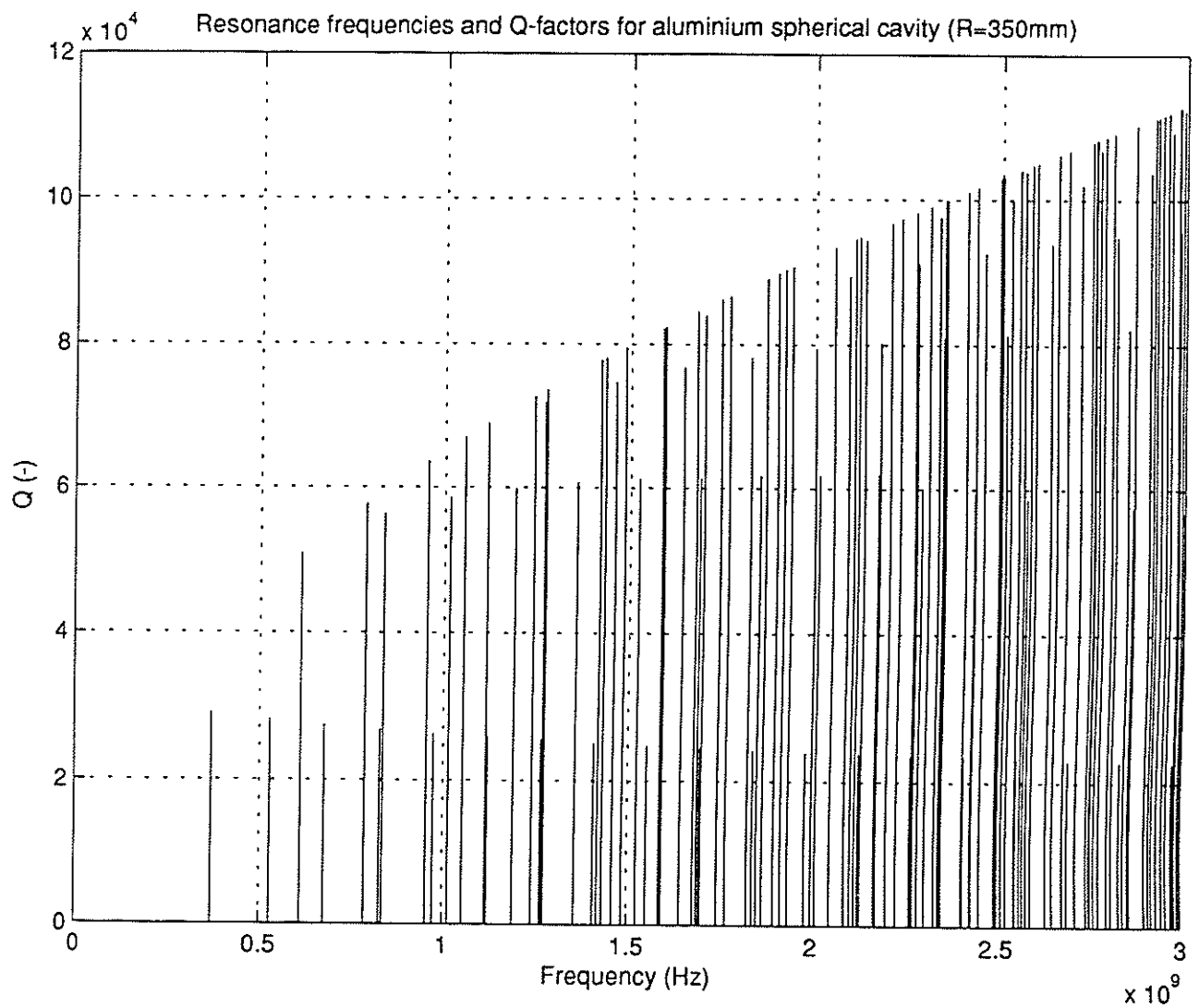


Fig A.6

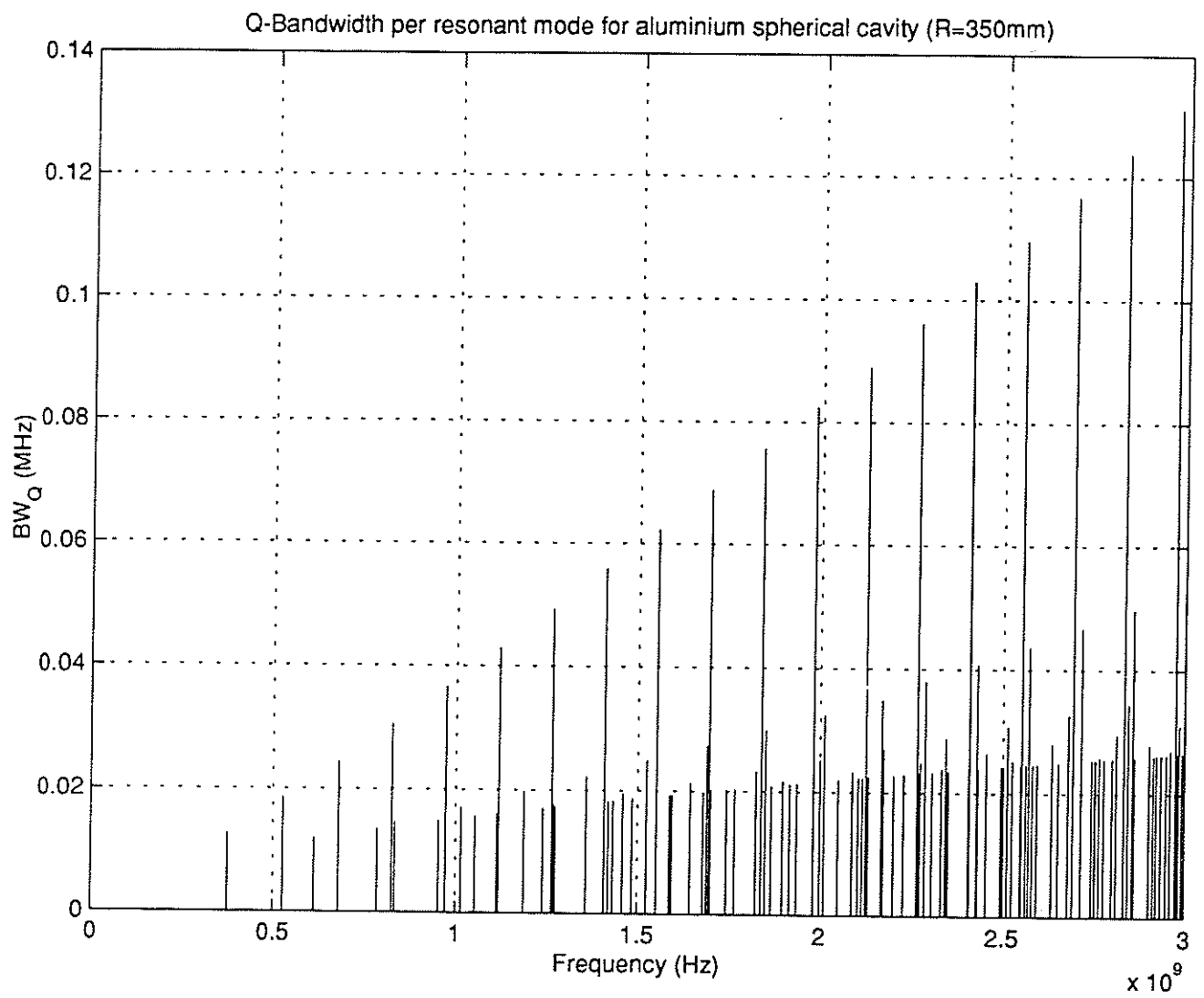


Fig A.7

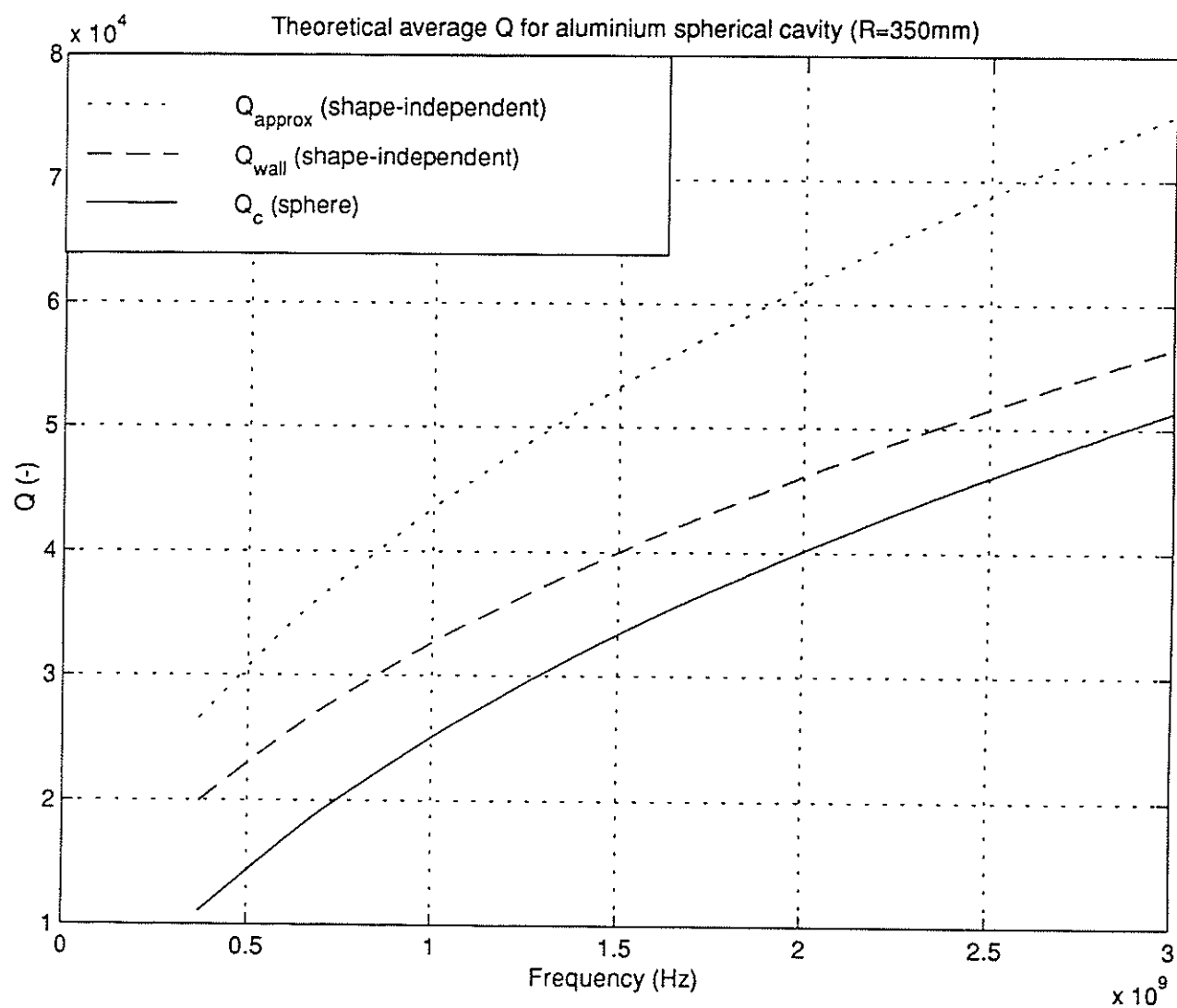


Fig A.8

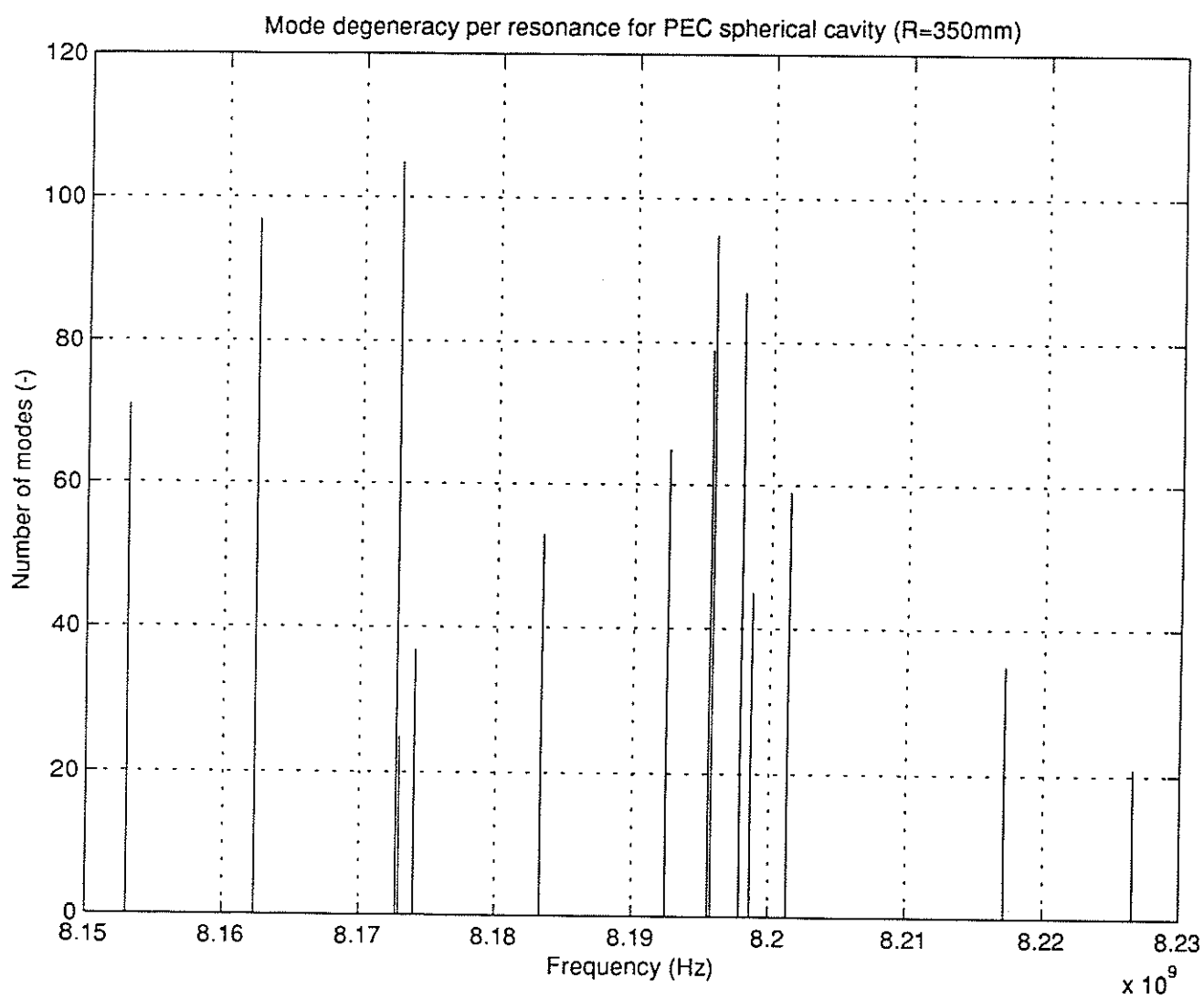


Fig A.9

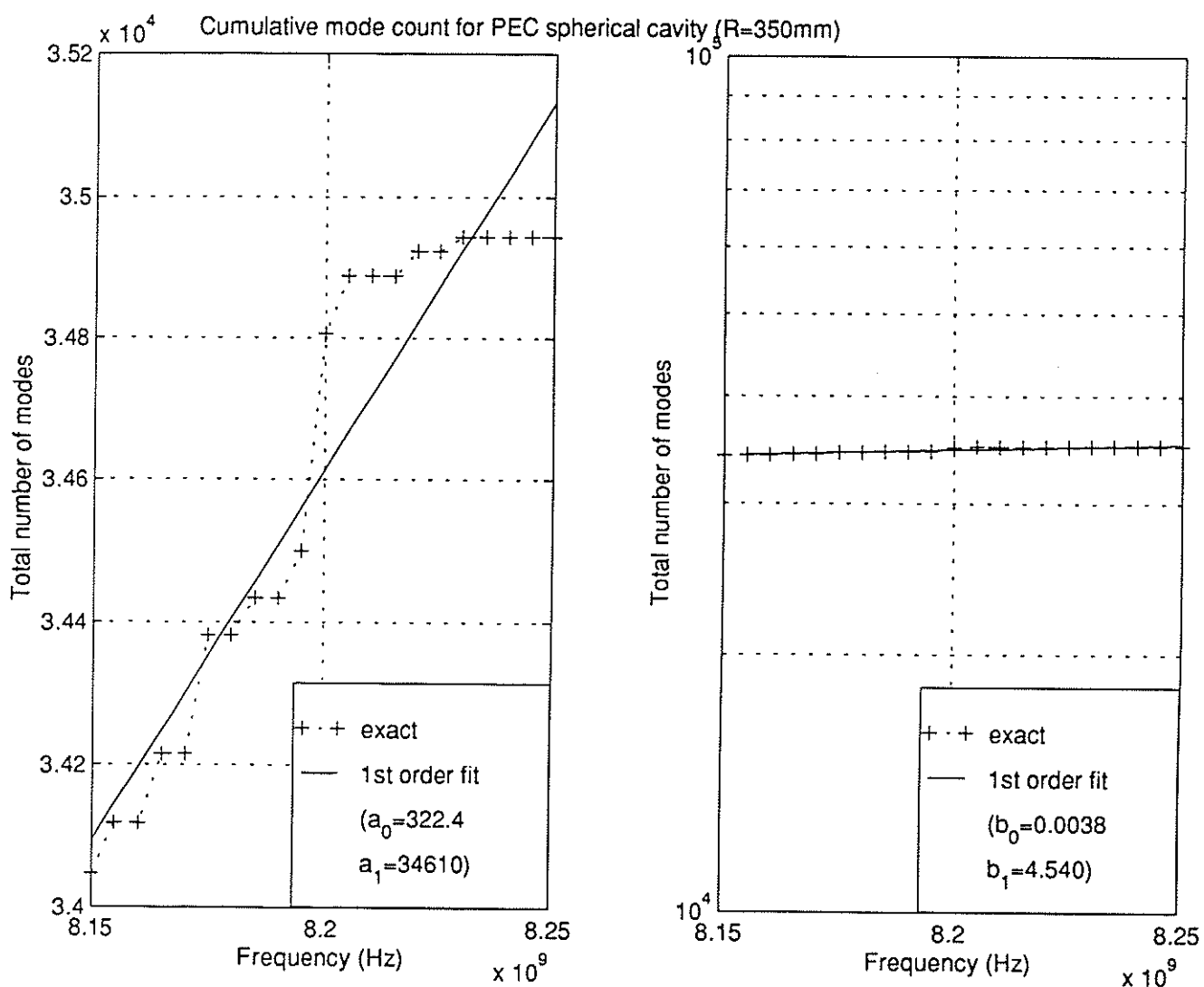


Fig A.10

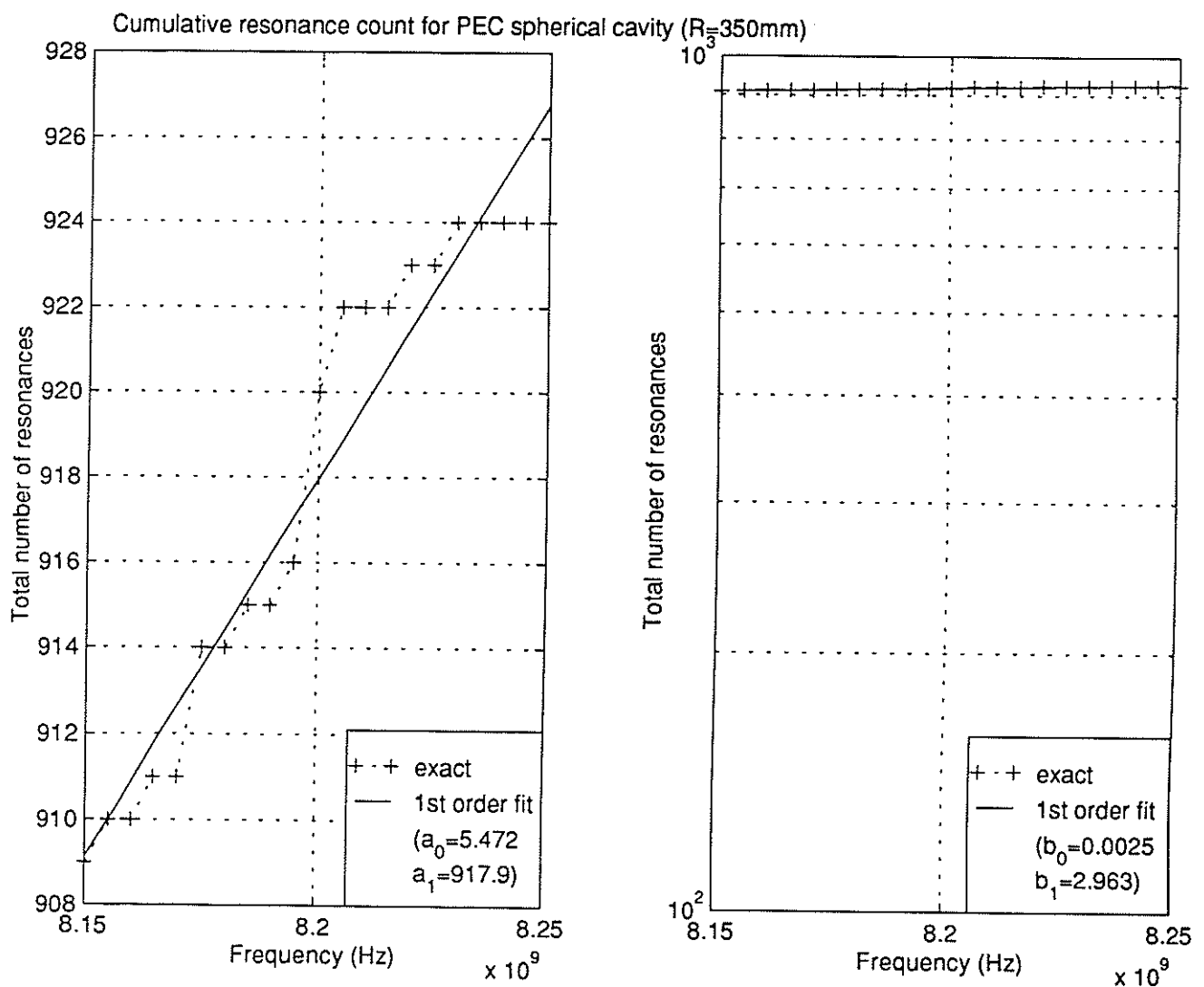


Fig A.11

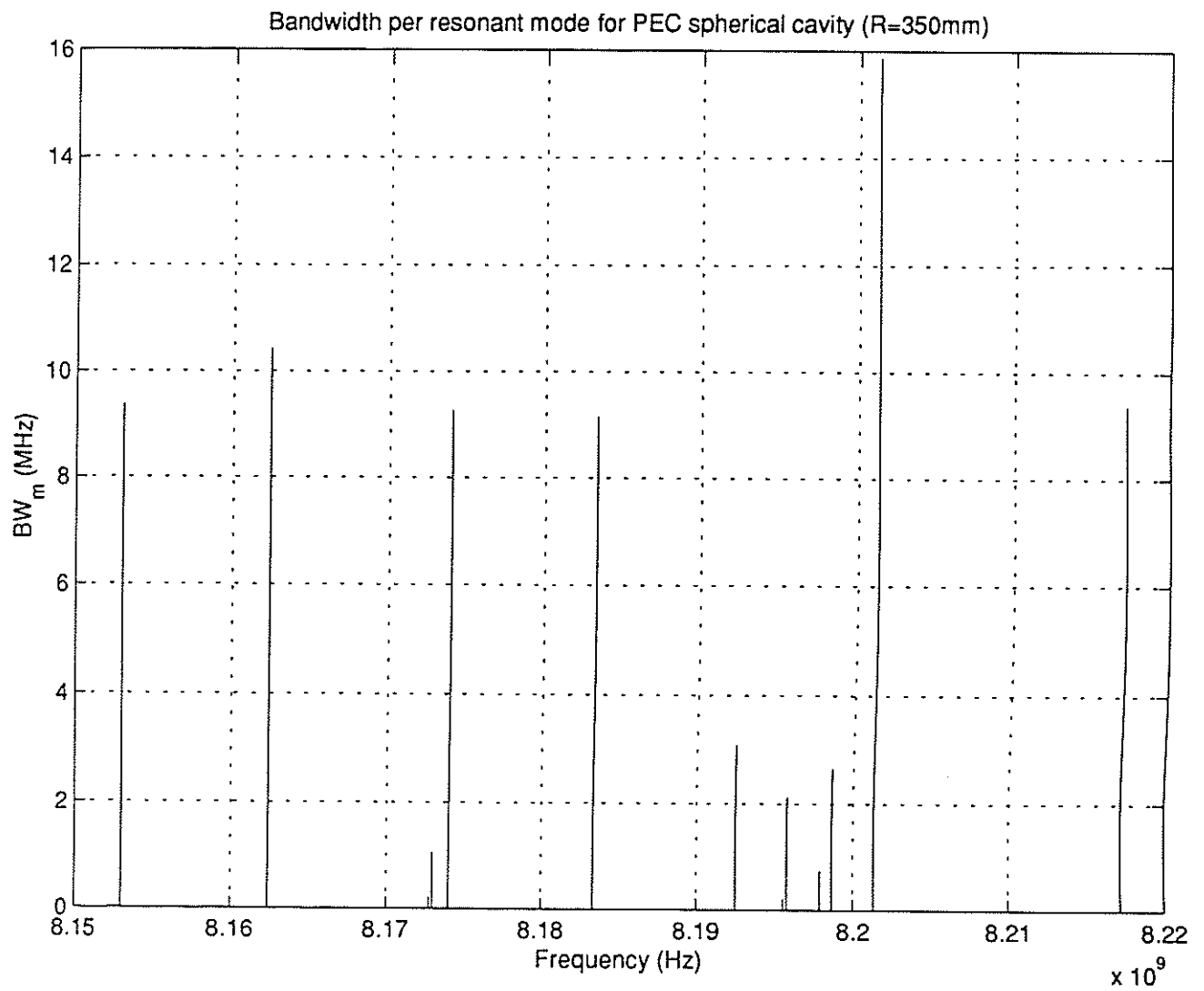


Fig A.12

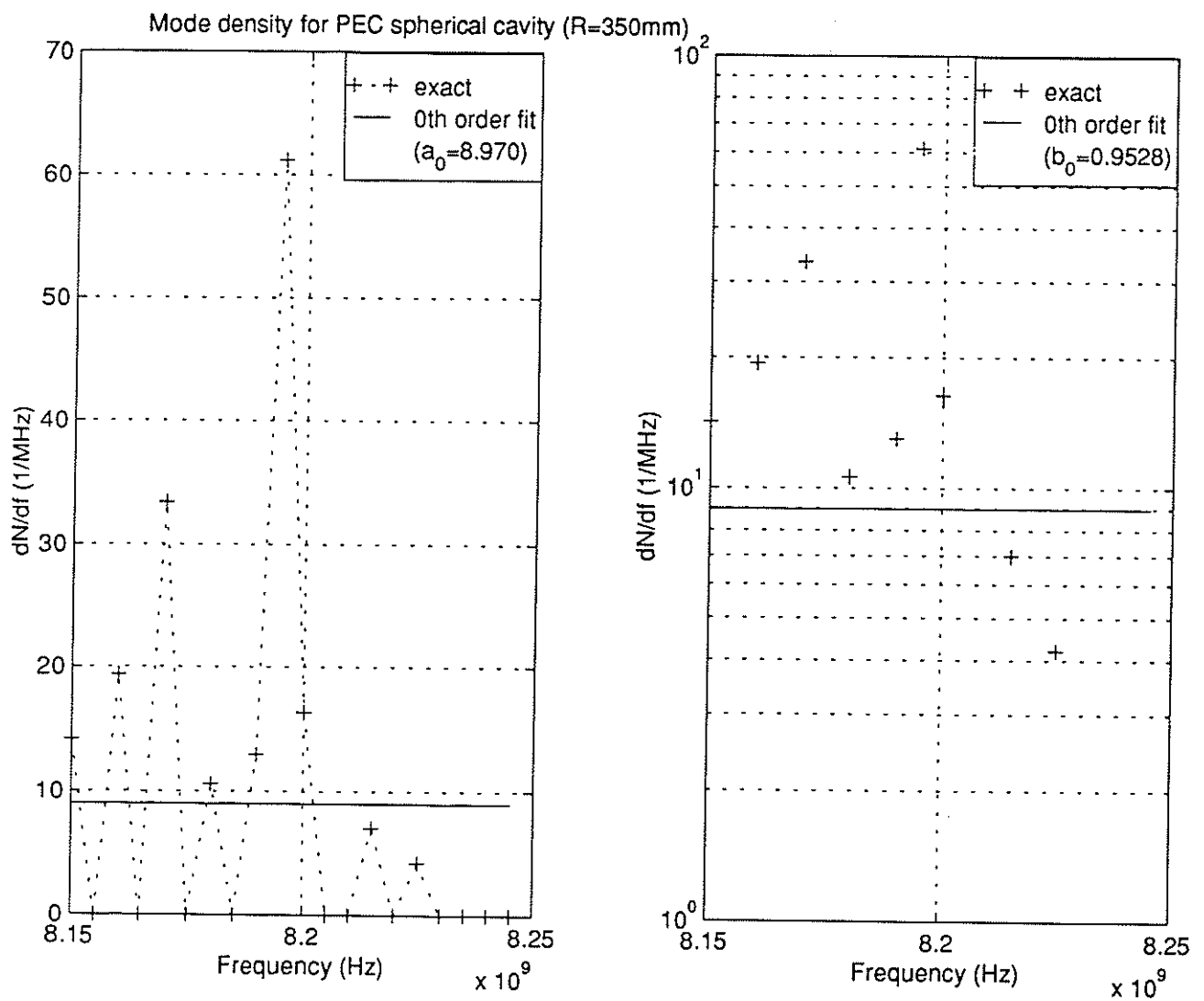


Fig A.13

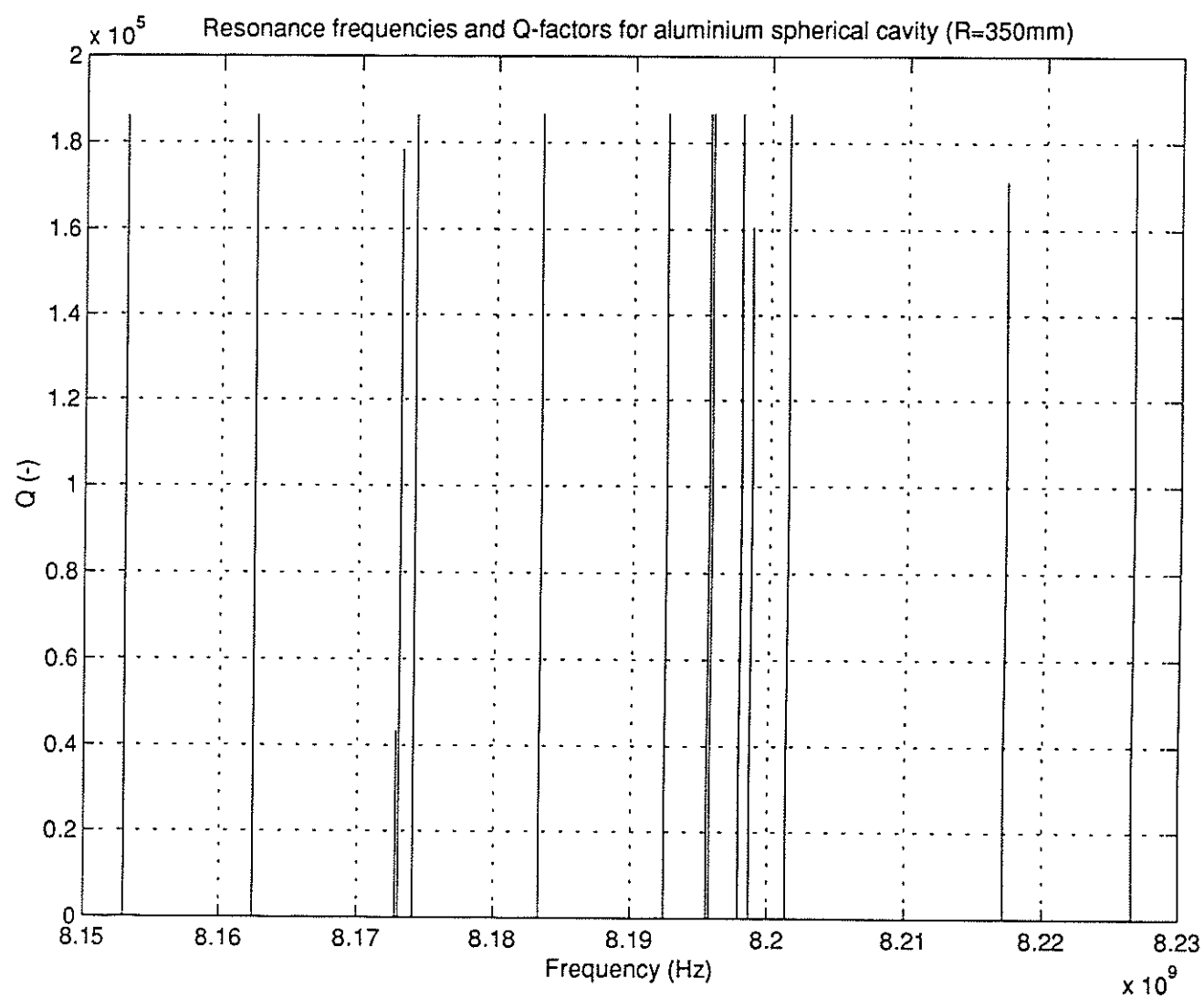


Fig A.14

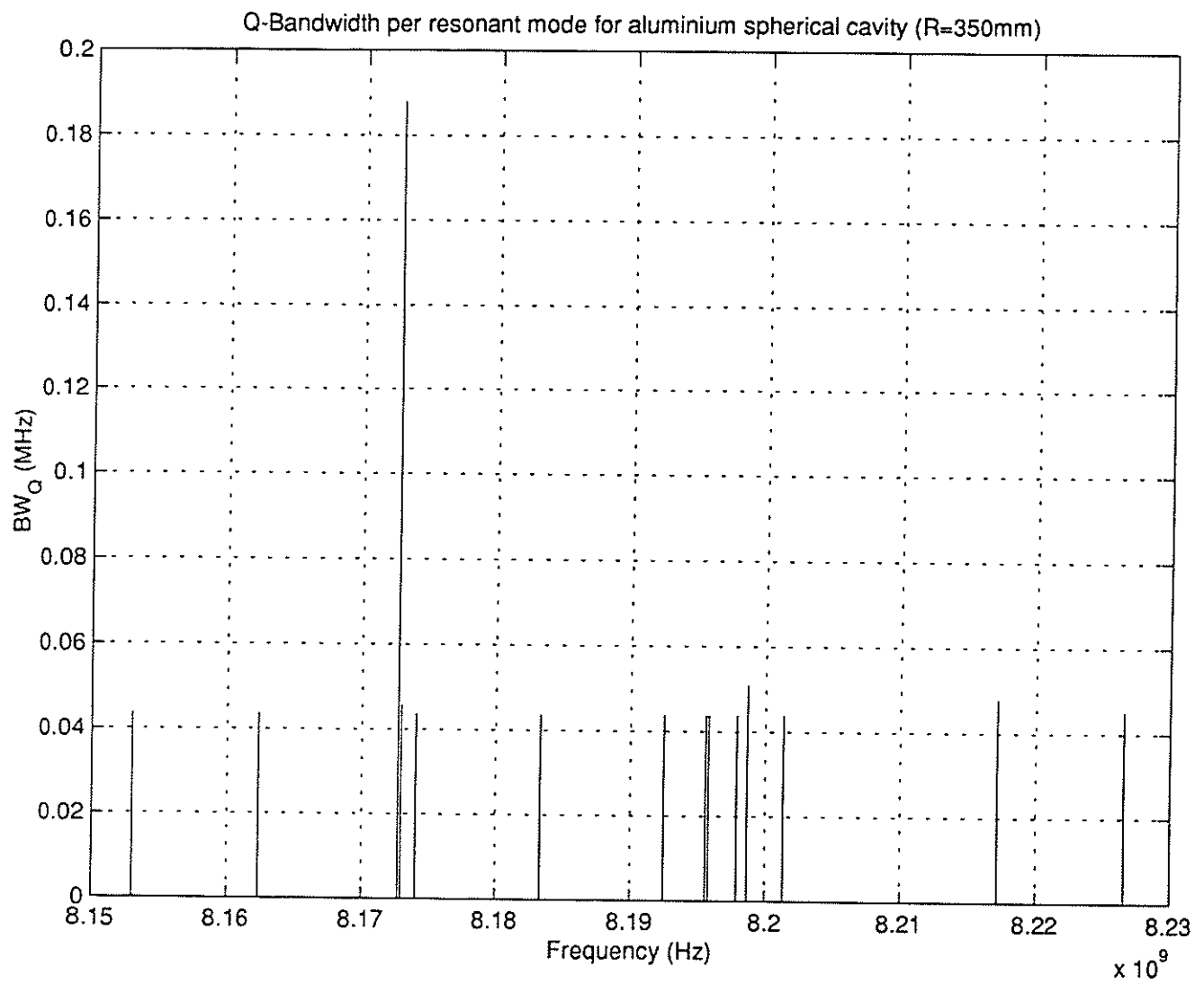


Fig A.15

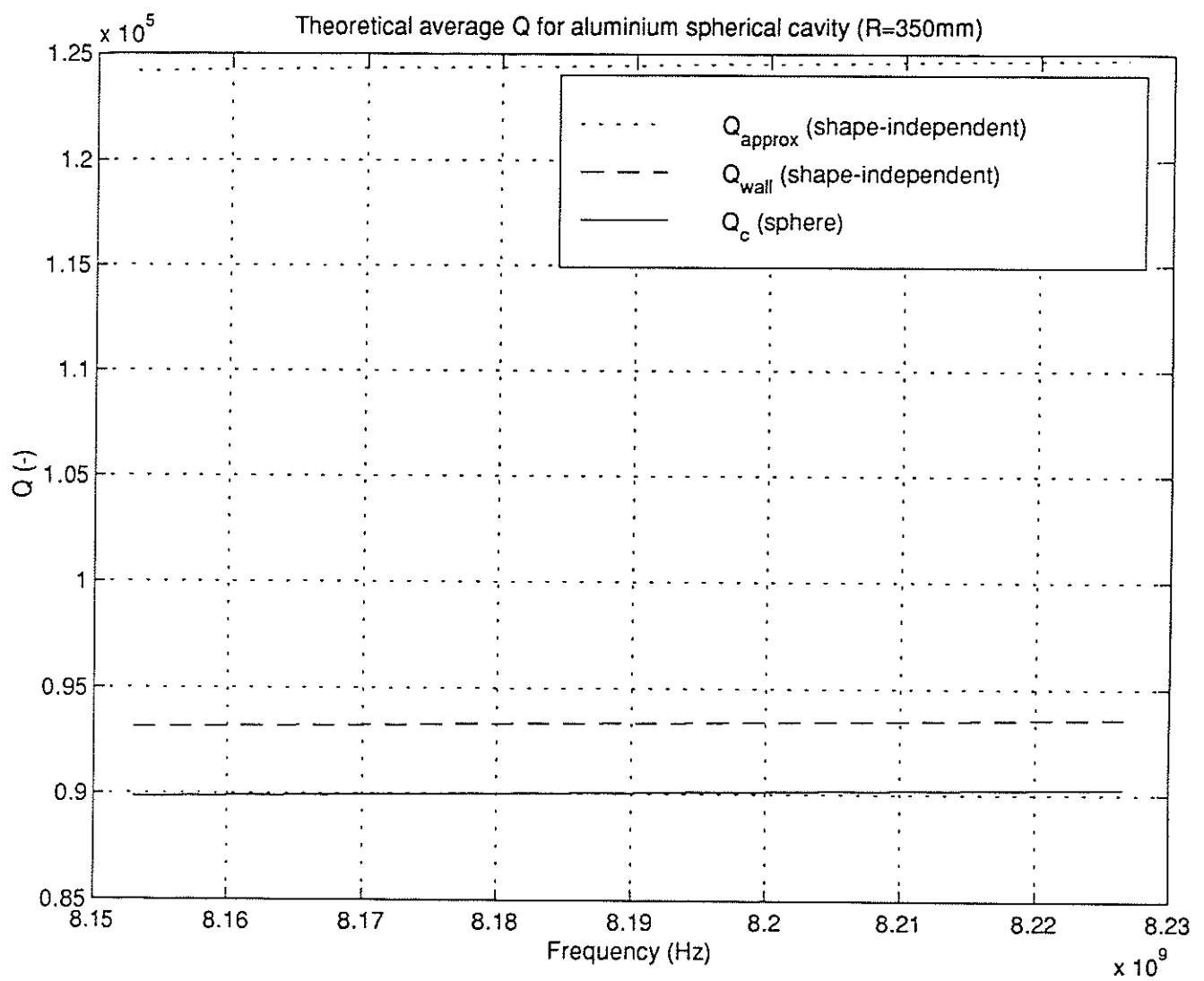


Fig A.16

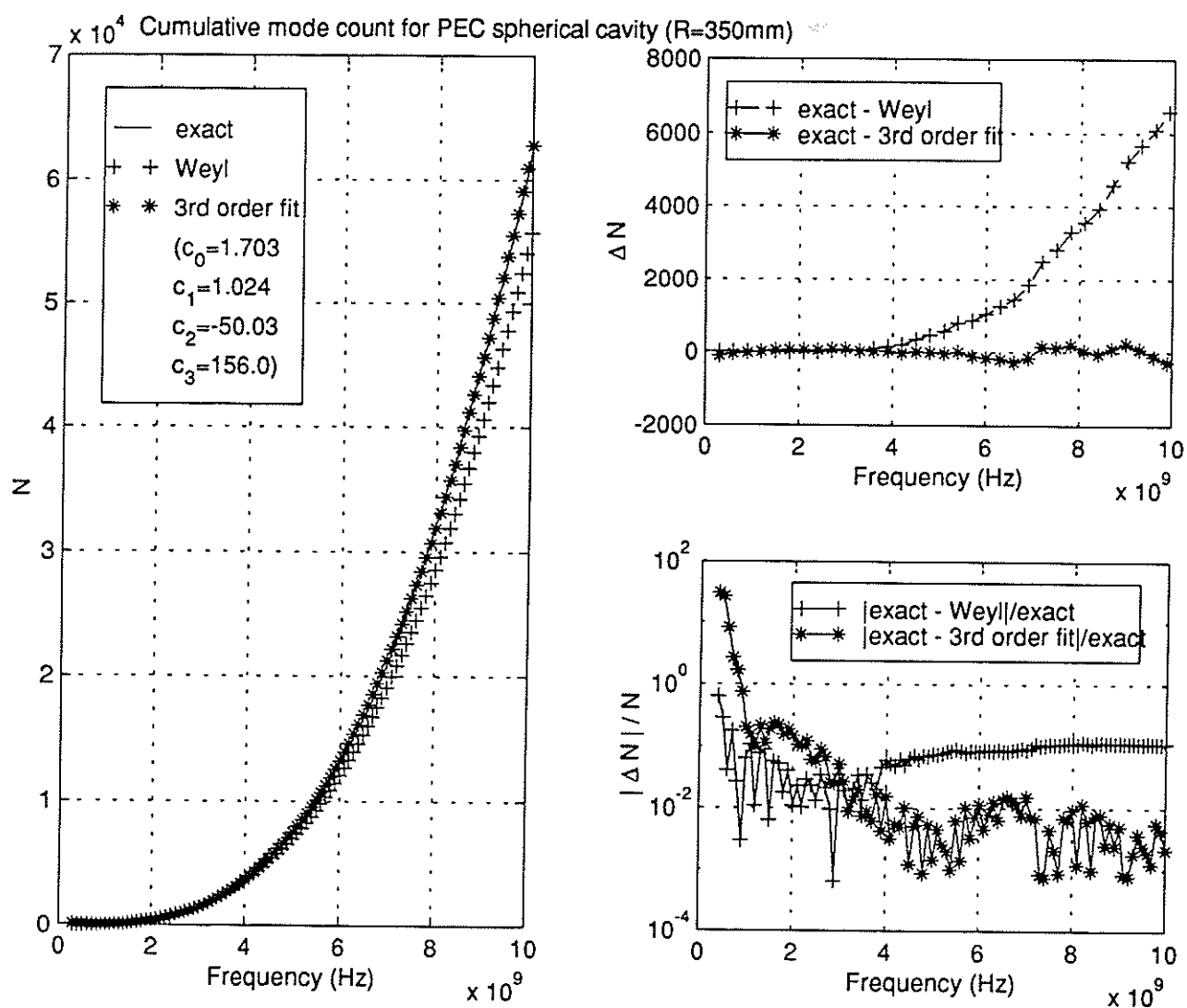


Fig A.17

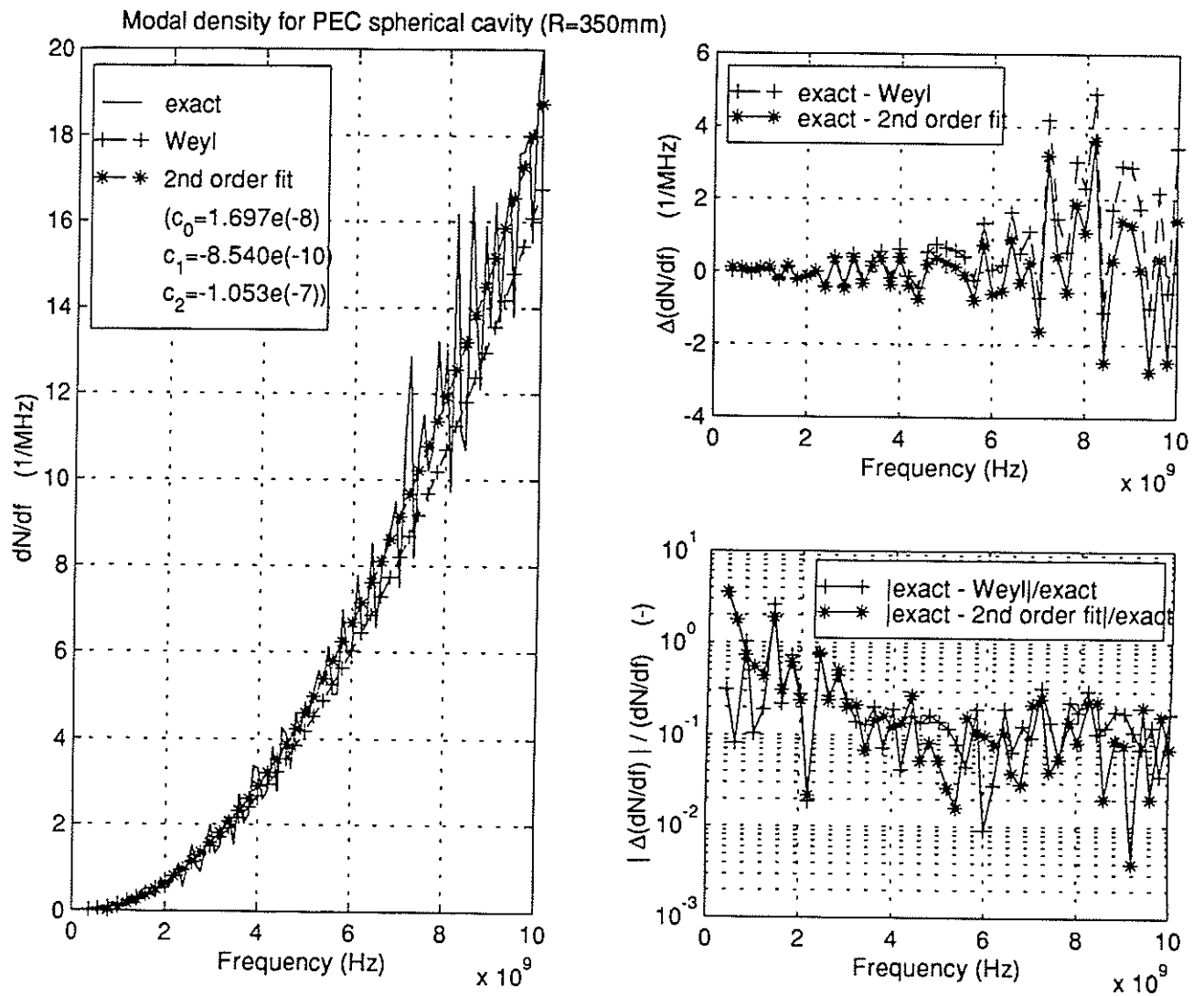


Fig A.18

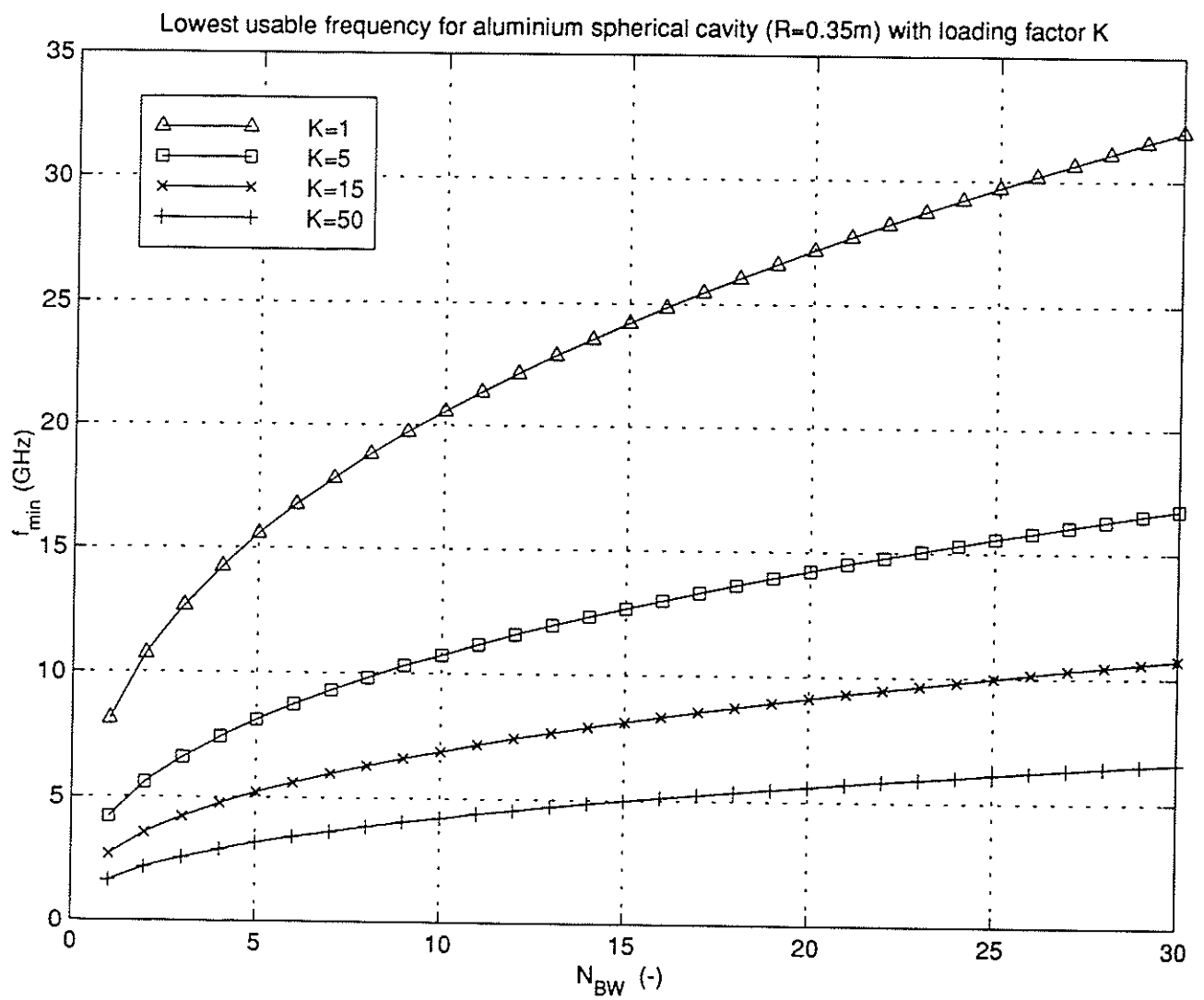


Fig A.19

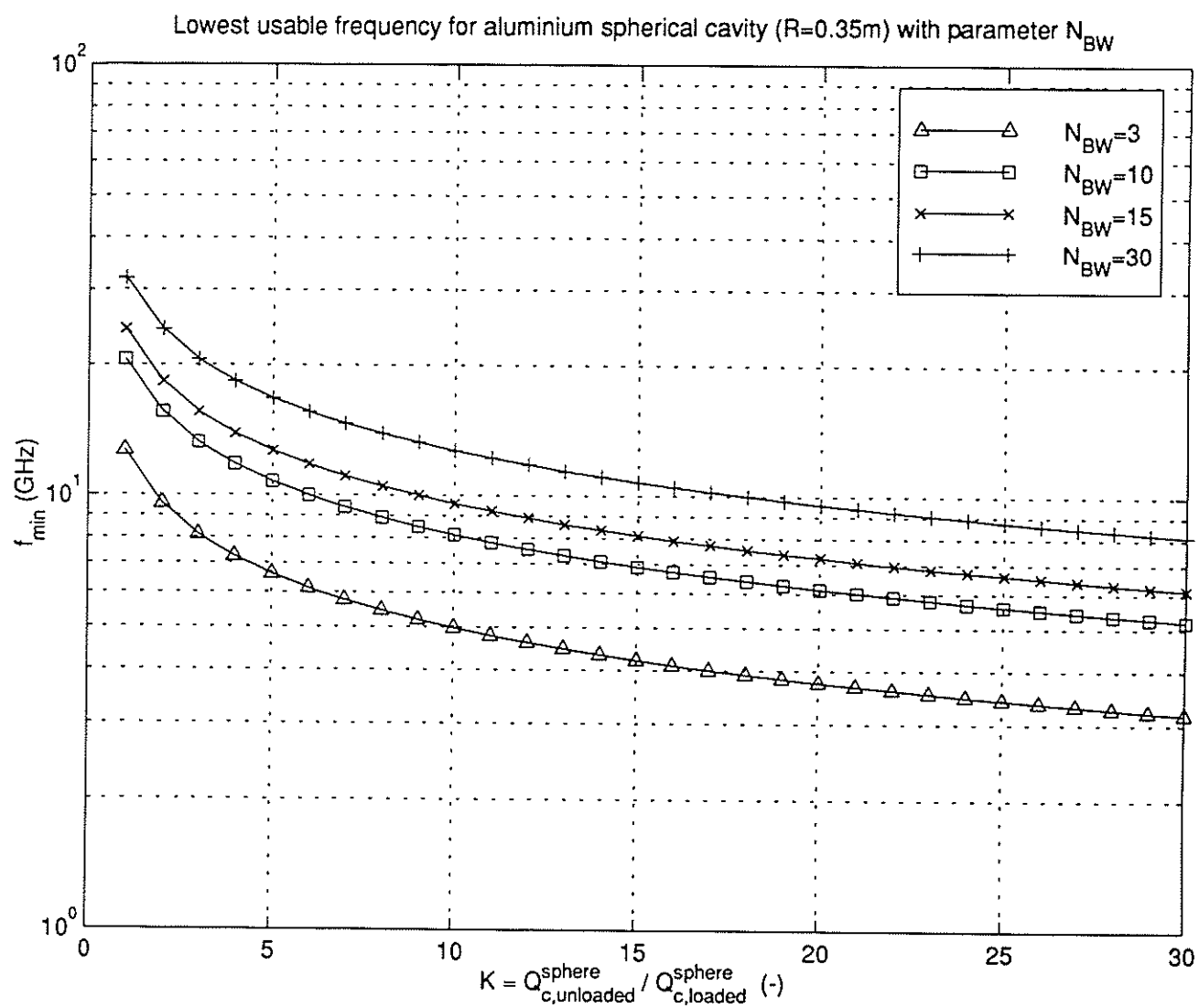
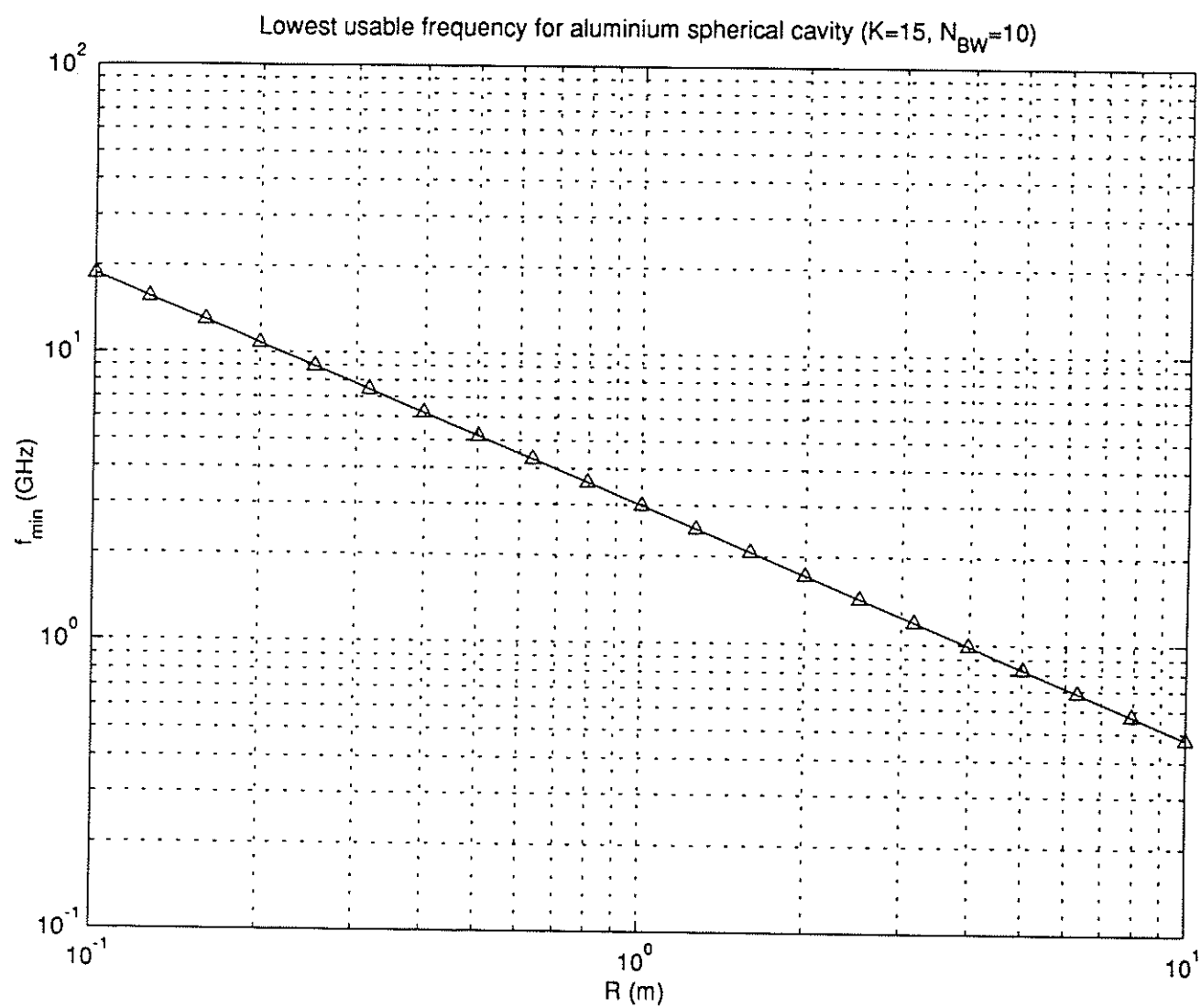


Fig A.20

**Fig A.21**

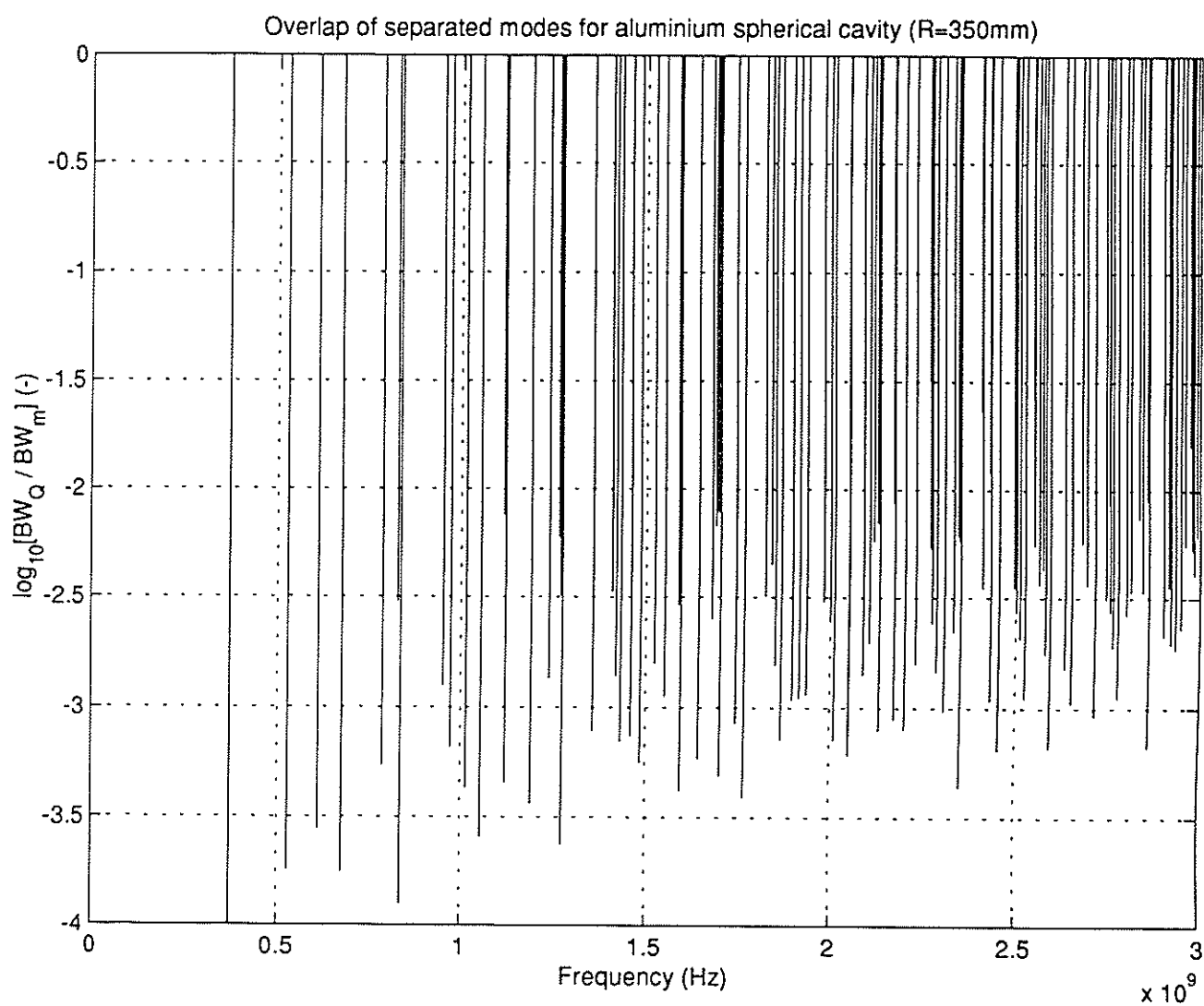


Fig A.22

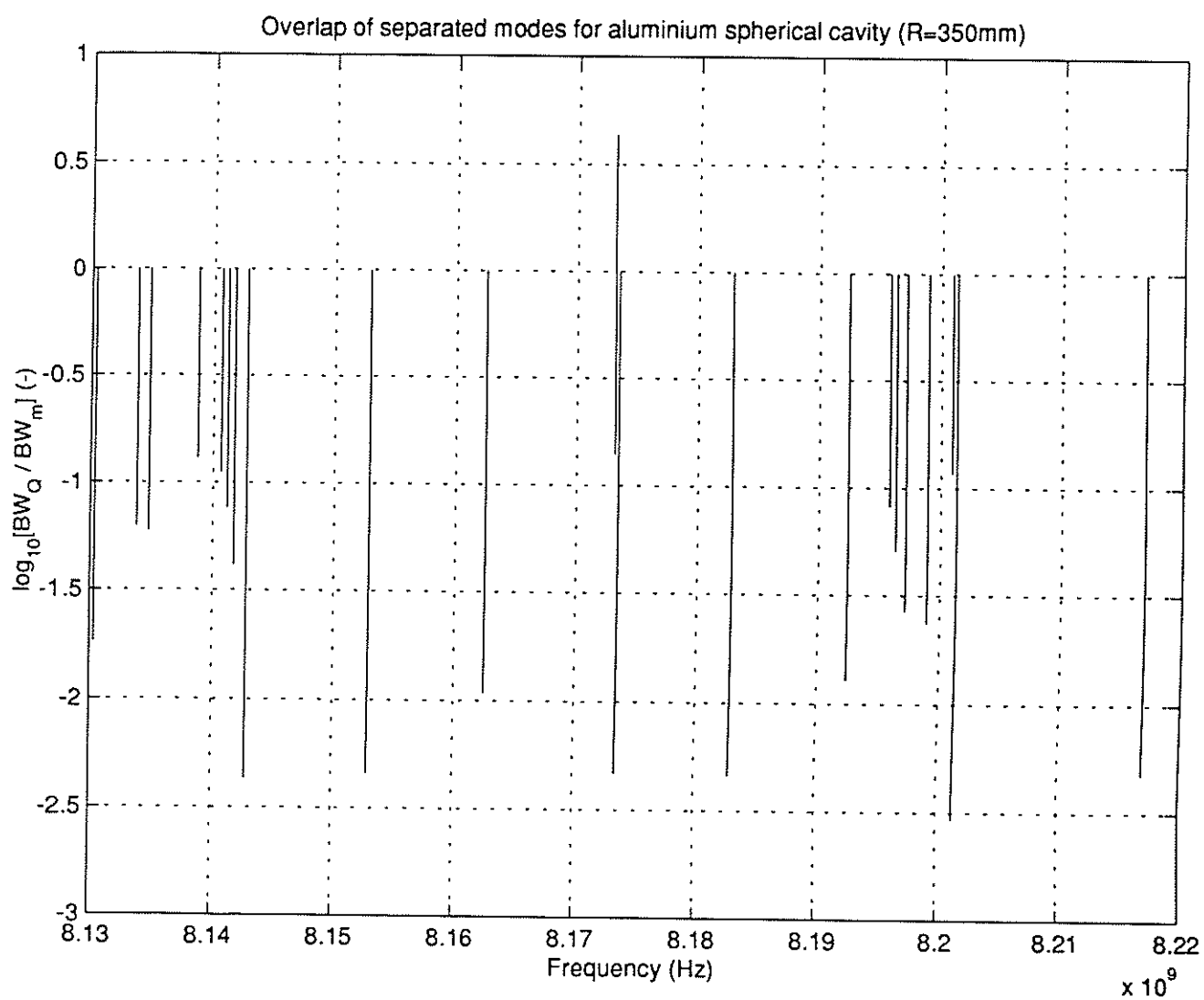


Fig A.23

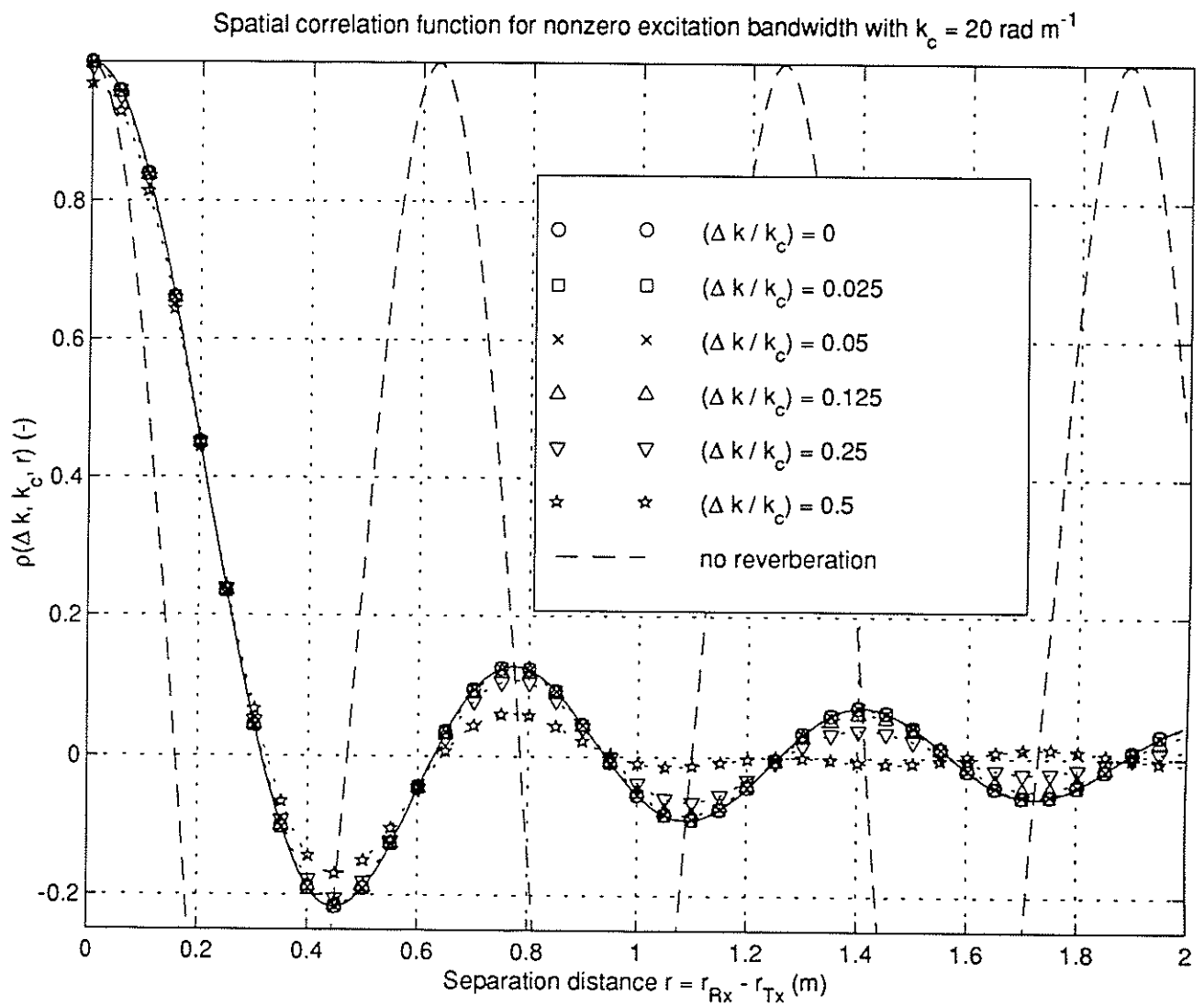


Fig B.1

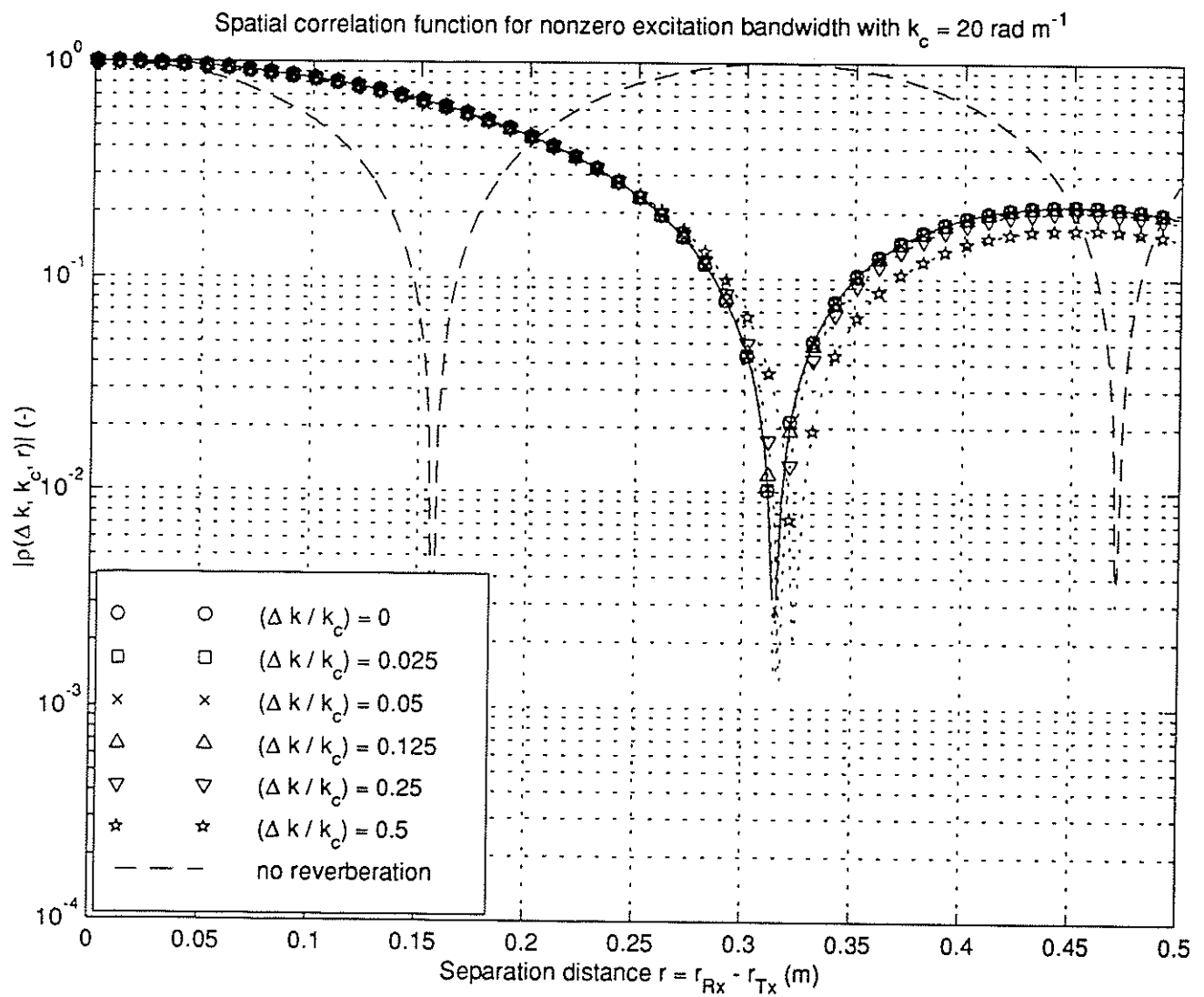


Fig B.2

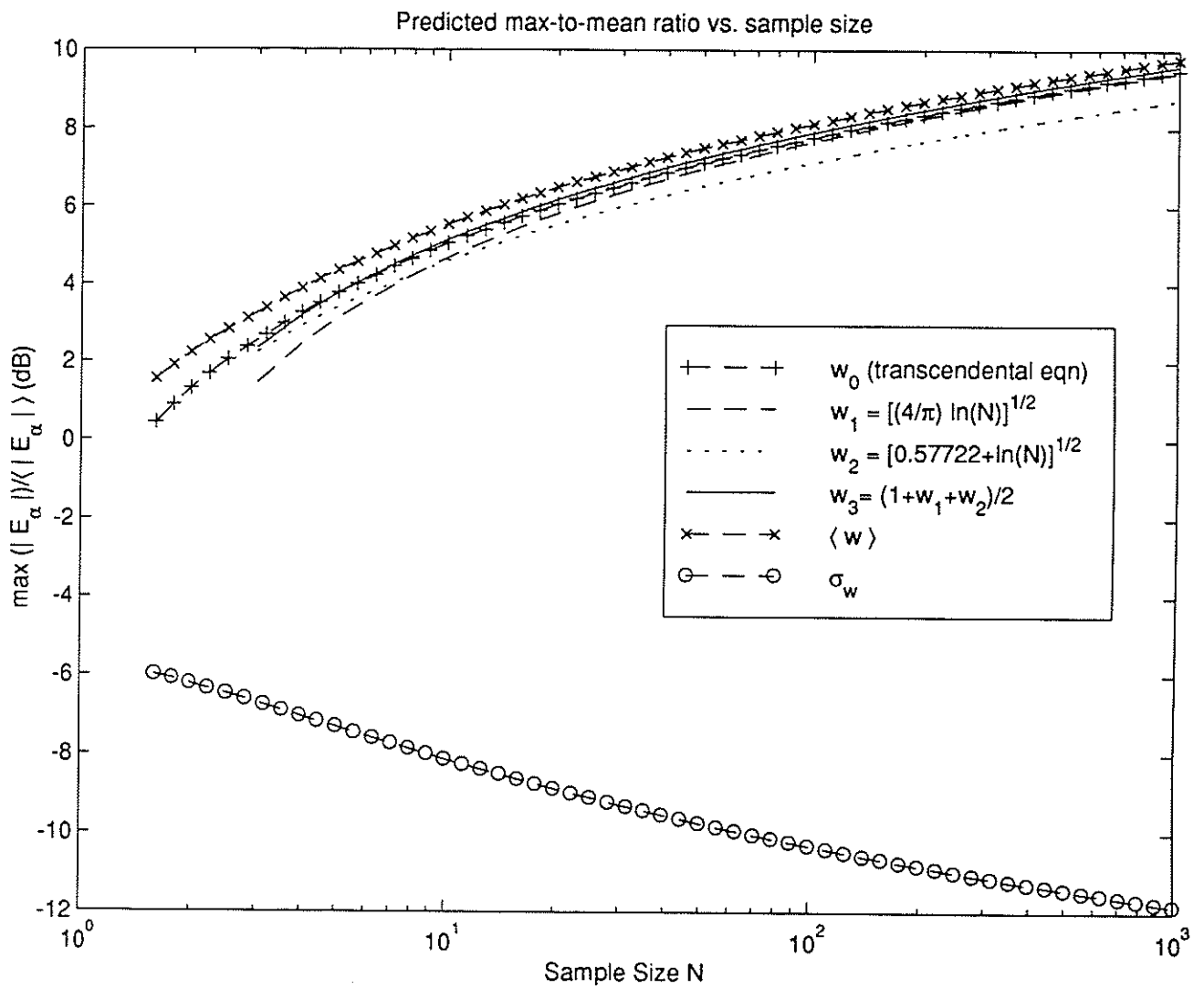


Fig C.1

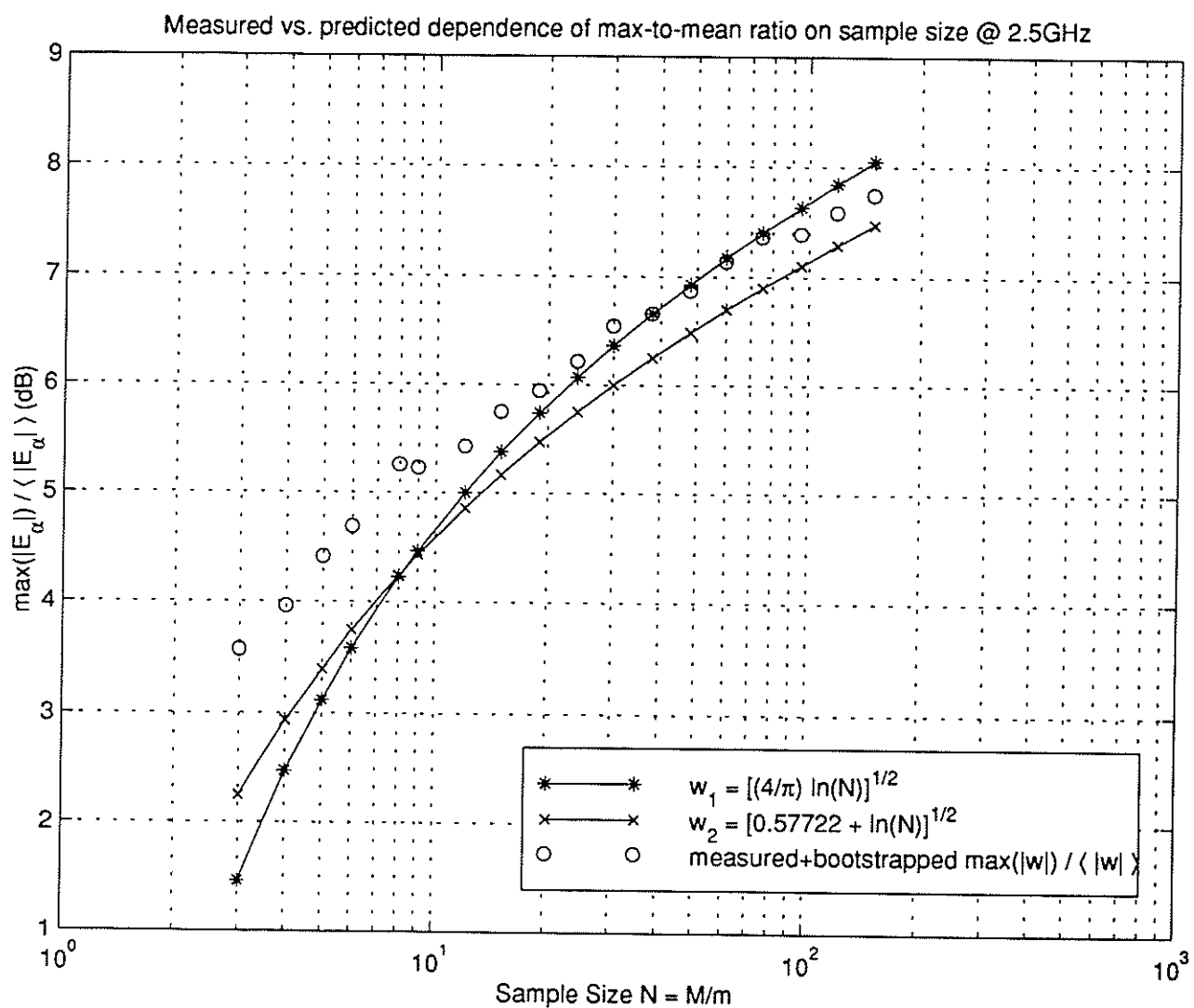


Fig C.2

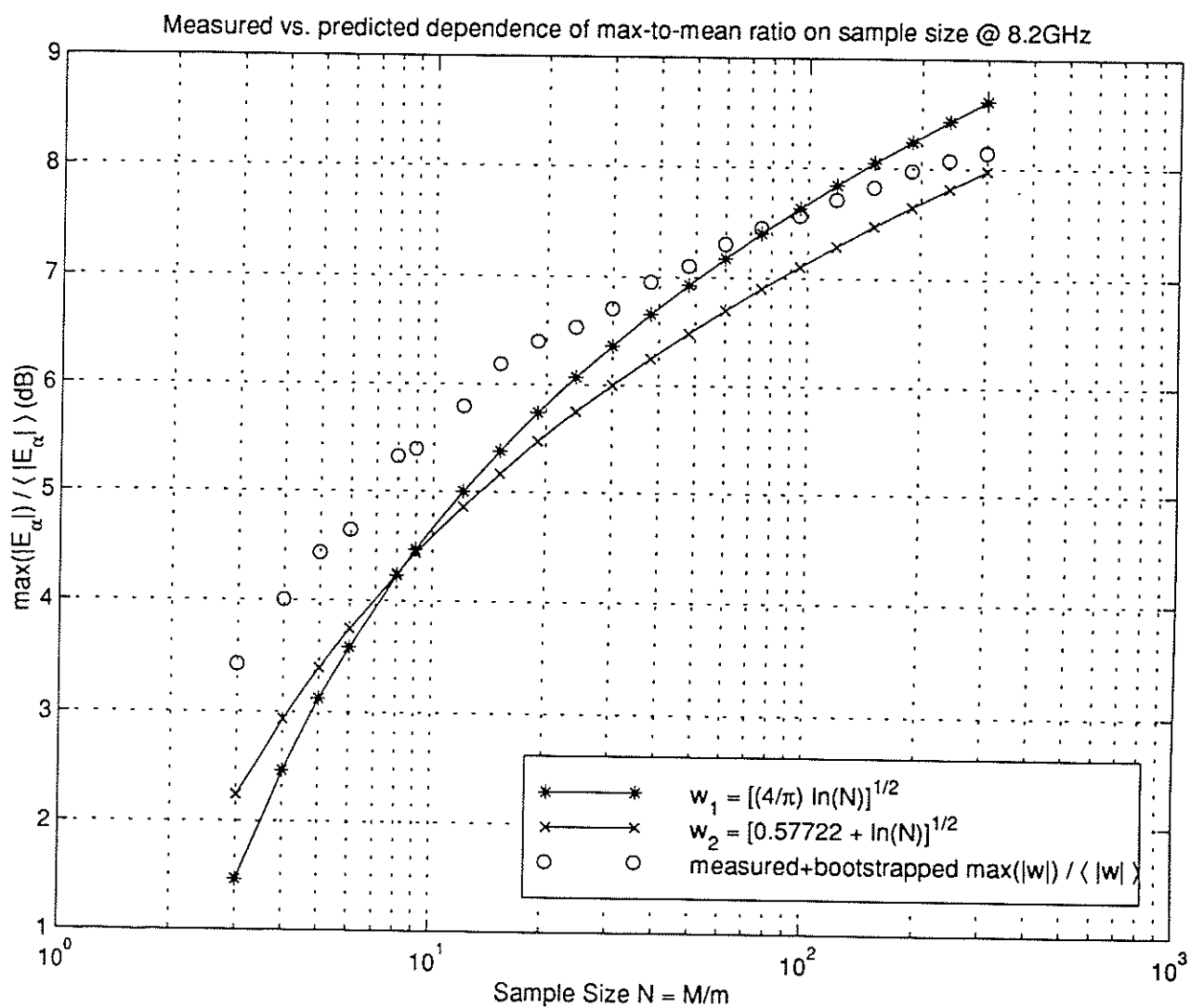


Fig C.3

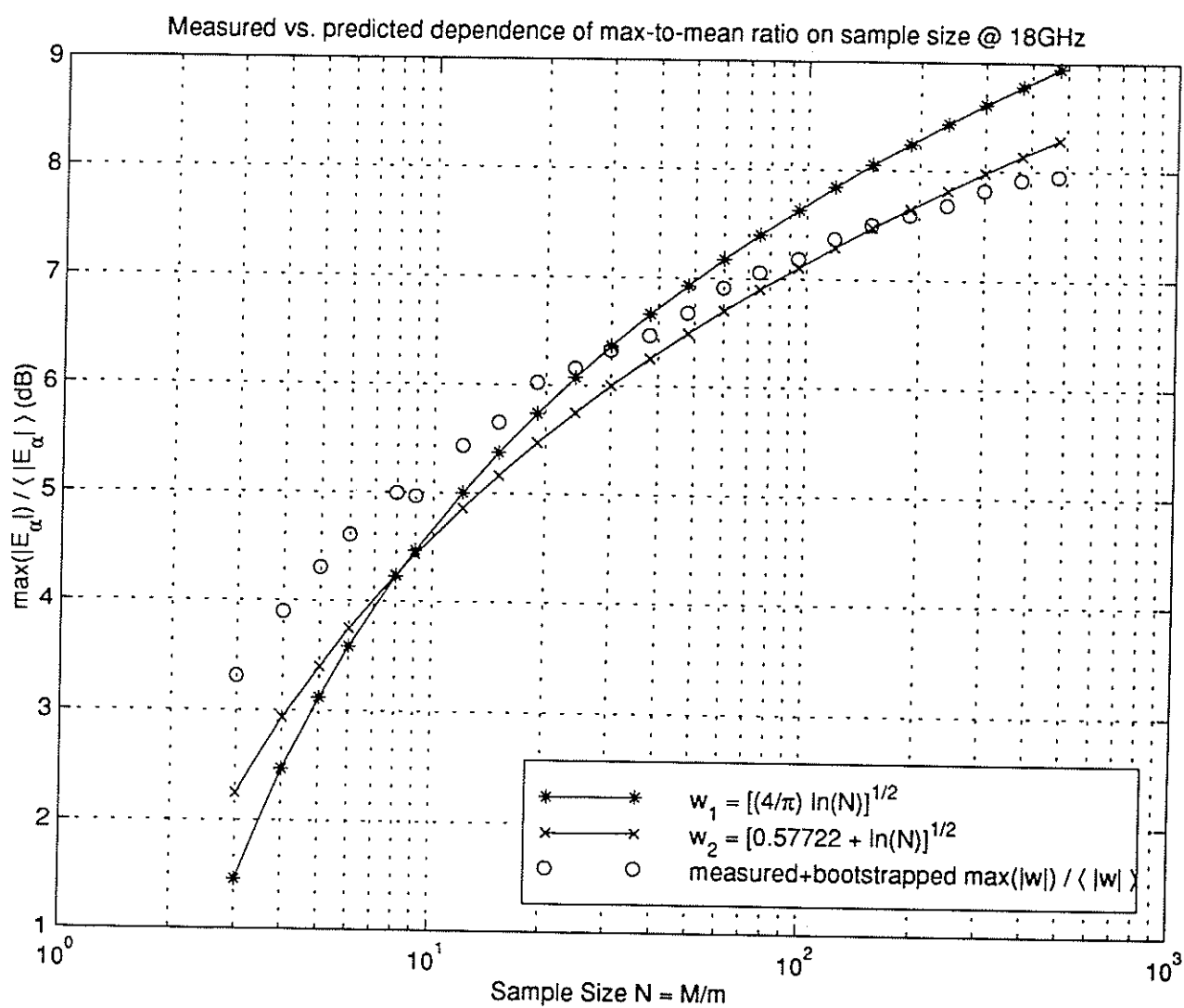


Fig C.4

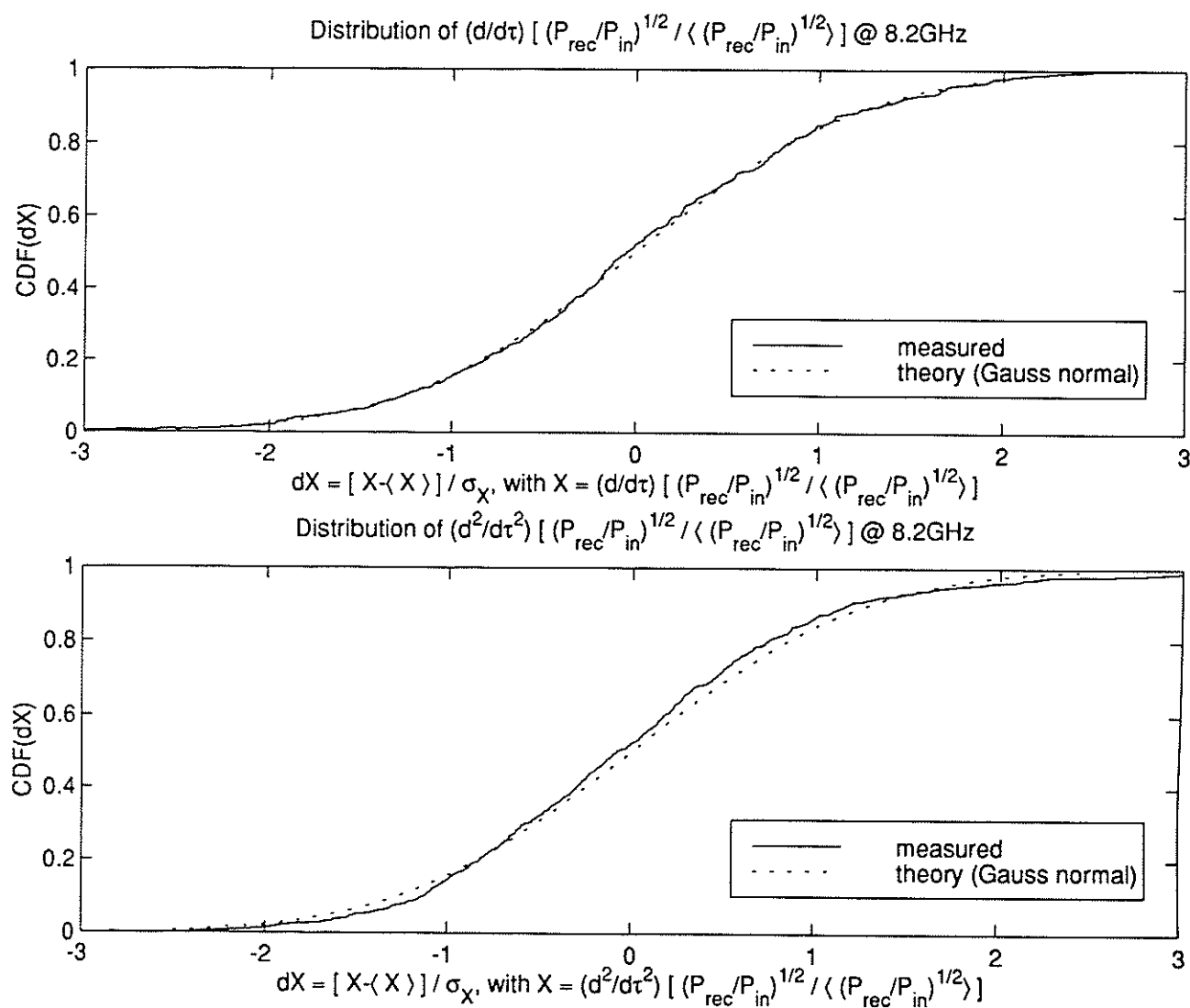


Fig C.5