

NPL Report CMMT(A)117

**Project PAJ2
Report No 7**

**An Evaluation of Yield
Criteria for Adhesives
for Finite Element
Analysis**

G D Dean, B E Read and B C Duncan

January 1999

An Evaluation of Yield Criteria for Adhesives for Finite Element Analysis

G D Dean, B E Read and B C Duncan
Centre for Materials Measurement & Technology
National Physical Laboratory
Queens Road, Teddington, Middlesex TW11 0LW

SUMMARY

Tensile and shear tests on bulk specimens of adhesives and tensile tests on butt-joint specimens have been employed to obtain stress/strain data for use with elastic-plastic models in finite element analyses of bonded joints. The adhesives investigated were a toughened epoxy and a toughened acrylic. The tests were carried out in an Instron machine at different crosshead speeds chosen to give the same effective plastic strain-rate for each mode of deformation.

The results of the tensile and shear tests reveal that the von Mises yield criterion is not applicable to these materials except possibly at small plastic strains. The test results were therefore used to derive parameters for the linear Drucker-Prager and the exponent Drucker-Prager materials models. These models were then used in finite element analyses to predict force-extension curves for the butt-joint specimens. It was observed that the exponent Drucker-Prager model appeared to accurately describe the deformation behaviour of the epoxy whilst the linear Drucker-Prager model was more appropriate for the acrylic.

These are believed to be chance observations, and a more detailed analysis of results has indicated that none of these models are suitable for toughened adhesives under all stress states. It is concluded that the von Mises criterion, possibly modified to include a small dependence on hydrostatic stress, is suitable for stress states and magnitudes in which the hydrostatic tensile stress is insufficient to initiate cavitation in the rubber or crazing in the matrix polymer. When the stress state and magnitude induce cavitation or crazing in the adhesive, then an alternative or modified model must be used to describe the influence of these mechanisms on deformation behaviour.

The basis for the development of a new model is briefly discussed.

© Crown copyright 1998
Reproduced by permission of the Controller of HMSO

ISSN 1361-4061

National Physical Laboratory
Teddington, Middlesex, UK, TW11 0LW

Extracts from this report may be reproduced provided
that the source is acknowledged.

Approved on behalf of Managing Director, NPL, by Dr C Lea,
Head of Centre for Materials Measurement & Technology

[LJN7:CMMTA117/V2]

CONTENTS

INTRODUCTION	1
2 ELASTIC-PLASTIC MODELS	2
2.1 YIELD CRITERIA	2
2.1.1 The Linear Drucker-Prager Criterion	2
2.1.2 The Exponent Drucker-Prager Criterion	3
2.1.3 Differences between the modified von Mises criteria	4
2.2 STRAIN HARDENING AND FLOW	5
3 DETERMINATION OF MODEL PARAMETERS	5
3.1 ELASTIC PROPERTIES (E , ν_e)	6
3.2 STRAIN-HARDENING FUNCTION ($\sigma_T(\epsilon_{pT})$)	6
3.3 HYDROSTATIC-STRESS SENSITIVITY PARAMETERS	6
3.3.1 Determination of λ	7
3.3.2 Determination of a	7
3.4 FLOW PARAMETER Ψ	8
4 EXPERIMENTAL	8
4.1 MATERIALS	8
4.2 TENSILE MEASUREMENTS ON BULK SPECIMENS	9
4.3 SHEAR MEASUREMENTS	10
4.3.1 Corrections to shear data	10
4.4 TENSILE MEASUREMENTS ON BUTT-JOINT SPECIMENS	12
5 RESULTS AND DISCUSSION	13
5.1 STRESS/STRAIN CURVES AND MODEL PARAMETERS	13
5.2 RESULTS AND ANALYSIS OF BUTT-JOINT TENSION TESTS	14
5.2.1 The epoxy adhesive	14
5.2.2 The acrylic adhesive	15
5.3 A NEW YIELD CRITERION	16
6 CONCLUSIONS	17
7 ACKNOWLEDGEMENTS	18
8 REFERENCES	19
APPENDIX 1. TERMINOLOGY FOR STRESSES AND STRAINS	20

NOMENCLATURE

E	Young's modulus
ν_e	the elastic component of Poisson's ratio
σ_T, ϵ_T	stress and strain values, respectively, under uniaxial tension
σ_s, ϵ_s	stress and strain values, respectively, under shear
$\sigma_1, \sigma_2, \sigma_3$	principal stress components
$\bar{s}$	the effective stress
ϵ_{pT}	component of plastic strain under uniaxial tension
$\epsilon_{p1}, \epsilon_{p2}, \epsilon_{p3}$	components of principal plastic strain
$\bar{e}_p$	the effective plastic strain
ν_p	plastic component of Poisson's ratio
p	the hydrostatic component of stress (see equation (10b))
q	the shear component of the effective stress (see equation (10a))
λ	ratio of compressive to tensile yield stresses at the same effective plastic strain
$A, B, \tan \beta$	parameters in the linear Drucker-Prager yield criterion related to λ
a, b, p	parameters in the exponent Drucker-Prager yield criterion
Ψ	the flow parameter
E_s, G_s	secant moduli obtained from plots of stress against plastic strain under uniaxial tension and shear, respectively.

1 INTRODUCTION

A finite element analysis is a powerful tool in the design of adhesively bonded joints. An analysis is able to locate regions of stress and strain concentration in the adhesive, which will be sites for failure initiation, and to show how stress or strain levels can be reduced through changes to the size or geometry of the joint or the inclusion of adhesive fillets at the ends of the bond. In conjunction with a suitable criterion for failure of the adhesive, an analysis can be used to indicate when a joint will fail under different loading conditions such as impact, creep or fatigue. For this purpose, high accuracy is needed in the calculated stress and strain levels in the regions where failure initiates. The accuracy of the calculations depends on the validity of the materials model used by the analysis for describing the deformation behaviour of the adhesive. Since adhesives are generally tough materials, they can sustain large strains before failure and, under these conditions, relationships between stress and strain are highly non-linear.

The non-linear behaviour of plastics, and this includes adhesives, is generally associated with an enhanced stress-relaxation or creep arising from a stress-induced increase in molecular mobility. Much of the strain is recoverable at levels below the yield strain, which is usually taken to correspond to the peak or to the onset of a plateau in a stress-strain curve. However, satisfactory models of this non-linear viscoelasticity have yet to be developed, and most FE packages consider material non-linearity in terms of elastic-plastic models that were developed for metals and adapted for use with adhesives and other materials.

A survey previously presented¹ of elastic-plastic models included proposed methods for determining relevant material parameters from tensile and shear stress-strain data on bulk adhesive specimens. In this report, details are given of the experimental techniques and data analyses to obtain these parameters for some additional adhesives at specified strain-rates. The tensile and shear data confirm results on other adhesives and show that a materials model based on the von Mises criterion is generally not suitable for toughened adhesives. Results have therefore been analysed to obtain parameters for models that include a dependence of yield behaviour on the hydrostatic component of stress. Using these models, predictions have been made of the deformation of butt-joint specimens loaded in tension in which the stress state is predominantly hydrostatic tension. Comparisons with experimental results are made which reveal limitations in these materials models. Modifications are suggested that should improve their overall applicability to adhesives.

2 ELASTIC-PLASTIC MODELS

2.1 YIELD CRITERIA

With elastic-plastic models, calculations of stress and strain distributions at low strains (≤ 0.01) are based on the theory of linear elasticity using, for isotropic materials, values of the Young's modulus E and elastic Poisson's ratio ν_e . The onset of non-linearity in a stress-strain curve is then ascribed to plastic deformation and occurs at a stress level regarded as the first yield stress. The subsequent increase in stress with strain is associated with the effects of strain hardening. In this non-linear region the total strain is considered as the sum of a recoverable, elastic

component and a plastic component which is non-recoverable. Stress analysis calculations now involve the use of multi-axial yield criteria, usually expressed in terms of the principal stress components σ_1 , σ_2 and σ_3 .

2.1.1 The Linear Drucker-Prager Criterion

The von Mises yield criterion specifies that yielding occurs when the second invariant of the deviatoric (shear) stress tensor

$$J_{2D} = \frac{1}{6} [(s_1 - s_2)^2 + (s_2 - s_3)^2 + (s_3 - s_1)^2] \quad (1)$$

reaches a critical value. However, for many types of adhesive it has been shown that this value further depends on the hydrostatic component of stress or stress invariant

$$J_1 = s_1 + s_2 + s_3 \quad (2)$$

A relatively simple criterion for adhesives^{2,3} assumes that yielding will occur at an effective stress $\bar{s}$, numerically equal to the yield stress σ_T under uniaxial tension, that is linearly related to the critical $J_{2D}^{1/2}$ and J_1 values, so that

$$\bar{s} = s_T = AJ_{2D}^{1/2} + BJ_1 \quad (3)$$

where

$$A = \frac{\sqrt{3}(l+1)}{2l}; \quad B = \frac{(l-1)}{2l} \quad (3a)$$

We note that σ_T is a true stress (see Appendix 1) which for a strain hardening material will increase with the plastic component of tensile strain. Equation (3) then defines a yield surface that will expand as σ_C increases with strain. Different stress states with the same effective stress will lie on the same yield surface and have the same effective plastic strain (see below). The parameter λ equals the ratio σ_C/σ_T where σ_C is the yield stress under uniaxial compression associated with the same effective plastic strain as σ_T . The value of λ provides a measure of the sensitivity of yield behaviour to the hydrostatic component of stress. It is related as follows to a parameter $\tan \beta$ which gives the change in shear yield stress with change in hydrostatic pressure and appears in an equivalent version of the above yield criterion due to Drucker and Prager (see ABAQUS technical manual⁴)

$$\tan \beta = 3(l-1)/(l+1) \quad (4)$$

An effective plastic strain $\bar{e}_p$ has also been defined in the form of equation (3) that is numerically equal to the plastic component of strain ϵ_{pT} under uniaxial tension, ie

$$\bar{e}_p = \frac{\sqrt{3}(l+1)}{2l(1+n_p)} I_{2D}^{1/2} + \frac{(l-1)}{2l(1-2n_p)} I_1 = e_{pT} \quad (5)$$

$$I_{2D} = \frac{1}{6} \left[(e_{p1} - e_{p2})^2 + (e_{p2} - e_{p3})^2 + (e_{p3} - e_{p1})^2 \right] \quad (6)$$

$$I_1 = e_{p1} + e_{p2} + e_{p3} \quad (7)$$

Here ε_{p1} , ε_{p2} and ε_{p3} are the plastic components of the principal strains and v_p is the plastic component of Poisson's ratio. Note that $\bar{e}_p$, ε_{pT} and the ε_{pn} ($n = 1,2,3$) are all true strains and that v_p is defined in terms of true strain components (see Appendix 1). With these definitions the sum of principal strains I_1 is zero and $v_p = 0.5$ for a plastic deformation at constant volume.

2.1.2 The Exponent Drucker-Prager Criterion

An alternative form for a modified von Mises yield criterion is given by

$$s_T^2 = \frac{3J_{2D}}{l} + \frac{(l-1)s_T J_1}{l} \quad (8)$$

This has been used with some success^{5,6} to describe the influence of the hydrostatic component of stress on the yield behaviour of plastics. This criterion has been implemented in ABAQUS as a particular case of the exponent Drucker-Prager model, which is expressed in the form

$$aq^b = p + p_t \quad (9)$$

Here

$$q = \sqrt{3} J_{2D}^{1/2} \quad (10a)$$

is the shear component of the effective stress, and

$$p = \frac{J_1}{3} \quad (10b)$$

is the hydrostatic pressure.

Equation (9) is the same as the criterion in equation (8) if the parameters a , b and p_t are given by

$$B = 2 \quad (11a)$$

$$a = \frac{1}{3(s_C - s_T)} \quad (11b)$$

$$\text{and } p_t = as_C s_T \quad (11c)$$

In the Abaqus model λ is a constant. This requires that the quantity $(\sigma_C - \sigma_T)$ should also be constant which implies that λ is dependent upon the plastic strain ϵ_p . Clearly the parameter p increases with plastic strain for a strain hardening material.

2.1.3 Differences between the modified von Mises criteria

In a principal stress coordinate system, the yield surface given by equation (3) for a particular value of σ_T is a cone with an axis that coincides with the hydrostatic stress axis. As σ_T increases with plastic strain, the cone expands uniformly. The yield surface given by equation (8) is a paraboloid, again with its axis coinciding with the hydrostatic stress axis. These surfaces intersect a plane of principal stress in an ellipse and either criterion can be used with approximately equal success to describe yield under biaxial stress states⁶. The important differences between these criteria are apparent under triaxial stress states as is illustrated in figure 1. Here the criteria are plotted on axes defined by the quantities p and q which are related to principle stress components by equations (10), (1) and (2). The plots are related to sections of the cone and paraboloid in principal stress space in a plane containing the hydrostatic stress axis. Both curves pass through the same values of yield stress under shear and uniaxial tension. The differences become significant under stress states which have a high component of hydrostatic tensile stress ($-p$). This is highly relevant for the stress analysis of adhesive joints, as will be discussed later in this report, since hydrostatic stress states are generated in the adhesive layer by the constraint imposed by a stiff adherend and are generally sites for fracture initiation.

2.2 STRAIN HARDENING AND FLOW

Strain hardening may now be represented as the dependence of effective stress on effective plastic strain ($\bar{s} = f(\bar{e}_p)$). Values of v_p are also employed in the calculation of the flow parameter $\tan \Psi$ which is the ratio of the change in volumetric plastic strain to the change in deviatoric plastic strain during yield. The magnitude of $\tan \Psi$ is determined by the direction of plastic flow and is required by the flow law¹ for relating increments of plastic strain $d\epsilon_{pn}$ to components of stress σ_n . Existing FE packages make no allowance for changes in v_p or $\tan \Psi$ with strain, in which case we have

$$\tan \Psi = \frac{3(1 - 2n_p)}{2(1 + n_p)} \quad (12)$$

If the yielding is insensitive to the hydrostatic stress component then $\sigma_C = \sigma_T$, $\lambda = 1$ and we obtain from equation (3) the von Mises criterion

$$\bar{s} = s_T = \sqrt{3} J_{2D}^{1/2} \quad (13a)$$

and

$$\bar{e}_p = \frac{\sqrt{3}}{(1 + n_p)} I_{2D}^{1/2} \quad (13b)$$

In this case it may be assumed that the yield process occurs at constant volume so that $v_p = 0.5$ and $\psi = \beta = 0$.

3 DETERMINATION OF MODEL PARAMETERS

In this section, methods are described that were used to obtain the data required by the elastic-plastic models. Bulk specimens of the adhesives were prepared for these tests⁷. It should be noted that each method refers to measurements at a specified strain-rate, or at specified strain-rates, and has been applied to obtain values of the parameters over a wide strain-rate range.

3.1 ELASTIC PROPERTIES (E , ν_e)

The Young's modulus and Poisson's ratio were determined from measurements of stress, strain and lateral strain in a tensile test at small strains where the stress-strain plot is linear. Appropriate relations are given in the Appendix.

3.2 STRAIN-HARDENING FUNCTION ($\bar{s}(\bar{e}_p) = \sigma_T(\epsilon_{pT})$)

The strain-hardening function was obtained from measurements of yield stress σ_T and plastic strain ϵ_{pT} in a uniaxial tension test at strains ranging from that associated with the onset of non-linearity ($\epsilon \approx 0.01$) up to the failure strain (typically $\epsilon \geq 0.2$). Owing to the high strains involved, true stresses and true strains (as defined in the Appendix) were determined over the entire strain range. For strains above the linear range, the stress σ is identified with the yield stress σ_T and the true plastic strain component was calculated using equation A7 of the Appendix.

3.3 HYDROSTATIC-STRESS SENSITIVITY PARAMETERS

The determination of the parameters λ or a requires measurements of the yield behaviour under two different stress states. If stress-strain data are available under both compression and tension, then these parameters may be obtained directly from values of the yield stresses σ_C and σ_T corresponding to the same effective plastic strain. However, owing to difficulties associated with uniaxial compression tests¹, values for the parameters have been determined in this project from a combination of tension and shear tests.

This relies on the determination of yield stresses in shear σ_S and tension σ_T having the same effective plastic strain $\bar{e}_p$ from stress/strain curves measured in shear and tension at the same effective plastic strain rate. The procedure for doing this was explained in an appendix to reference (1) and is summarised below.

- (i) From corresponding σ_T and ϵ_{pT} values on the strain-hardening curve, the secant Young's modulus was calculated from $E_s = \sigma_T / \epsilon_{pT}$.

- (ii) A secant shear modulus $G_s = E_s/2(1+\nu_p)$ was evaluated from E_s and the measured ν_p (see equation (A9)).
- (iii) Plastic strains γ_p were calculated as a function of σ_s from the shear data and the value of σ_s was identified for which the secant modulus $G_s = \sigma_s/\gamma_p$ was equal to that determined in (ii). This involved modelling the variation of σ_s with G_s using a polynomial function.

3.3.1 Determination of λ

It follows from equations (1) and (2), that in a tensile test

$$J_{2D}^{1/2} = \frac{s_T}{3} \quad \text{and} \quad J_1 = s_T \quad (14)$$

and in a shear test

$$J_{2D}^{1/2} = s_s \quad \text{and} \quad J_1 = 0 \quad (15)$$

Inserting equations (15) into equation (3) gives

$$s_T = \frac{\sqrt{3}(l+1)}{2l} s_s \quad (16)$$

from which

$$l = \frac{\sqrt{3}(s_s/s_T)}{2 - \sqrt{3}(s_s/s_T)} \quad (17)$$

If pairs of yield stresses σ_s and σ_T are determined over a range of values for the effective plastic strain $\bar{e}_p$, then any variation of λ with $\bar{e}_p$ can be recorded.

3.3.2 Determination of a and p

Inserting equations (14) and (15) into equation (9) gives

$$a s_T^2 + \frac{s_T}{3} = 3a s_s^2 \quad (18)$$

so

$$a = \frac{s_T}{3(s_s^2 - s_T^2)} \quad (19)$$

Also p_t , which characterises strain hardening behaviour, is given by

$$p_t = a s_T^2 + \frac{s_T}{3} \quad (20)$$

Note that σ_T is a function $\sigma_T(\epsilon_{pT})$ of plastic strain ϵ_{pT} .

3.4 FLOW PARAMETER ψ

The flow parameter was determined from the tensile data using equation (12) and the measured value of v_p (equation A9). These calculations were repeated for different ϵ_{pT} values in the non-linear range to ascertain whether Ψ varied with strain.

4 EXPERIMENTAL

4.1 MATERIALS

The adhesive materials investigated in this project are listed in Table 1 together with the trade name and suppliers.

Table 1 - The adhesives studied

Adhesive type	Supplier/code
1-part, toughened epoxy	Ciba/LMD 1142
2-part, ambient-temperature curing acrylic	ITW Plexus/MA310

Sheets of the bulk adhesives of thickness between 2 and 3 mm were prepared following procedures that are now standardised⁷. Sheets of the epoxy adhesive were cured at 150 °C for 2 hours. The acrylic adhesive has an ambient temperature cure but to minimise any changes in properties with time after moulding, sheets of this material were post cured at 40 °C for 2 hours.

For the tensile tests, standard dumbbell-shaped specimens (ISO 3167) of overall length 150 mm and width of the narrow gauge section 10 mm, were machined from the sheets. For the shear measurements, Arcan V-notched square plates⁸, having a notch separation of 12 mm, were also machined from the sheets. The measurement of tensile and shear properties on bulk specimens are described in sections 4.2 and 4.3.

In order to investigate the applicability of the different yield criteria over a wider range of stress states than tension and shear, tests have also been made on butt-joint specimens loaded in tension. These measurements are described in section 4.4.

4.2 TENSILE MEASUREMENTS ON BULK SPECIMENS

Tensile tests were carried out using an Instron universal testing machine type 4505. For the determination of Young's modulus, longitudinal strains were measured using extensometers, developed at the NPL⁸, that were mounted on opposite faces of the specimen. Each extensometer comprised an inductive displacement transducer that contacted the specimen via two knife edges, one attached to the body of the transducer and the other to its core. The gauge length between the knife edges was 50 mm. Measurements of transverse strain, also required for the determination of ν_e , were made using a commercial (Instron) clip-on extensometer mounted across the specimen width. Values of E and ν_e were usually evaluated from data over the strain range 0.0005→0.0025 and were reproducible to within $\pm 1\%$ and $\pm 3\%$ respectively.

For determining the strain-hardening function and associated yield parameters in the non-linear region, strains were determined by means of a Messphysik ME64 videoextensometer. With this method, a video camera (with zoom lens) is employed to generate an image of the specimen on a computer screen and the distance between gauge marks on the specimen surface is determined from the number of pixels on the screen between the marks. Strains are then calculated from the changes in the measured distance. For longitudinal strain measurements, the gauge marks were usually in the form of parallel black lines drawn across the width of the specimen with a marker pen. The distance between the lines was typically in the range 5-20 mm. For transverse strain measurements, the gauge marks were either in the form of black lines drawn along opposite edges of the specimen or were defined by the edges of the specimen against a black background. Distances between these gauge marks could then be determined along a chosen number of 'measuring axes' equally spaced between the longitudinal gauge marks. It was found necessary to paint the specimen surfaces white between gauge marks (see figure 2) to maintain a uniform contrast as the materials changed colour due to stress-whitening during the deformation.

4.3 SHEAR MEASUREMENTS

In the shear tests, details of which were reported previously⁹, notched-plate shear (Arcan) specimens were loaded in the Instron machine parallel to the (vertical) axis joining the V-notches (see figure 3). A state of pure shear was thereby produced in regions close to this axis away from the notch roots. Shear stresses were calculated approximately from the applied load F divided by the cross-sectional area A between notches (notch separation times specimen thickness). Shear strains were obtained from the relative vertical displacement u of two points located on opposite sides of the central axis (figure 3) and separated by a known horizontal distance d of about 3 mm. This displacement was determined by an extensometer containing two pivoted lever arms each of which contacted the specimen at one of the points shown via a needle point attached at one end⁹. During the specimen deformation, the relative displacement of the needle points (which produces independent rotations of the two lever arms) was monitored through the output of inductive displacement transducers in contact with the opposite ends of the lever arms. To compensate for twisting motions of the specimen, average displacements were determined for opposite faces of the specimen using two extensometers each mounted on the lower grip of the specimen.

The shear tests were performed at the crosshead speeds of 0.2, 2 and 20 mm/min which were chosen to give shear strain-rates at low strains approximately equal to the tensile strain-rates in the tensile tests for speeds of 1, 10 and 100 mm/min respectively. They also gave shear strain-rates about 1.8 times the tensile strain-rates in the respective tensile tests at strains associated with the peaks in stress. It follows that the effective plastic strain-rates in the shear and corresponding tensile tests were closely similar, as required in the determination of the parameters in the yield criteria (section 3.3).

4.3.1 Corrections to shear data

Sources of error in the shear data were previously discussed in some detail⁹ and it was concluded that uncertainties in the shear stresses and strains are substantially larger than those in the corresponding tensile quantities. It has since been found that measured shear moduli G may typically be up to 10% lower than values calculated from tensile data using

$$G = \frac{E}{2(1+\nu_e)} \quad (21)$$

Since accurate values of G are required in calculating the elastic shear strain ϵ_e and subsequently deriving related quantities such as λ and a , further studies have been undertaken to ascertain the origin of the discrepancies and provide corrections to the data.

As part of this investigation, Arcan shear tests were carried out on polycarbonate at crosshead speeds of 0.2, 2 and 20 mm/min using different methods of coupling the specimen grips to the Instron pullrods. For each crosshead speed, shear moduli were determined from the stress-strain data in the strain range 0.003 to 0.006 and were compared with values calculated from tensile data (using equation (21)) over the same strain range and at a corresponding crosshead speed. It was noted that the shear data were highly scattered at the lower strains (0.0005-0.0025) frequently used for determining E and ν_e and that the shear and tensile stress-strain curves were each quite linear for strains in the range 0.003-0.006. For all modes of coupling, the measured G values were lower than those calculated from tensile data. However, the discrepancies were minimised by employing universal joints to obtain flexible couplings between the grips and pullrods, which may have resulted from a reduction in in-plane or out-of-plane specimen distortions.

Finite element calculations have shown that the above discrepancies, as well as errors in the high-strain data, can be traced to stress and strain distributions in the loaded Arcan specimen. Results of these calculations have provided correction factors for converting the respective F/A and u/d values to the true shear stress σ_s and true shear strain γ at the mid-point of the test specimen. In the low-strain, elastic region, we obtained $\sigma_s/(F/A) = 1.03$ and $\gamma/(u/d) = 0.94$ for a propylene-ethylene copolymer and an epoxy adhesive. The correction factor for stress undoubtedly arises from a marked fall-off in shear stress along the vertical centre line of the specimen within about 1 mm of each notch root. The correction factor for shear strain is less well understood but may reflect contributions from bending deformations to the measured displacement u . The combined factors for stress and strain represent a correction of +9.5% to

the measured shear modulus which largely accounts for the above discrepancies between the measured and calculated values of G .

With increasing strain, the stress correction factor $\sigma_s/(F/A)$ decreases only slightly, values around 1.015 being obtained for shear strains at which substantial yielding has occurred. However, the strain correction factor $\gamma/(u/d)$ exhibits a marked increase with strain, and values as high as 1.97 were obtained for the propylene-ethylene copolymer for $\gamma \approx 0.3$. This result largely arises from the localised plastic deformation in the vicinity of the centre line, as evidenced by the appearance of a shear band having a width less than the 3 mm separation between the extensometer arms. The non-uniform shear yielding outweighs other effects such as that associated with bending contributions to the measured u , or with the fact that, at high strains, differences will exist between the true shear strain and the measured engineering strain in any local region. Furthermore the shear banding differs for different materials and will depend on strain-rate and temperature. Finite element calculations of strain correction factors will therefore be required for each shear test or a different technique used to determine local shear strains. It should be emphasised, however, that shear banding is usually associated with a flat region of the stress-strain curve so that relatively large corrections to the shear strain have little effect on the curve. Such corrections have not therefore been applied so far in this work. Corrections to the shear stress and strain values have, however, been made using the factors 1.03 and 0.94 respectively. These factors seem to be independent of the material and test conditions, and give reliable values of G at low strains.

4.4 TENSILE MEASUREMENTS ON BUTT-JOINT SPECIMENS

The loading assembly for tensile tests on butt-joint specimens is shown in figure 4. The specimen consists of two 25 mm diameter hardened steel rods that are bonded together at end faces. An alignment jig was used during the bonding to ensure that the adherends were accurately coaxial. A v-shaped groove was machined in the surface of each adherend at a distance of 2.75 mm from the face to be bonded. The depth was 1.0 mm. These served to locate an extensometer consisting of an assembly of displacement transducers that were used to determine the axial deformation around the adhesive bond in response to an applied load. The extensometer is shown in figure 5. It consists of two rings with knife edges on their inner surfaces. Each ring locates in one of the grooves in a bonded specimen. One of the rings supports three precision inductive displacement transducers that are equally spaced around the ring. The core of each transducer is supported by an elastic suspension that permits friction-free movement of the core and supplies a restoring force to any displacement. The core of each transducer contacts the surface of the second ring. The transducers record the combined axial deformation of the adhesive and the distortion of the region of the adherend between each bond face and the corresponding v-groove. The strain in the adhesive can be deduced from a finite element analysis as discussed in section 5.2.

The specimen is loaded through universal joints linked to each adherend. These are intended to minimise any bending forces applied to the specimen arising from misalignment of the test assembly. Any non-uniform loading is recognised by differences in the transducer outputs. Differences were generally small under the application of force levels where behaviour was linear. However, as the adhesive starts to soften at higher strains, the strain would increase

substantially in one region of the adhesive and rise more slowly or even decrease in other regions. This is an unstable situation and gives rise to an increasing bending moment in the specimen with further loading. In order to maintain a uniform axial strain in the adhesive it was necessary to manually apply a counteracting moment throughout a test that was sufficient to ensure that the output from each transducer increased uniformly.

In the design stage of the butt-joint specimen, finite element analyses were carried out to explore the location of stress and strain concentrations in the adhesive layer. An elastic-plastic model was used with the assumption of von Mises yielding. Regions of high strain, are localised at the top and bottom circumferences of the adhesive layer at the interface with the corner of each adherend. The strains in these regions are singular if the corners of the adherends are sharp. The singularities could be removed by adding a radius to the corners but this only moved the regions of strain concentration closer to the specimen axis and had the disadvantage that the strain distribution in the plane of the adhesive layer became less uniform. Plane adherend faces have therefore been used so far, although further studies will be made to investigate the influence of local strain singularities on joint performance.

Results of butt-joint tensile tests for each adhesive studied are shown in figures 13 and 15 and are discussed in section 5.2

5 RESULTS AND DISCUSSION

5.1 STRESS/STRAIN CURVES AND MODEL PARAMETERS

Figures 6 and 7 show tensile and shear stress/strain curves and the variation of Poisson's ratio with strain obtained on bulk specimens of both adhesives at 23 °C and at an effective plastic strain rate of about 0.0025. Values for E and G were derived from these plots and used to obtain the variations of yield stresses with the plastic component of strain by means of equations A5 to A7 in the Appendix. These curves are shown in figures 8 and 9 for both adhesives. From these

curves, yield stress pairs σ_T and σ_S corresponding to the same equivalent plastic strain $\bar{e}_p$ were determined using the procedure described in section 3.3. The variation of the parameter λ with plastic strain was then derived for both materials using equation (17) and is shown in figure 10. Similarly, equation (19) was used to determine the parameter a whose variation with plastic strain is shown in figure 11. Finally, from Poisson's ratio measurements shown in figures 6 and 7, the plastic component v_p was derived using equations A5 to A9, and, by means of equation (12), values for the flow parameter ψ were calculated and are shown with v_p values in figure 12.

In figure 9, it is seen that the hardening behaviour of the acrylic adhesive is clearly different in tension from that in shear. The sharp change in the slope of the tensile curve at a strain of around 0.02 is accompanied by observed stress whitening and a gradual decrease in Poisson's ratio. The small increase in Poisson's ratio seen in figure 7 in the region of strain between 0.01 and 0.02 is consistent with the onset of plastic deformation involving a process in which the volume is essentially constant with $v_p = 0.5$. At strains above 0.02 in tension, there is an abrupt change in the deformation mechanism to a process involving an increase in volume presumably

associated with crazing or cavitation. Since this process requires a dilatational component of stress, it does not occur in a shear test, and this explains why the hardening curve in shear is different from that in tension (see figure 9).

The epoxy adhesive also shows stress whitening under tension and a reduction in Poisson's ratio at strains above 0.02 as shown in figure 6 but the hardening behaviour in tension and shear appear similar (figure 8). The parameter λ is consequently far less dependent upon plastic strain than for the acrylic as seen in figure 10.

Cavitation in the rubber phase is known to contribute to plastic deformation in most (or all) toughened adhesives¹⁰. In order to use FE methods with these materials, it is necessary to employ a yield criterion in the materials model that explains the contribution from cavitation to the yield process. The deformation behaviour of the butt-joint specimens loaded in tension was analysed to assess the suitability of available criteria and whether there is a need to develop an alternative description. The analysis of results from these tests is discussed next.

5.2 RESULTS AND ANALYSIS OF BUTT-JOINT TENSION TESTS

5.2.1 The epoxy adhesive

Finite element analyses were used to calculate the variation of the applied force with the change in the separation of the extensometer contact points for the butt-joint specimen shown in figure 5. Calculations were made using the von Mises, the linear Drucker Prager and the exponent Drucker-Prager yield criteria. The parameters used for the epoxy adhesive are given in table 2. Parameters λ , a , v_p and Ψ are mean values of their variations with plastic strain obtained from figures 10, 11 and 12.

**Table 2 - Properties of the epoxy adhesive used for
FE analyses**

E (GPa)	v_e	λ	a (MPa ⁻¹)	v_p	Ψ	$\sigma_T(\epsilon_p)$ (MPa)
1.68	0.44	1.55	0.018	0.3	250	fig. 8

Comparisons of the measured and calculated force/extension curves are shown in figure 13. It is evident that there are significant differences between the predicted performance obtained using the three models. The reason for this, as indicated in section 2.1.3, is that the adhesive in a butt-joint loaded in tension is subjected to a predominantly triaxial tensile stress. At low stresses where the behaviour is linear, the lateral stresses in the adhesive are calculable from a knowledge of the Young's moduli and the Poisson's ratios of the adhesive and steel. The lateral stresses are about 80% of the axial stress. According to the FE analyses using both Drucker-Prager models, as the strain in the adhesive increases in the non-linear region, the ratio of lateral to axial stresses increases and is close to unity for plastic strains above 0.03 (see figure 14). Thus in this region of strain, a state of almost pure equi-triaxial (hydrostatic) tension is established in the adhesive. These yield criteria differ substantially in their prediction of yield

behaviour under this stress state as illustrated in figure 1. An equi-triaxial tensile stress corresponds to a state of zero shear stress and thus to points on the $-p$ axis ($-ve$ because p is a hydrostatic pressure). According to the von-Mises criterion, yielding will only occur at infinite stress under this stress state, and this gives rise to the linear elastic prediction in figure 13. With both of the hydrostatic stress sensitive yield criteria, yielding is possible under hydrostatic tension, but with the linear function, the yield stress is higher than with the parabolic (exponent) function. By virtue of the close agreement with experimental data, it would appear that the parabolic function is a more realistic criterion for this adhesive. Furthermore, if a value for the parameter a of 0.165 were used in place of that given in table 2, then the calculated force/extension curve agrees very closely with the experimental data in figure 13.

5.2.2 The acrylic adhesive

The apparent success of the exponent Drucker-Prager model to describe deformation of the butt-joint specimen of the epoxy adhesive is not repeated with specimens of the acrylic adhesive. The measured force/extension curve for the butt-joint specimen is shown in figure 15. From the peak force, a value of 29 MPa is estimated to be the flow stress for the acrylic adhesive under butt-joint tension. (A small correction has been made to allow for the non-uniform stress near the circumference of the bond based on the FE analysis for the epoxy adhesive.) Assuming the stress state in the acrylic is pure triaxial at these strain levels (as was predicted for the epoxy adhesive) it is possible to use this value to help visualise the shape of the yield surface for this adhesive in the limit of large strains. It is plotted with the uniaxial tensile and shear yield stress values corresponding to large plastic strains (see figure 9) on a qp diagram in figure 16. In contrast to the epoxy, the yield behaviour of this material appears to be modelled closely by the linear Drucker-Prager criterion. This is believed to be fortuitous and results from the softening observed in the tensile stress/strain curve (figure 9) which is not observed in either the shear (figure 9) or butt-tension data (figure 15). The most plausible interpretation of the data in figure 16, and the equivalent data for the epoxy adhesive, is that the yield stress values lie on two surfaces characterised by different yield criteria. This is discussed further in the next section.

5.3 A NEW YIELD CRITERION

The analysis of results described in sections 5.1 and 5.2 suggests that, in terms of an elastic-plastic model, the onset of non-linear deformation (plastic strains in the region of 0.01 or 0.02) is controlled by a predominantly shear yielding mechanism. The plastic component of strain is then associated with a deformation mechanism that occurs under constant volume and gives rise to the observed increase in Poisson's ratio with strain. Deformation in this restricted strain range can be modelled using the von Mises yield criterion. At larger strains however, and under loading conditions where there is a significant component of hydrostatic stress, plastic deformation is associated with a dilatational yield mechanism involving the formation of microvoids by cavitation in the rubber or crazing in the matrix polymer. As both of these mechanisms require a dilatational stress component, and do not therefore occur under shear, the effect is to lower the tensile yield stresses relative to the shear yield stresses. This gives rise to different shapes for the tensile and shear stress/strain curves which is particularly noticeable for the acrylic material.

To allow for the effects of cavitation, there is a need to modify or extend the yield criteria represented by equations (3) and (8). This is outside the scope of this report and requires further investigation. The recent work of Bucknall^(11,12) forms a relevant basis for such an investigation. He has modified a model proposed by Gurson to describe yield in a ductile metal containing voids. Bucknall's analysis assumes that the yield behaviour of a toughened plastic is described by the pressure sensitive criterion given in equation (3) until a critical volumetric strain is reached in the rubber under the applied stress. At this strain, cavitation of rubber particles is initiated. The cavitation is progressive, starting in the larger particles, and yielding takes place by shear deformation of the material surrounding the particles. Bucknall proposes that the yield surface is a cone but with the apex being a smooth curve similar to the paraboloid of the exponent Drucker-Prager criterion (equation (8)). This model is therefore potentially suitable for describing yielding of the epoxy adhesive.

It does not however seem applicable to the acrylic adhesive, despite the fact that the linear representation shown in figure 15 is not considered to be realistic. Firstly, the yield criterion for stresses around $p=0$ and p positive is likely to be close to von-Mises, possibly with a small hydrostatic stress sensitivity. The large reduction in the effective stress to cause yield under uniaxial tension may be caused by the initiation of crazes in the acrylic polymer. Cavitation of rubber particles may also contribute. Crazing may thus be responsible for the stress softening observed in the acrylic under uniaxial tension in figure 7. The hardening behaviours in tension and shear (figure 9) are thus distinctly different in contrast to the epoxy (figure 8) where the curves are similar in shape. This may be because the epoxy matrix deforms by a shear yielding mechanism, even when the rubber particles cavitate, rather than by crazing. The hardening behaviour of the acrylic materials under the stress state generated in the butt-joint tensile test appears different from that under uniaxial tension. This may indicate that, in the butt-joint test, crazing does not contribute significantly to the yielding process but that, as with the epoxy, yield deformation is by a shear mechanism in the matrix around cavitated particles.

6 CONCLUSIONS

- Elastic-plastic materials models are used in finite element analyses for predicting stresses, strains and deformations in an adhesive joint when the adhesive has been loaded to strain levels where its behaviour is non-linear. These models require a valid yield criterion for the adhesive.
- The results of tests on adhesive specimens carried out on bulk specimens in tension and shear and on a butt-joint specimen in tension enable the applicability of different yield criteria to be assessed. The latter test is particularly important for this purpose. From results on a toughened epoxy and a toughened acrylic, it is evident that the von Mises criterion is not suitable. The linear Drucker-Prager criterion is not suitable for the epoxy but appears to fit results for the acrylic. The exponent Drucker-Prager is not suitable for the acrylic but appears to describe yielding in the epoxy.
- These observations are believed to be fortuitous.

- Further analysis of the results of these tests supports the interpretation that under stress states and stress levels that are insufficient to initiate cavitation in the rubber phase or crazing in the matrix polymer, then the von Mises criterion is satisfactory. When there is a hydrostatic component of stress that is sufficient to cause cavitation or crazing, then an alternative criterion or a modification of the von Mises criterion must be used.
- Further work is needed to establish the form of a suitable criterion or criteria. Ideally, the new criterion will be able to describe the different hardening behaviours observed in tension and shear.

7 ACKNOWLEDGEMENTS

The work reported here forms part of a research programme on materials measurement and technology funded by the Department of Trade and Industry.

8 REFERENCES

1. M.N. Charalambides and G.D. Dean, Constitutive models and their data requirements for use in finite element analysis of adhesives under impact loading. NPL report CMMT(A)59 (1997).
2. S.S. Sternstein and L. Ongchin, American Chemical Society Polymer Preprints, 10 (1969), p 1117.
3. S. Gali, G. Dolev and O. Ishai, Int. J. Adhesion and Adhesives, (1981) p 135.
4. ABAQUS User's and Theory Manuals, Version 5.5, Hibbit, Karlsson and Sorensen Inc., USA, 1995.
5. R. Raghava, R. Caddell and G. Yeh, J. Mat. Sci, 8 (1973) p 225.
6. R. Caddell, R. Raghava and A. Atkins, Mater. Sci. Engng, 13 (1974) p 113.
7. ISO 15166:1998, Adhesives - Methods for preparing bulk specimens.
Part 1: Two-part systems.
Part 2: Elevated-temperature curing, one-part systems.
8. B.C. Duncan and P.E. Tomlins, Measurement of strain in bulk adhesive testpieces. NPL report DMM(B)398 (1994).
9. B.C. Duncan and G.D. Dean, Test methods for determining shear property data for adhesives suitable for design. NPL report CMMT(B)56 (1996).
10. Y. Huang, D.L. Hunston, A.J. Kinloch and C.K. Riew, Toughened Plastics I: Science and Engineering. Advances in Chemistry Series 233 (1993) pp 1-35.

11. A. Lazzeri and C.B. Bucknall, J. Mater. Sci. 28 (1993) p 6799.
12. A. Lazzeri and C.B. Bucknall, Polymer 36 (1995) p 2895.

APPENDIX 1. TERMINOLOGY FOR STRESSES AND STRAINS

To account for significant variations in specimen dimensions at high strains, the stress-strain behaviour has been analysed on the basis of true principal stresses and strains for all strain levels. Some relevant definitions and relationships are given in this Appendix.

Uniaxial tension

True normal stress components are defined with reference to the deformed cross-sectional areas. Under uniaxial tension, the true stress is therefore calculated using

$$s = s' / (1 + e_t')^2 \quad (A1)$$

where the nominal (engineering) stress σ' equals the load divided by the unstretched cross-sectional area and ϵ_t' is the transverse engineering strain (fractional change in width or thickness). The true (natural) longitudinal strain ϵ is defined by

$$e = \ln(1 + e') \quad (A2)$$

where ϵ' is the engineering strain (fractional change in length) and the true transverse strain ϵ_t by

$$e_t = \ln(1 + e_t') \quad (A3)$$

Using A2 and A3 we define a true Poisson's ratio

$$n = -e_t / e \quad (A4)$$

and note that the Young's modulus E and elastic Poisson's ratio ν_e represent the values of σ/ϵ and ν , respectively, in the linear region at small strains when true stresses and strains tend to the respective engineering quantities.

From the definitions A2 and A3 above, the following relations were employed to calculate the elastic and plastic components of the true strains ϵ and ϵ_t .

True elastic strain,

$$e_e = \ln\left(1 + \frac{s}{E}\right) \approx \frac{s}{E} \quad (\text{A5})$$

True transverse elastic strain,

$$e_{te} = \ln\left(1 - \frac{n_e s}{E}\right) \approx \frac{-n_e s}{E} \quad (\text{A6})$$

True plastic strain,

$$e_p = e - e_e \quad (\text{A7})$$

True transverse plastic strain,

$$e_{tp} = e_t - e_{te} \quad (\text{A8})$$

True plastic Poisson's ratio

$$n_p = -e_{tp} / e_p \quad (\text{A9})$$

A subscript T is usually added to σ and the ϵ components to indicate stresses and strains under tension. Note also that the transverse strains ϵ_t' , ϵ_t , ϵ_{te} and ϵ_{tp} are of negative sign whereas σ and the longitudinal strains are positive.

FIGURE CAPTIONS

- Fig 1 Schematic diagram of yield criteria plotted on axes of q against p (see equations (10)).
- Fig 2 Illustration of the gauge section and gauge marks on a tensile specimen for determining strains with the video extensometer.
- Fig 3 Illustration of the Arcan shear test specimen and grips showing the locations of contact points for the extensometer lever arms near the centre of the specimen.
- Fig 4 Test assembly for the butt-joint tensile test.
- Fig 5 Section detail of the extensometer for the butt-joint tensile test showing location grooves on the specimen.
- Fig 6 Plots of tensile stress, shear stress and Poisson's ratio with strain for the epoxy adhesive.
- Fig 7 Plots of tensile stress, shear stress and Poisson's ratio with strain for the acrylic adhesive.
- Fig 8 Plots of tensile yield stress and shear yield stress with effective plastic strain for the epoxy adhesive.
- Fig 9 Plots of tensile yield stress and shear yield stress with effective plastic strain for the acrylic adhesive.
- Fig 10 Plots of the parameter λ with the effective plastic strain for the epoxy and acrylic adhesives.
- Fig 11 Plots of the parameter a with the effective plastic strain for the epoxy and acrylic adhesives.
- Fig 12 Plots of the plastic component of Poisson's ratio and the flow parameter $\tan \Psi$ with the effective plastic strain for the epoxy and acrylic adhesives.
- Fig 13 Comparison of measured and calculate force against extension curves for a butt-joint specimen of the epoxy adhesive loaded in tension. The material parameters used in the FE analysis are given in table 4.
- Fig 14 Calculated variation of the axial, radial and circumferential components of stress with applied load in the epoxy adhesive in the central region of the butt-joint specimen. The exponent Drucker-Prager model was used for the calculation.

- Fig 15 Measured force against extension curve for a butt-joint specimen of the acrylic adhesive loaded in tension. The three traces correspond to output from the three transducers in the extensometer (see figure 5).
- Fig 16 Experimental yield stresses for the acrylic adhesive plotted on axes of q against p (see equations 10a and 10b). The yield stress values were obtained from the flow region of the curves in figures 7 and 15.

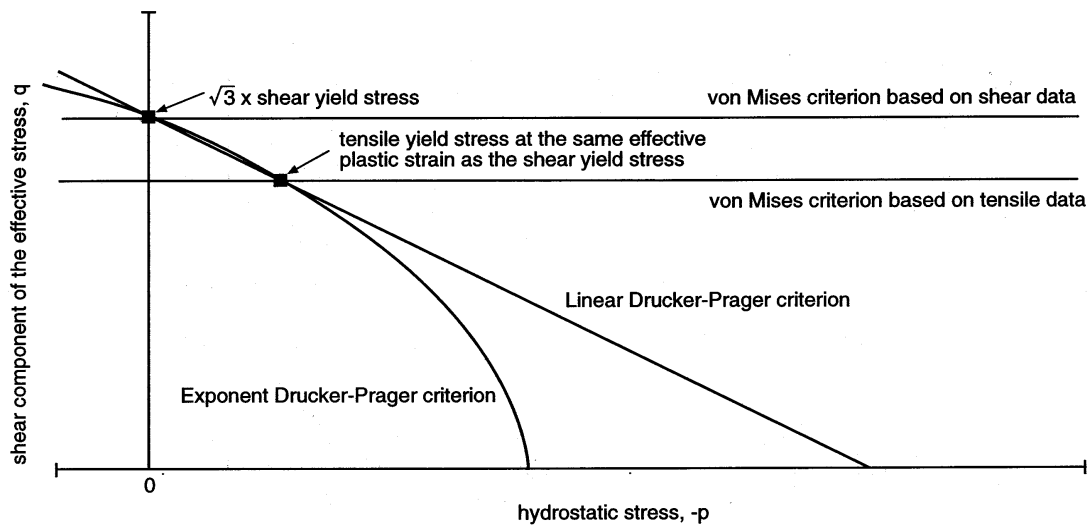


Fig 1. Schematic diagram of yield criteria plotted on axes of q against p (see equations [10])

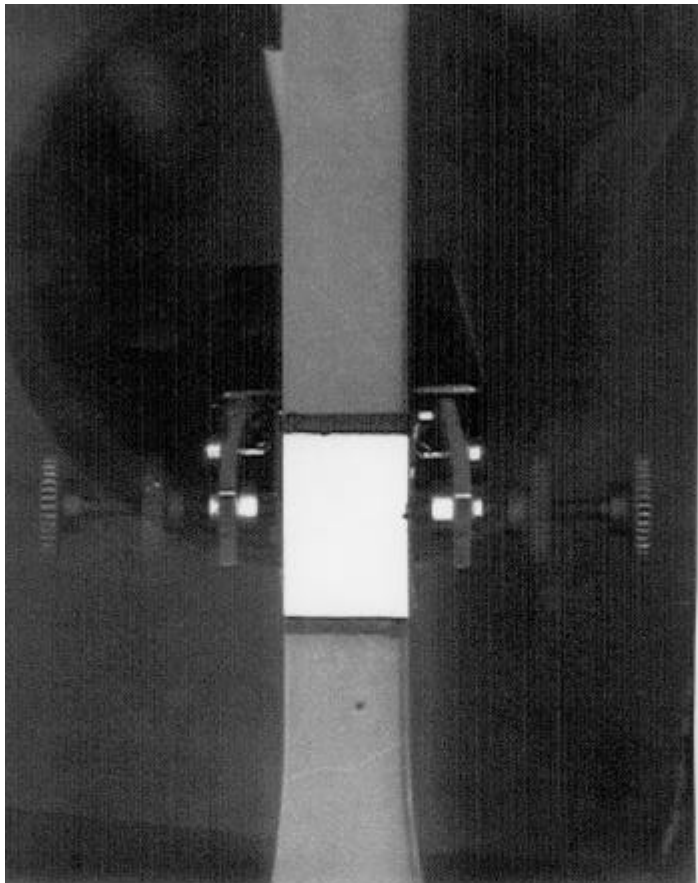


Fig 2 Illustration of the gauge section and gauge marks on a tensile specimen for determining strains with a video extensometer

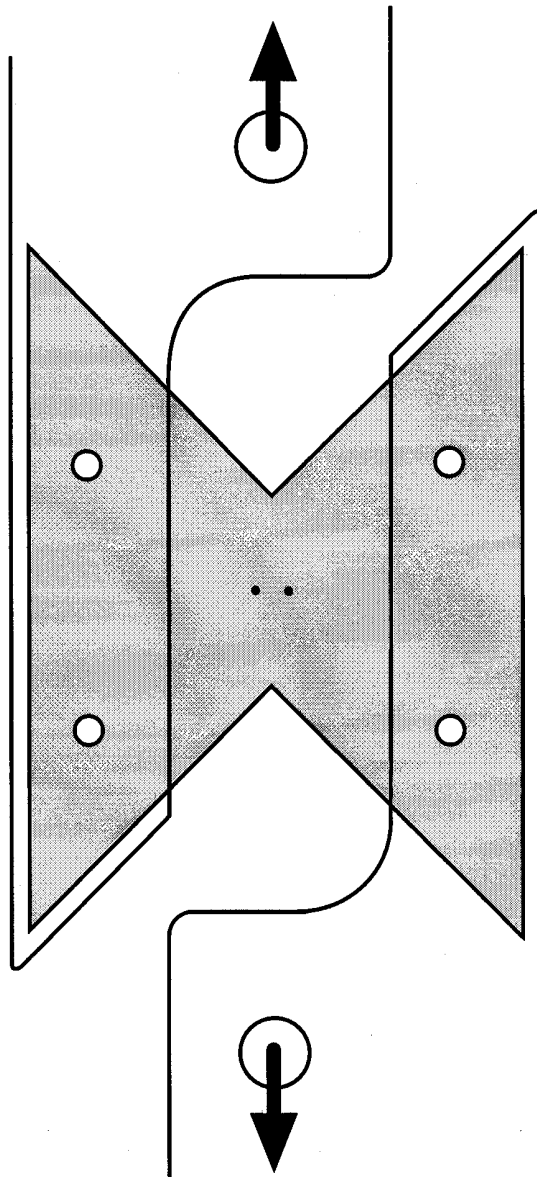


Fig 3 Illustration of the Arcan shear test specimen and grips showing the locations of contact points for the extensometer lever arms near the centre of the specimen.

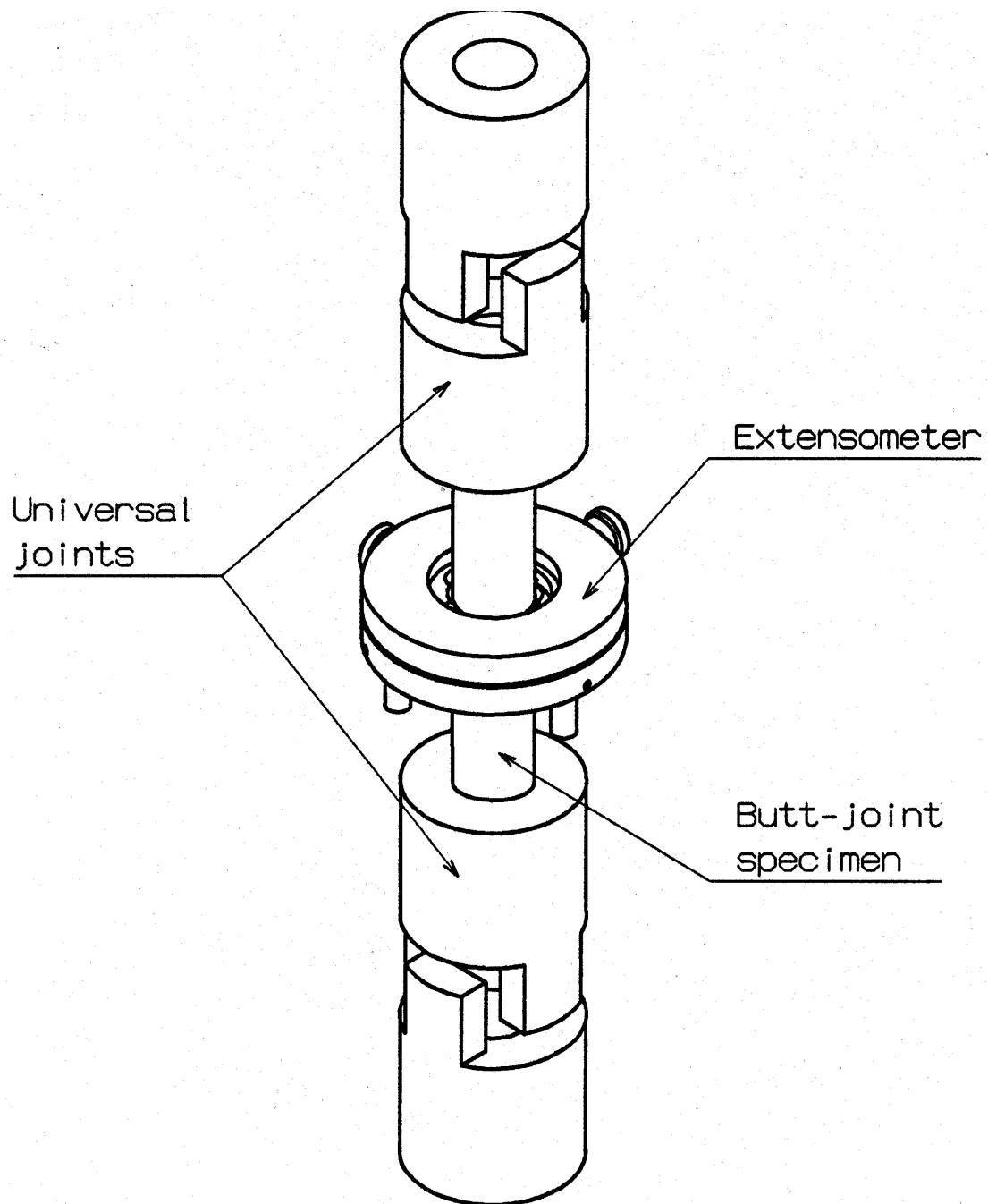


Fig 4 Test assembly for the butt-joint tensile test.

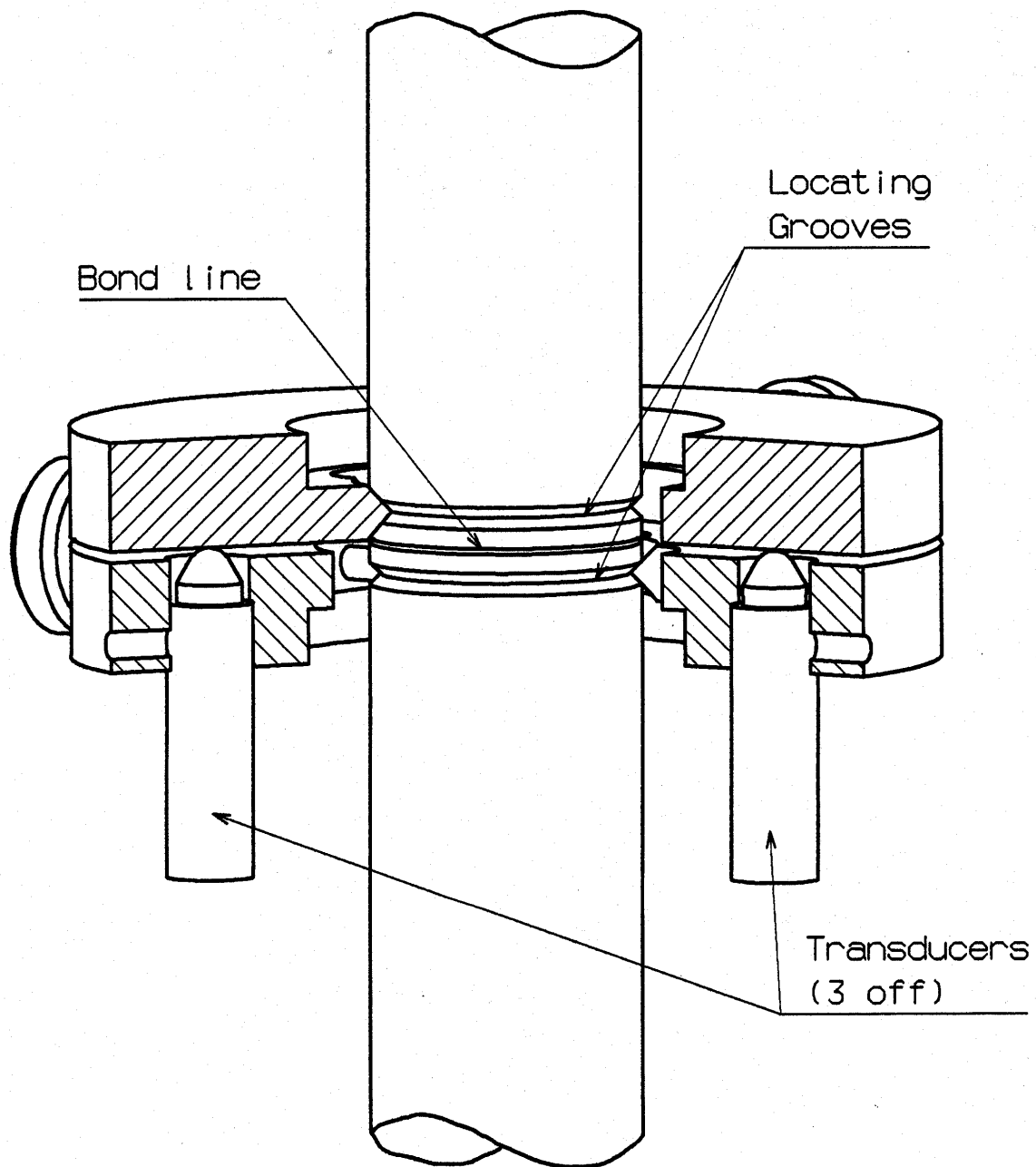


Fig 5 Section detail of the extensometer for the butt-joint tensile test showing location grooves on the specimen.

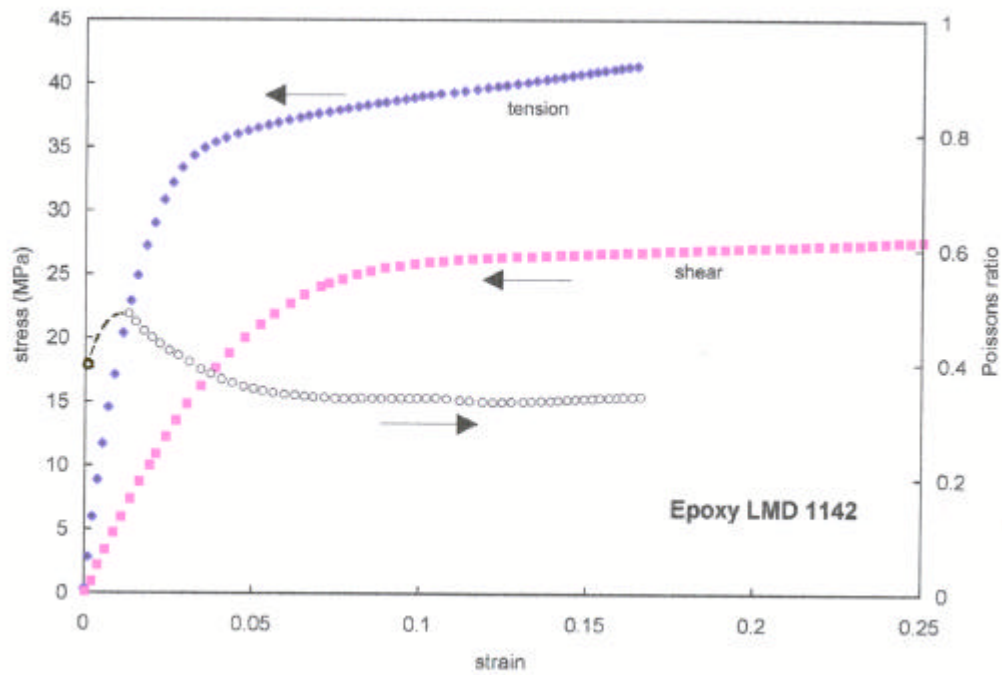


Fig 6 Plots of tensile stress, shear stress and Poisson's ratio with strain for the epoxy adhesive.

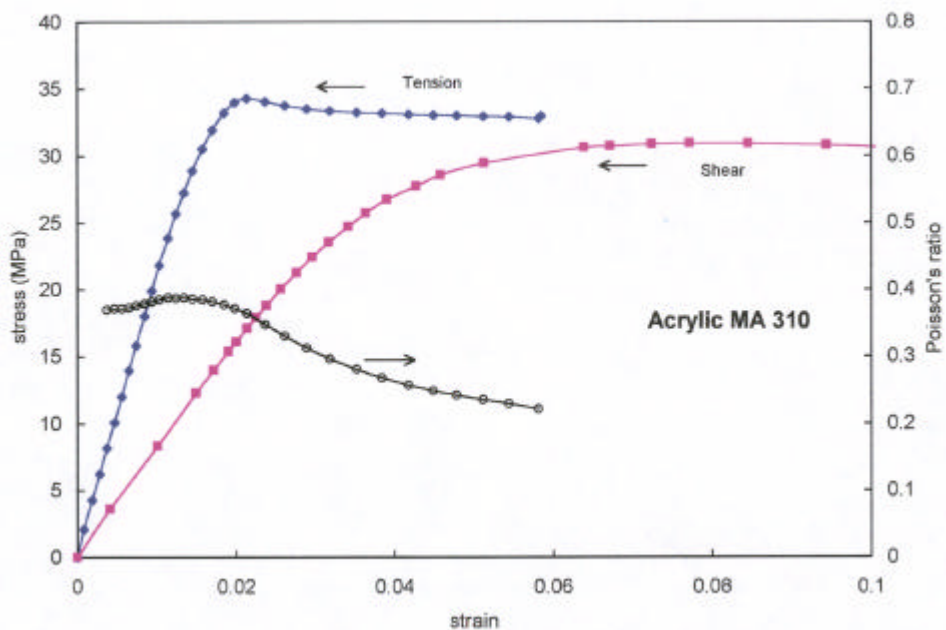


Fig 7 Plots of tensile stress, shear stress and Poisson's ratio with strain for the acrylic adhesive.

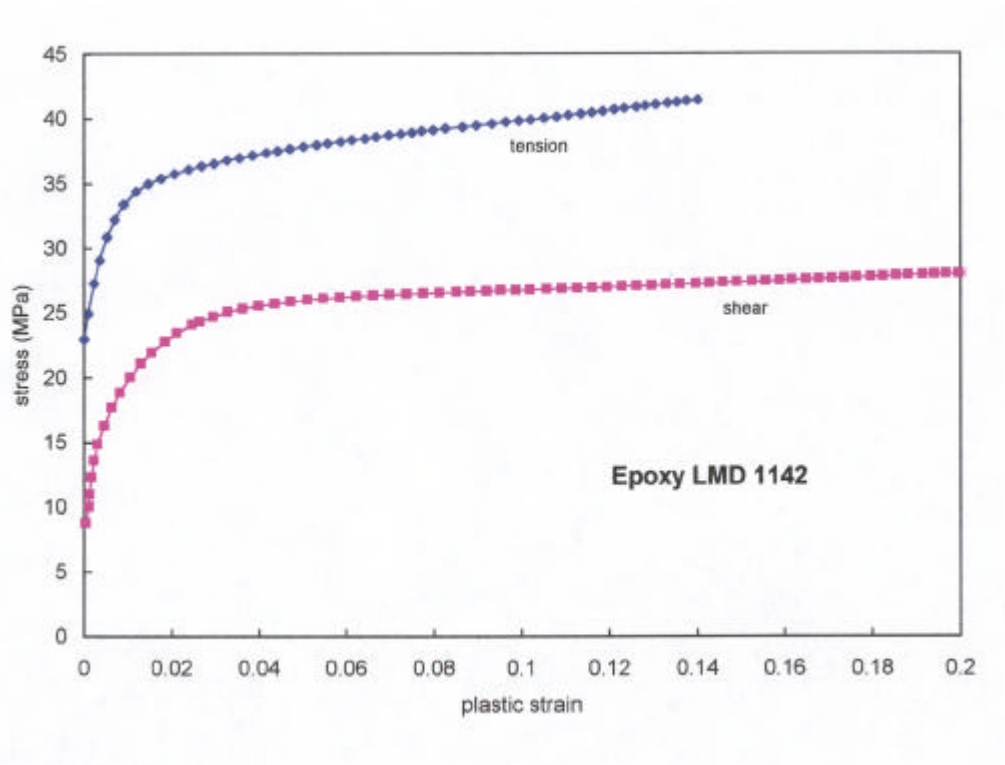


Fig 8 Plots of tensile yield stress and shear yield stress with effective plastic strain for the epoxy adhesive.

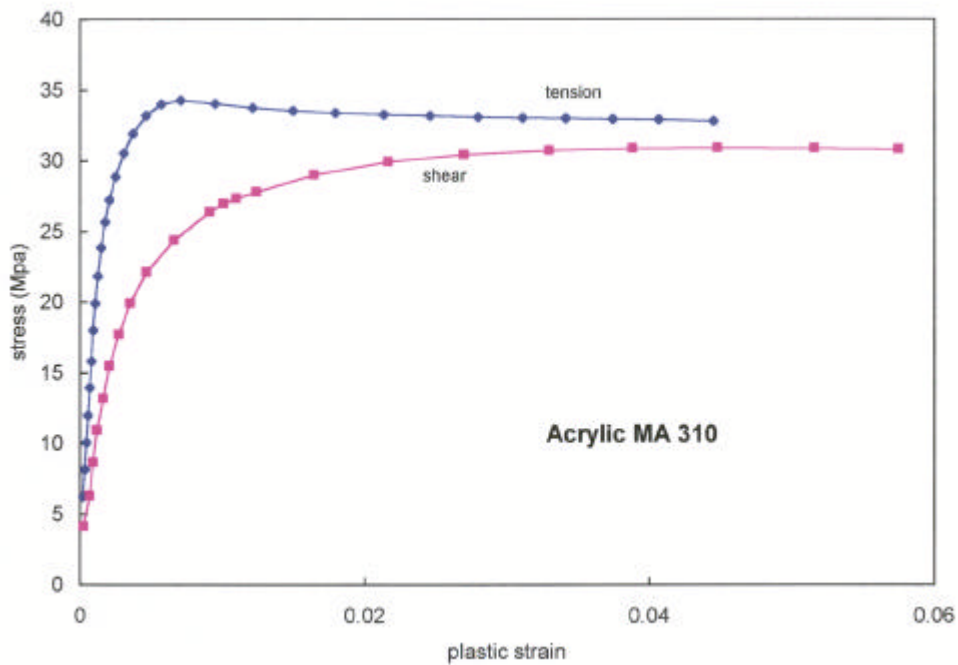


Fig 9 Plots of tensile yield stress and shear yield stress with effective plastic strain for the acrylic adhesive.

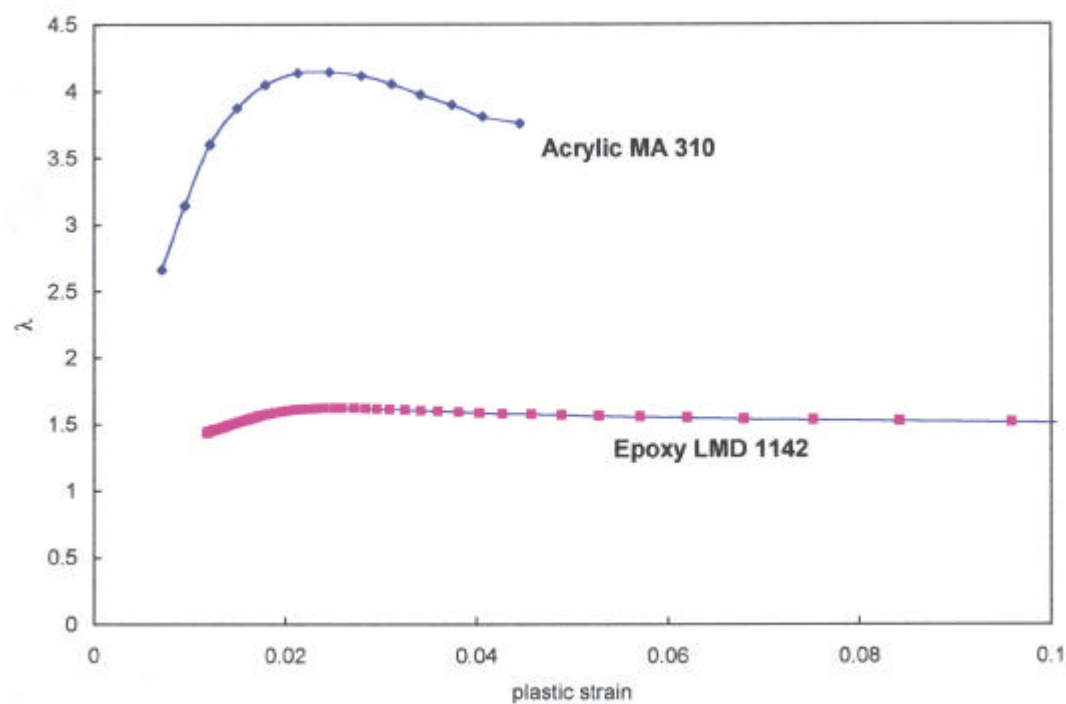


Fig 10 Plots of the parameter λ with the effective plastic strain for the epoxy and acrylic adhesives.

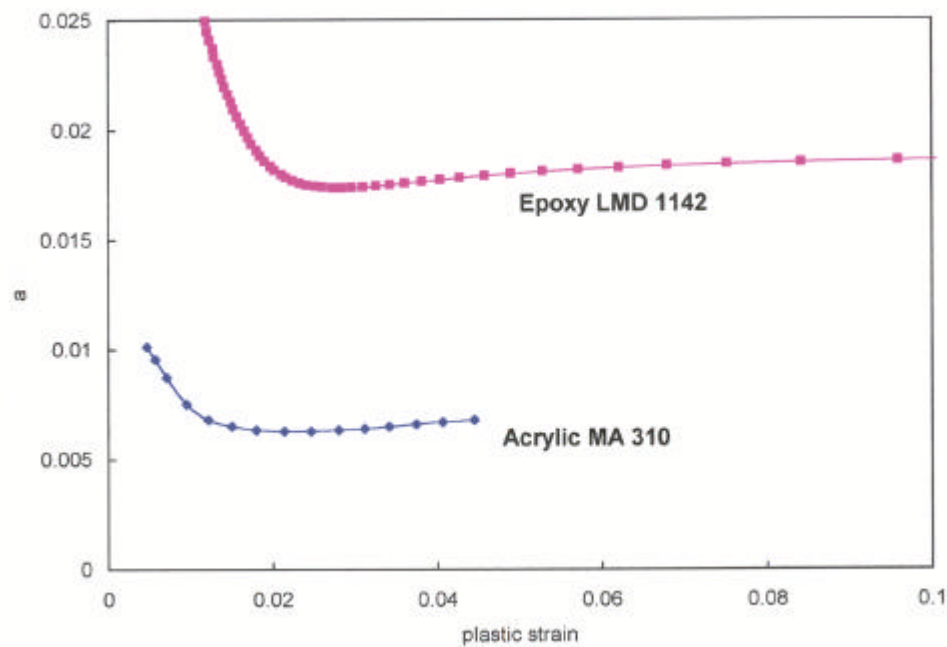


Fig 11 Plots of the parameter a with the effective plastic strain for the epoxy and acrylic adhesives.

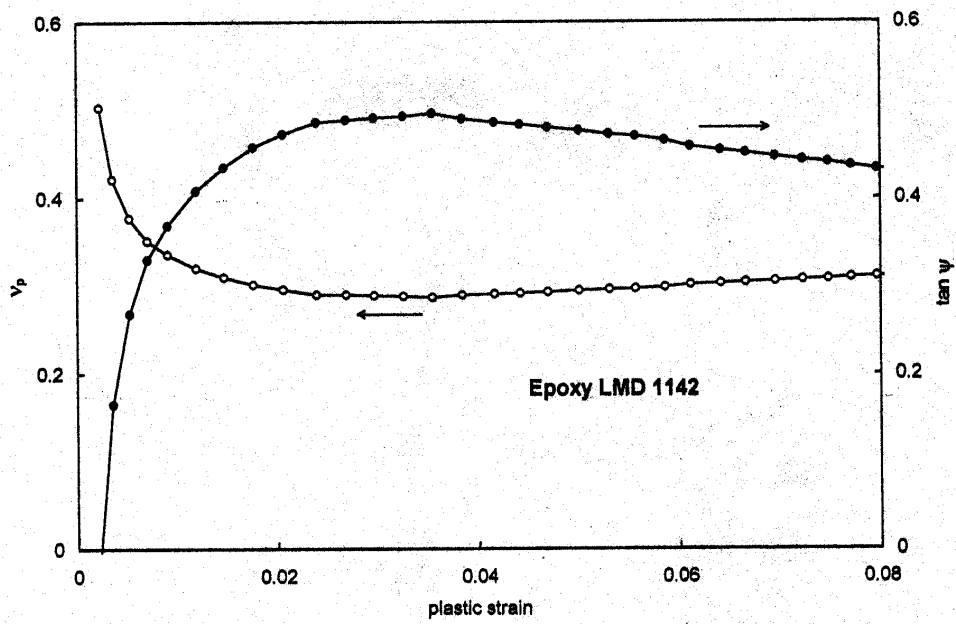


Fig. 12a

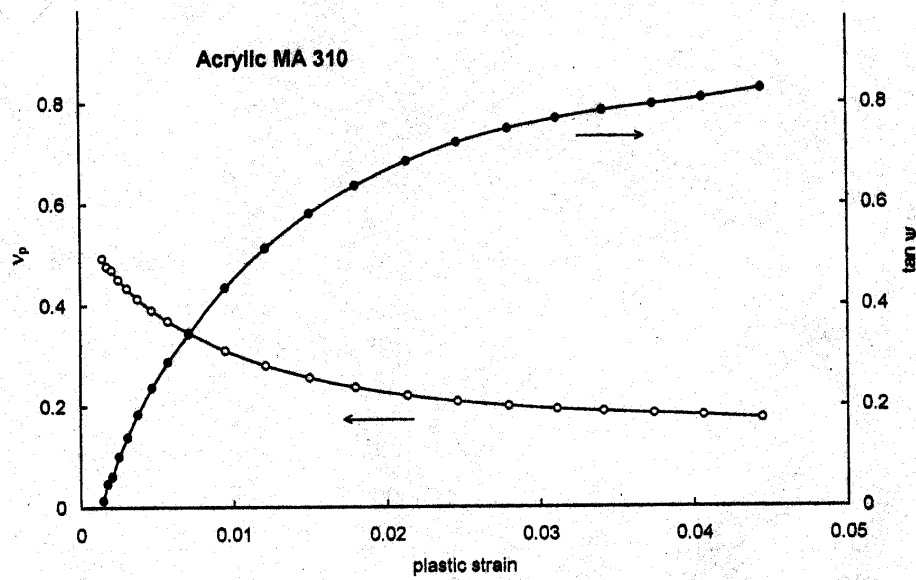


Fig. 12b

Fig 12 Plots of the plastic component of Poisson's ratio and the flow parameter $\tan \Psi$ with the effective plastic strain for the epoxy and acrylic adhesives.

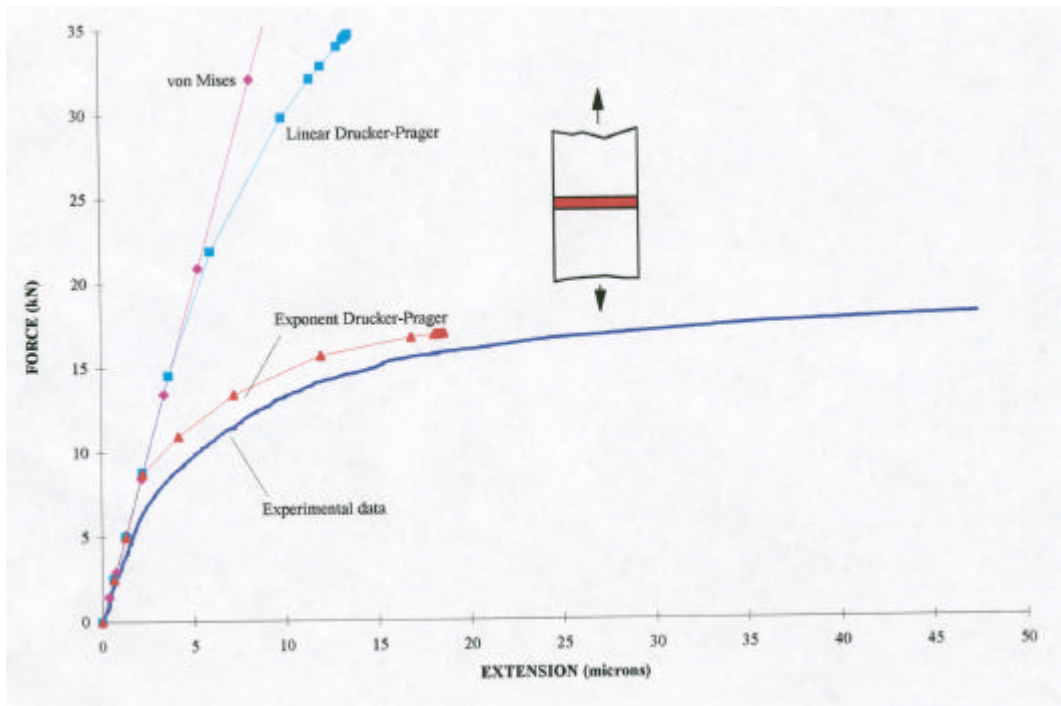


Fig 13 Comparison of measured and calculate force against extension curves for a butt-joint specimen of the epoxy adhesive loaded in tension. The material parameters used in the FE analysis are given in table 4.

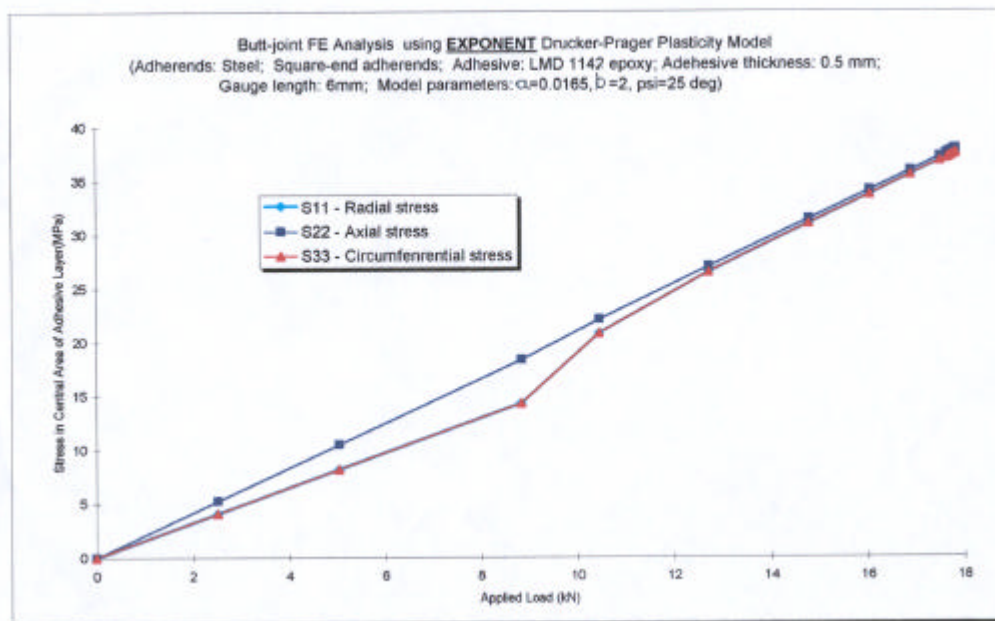


Fig 14 Calculated variation of the axial, radial and circumferential components of stress with applied load in the epoxy adhesive in the central region of the butt-joint specimen. The exponent Drucker-Prager model was used for the calculation.

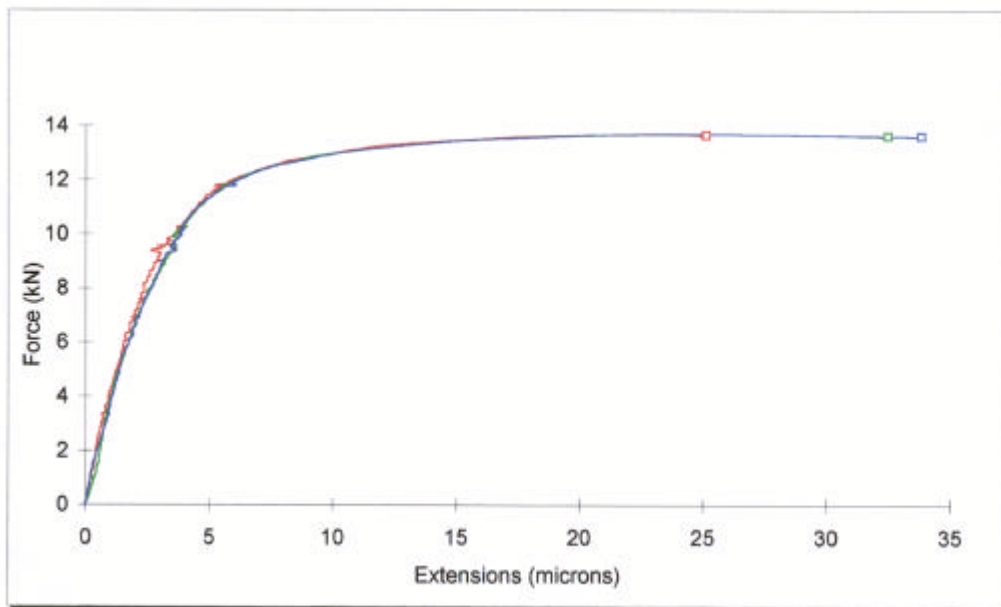


Fig 15 Measured force against extension curve for a butt-joint specimen of the acrylic adhesive loaded in tension. The three traces correspond to output from the three transducers in the extensometer (see figure 5).

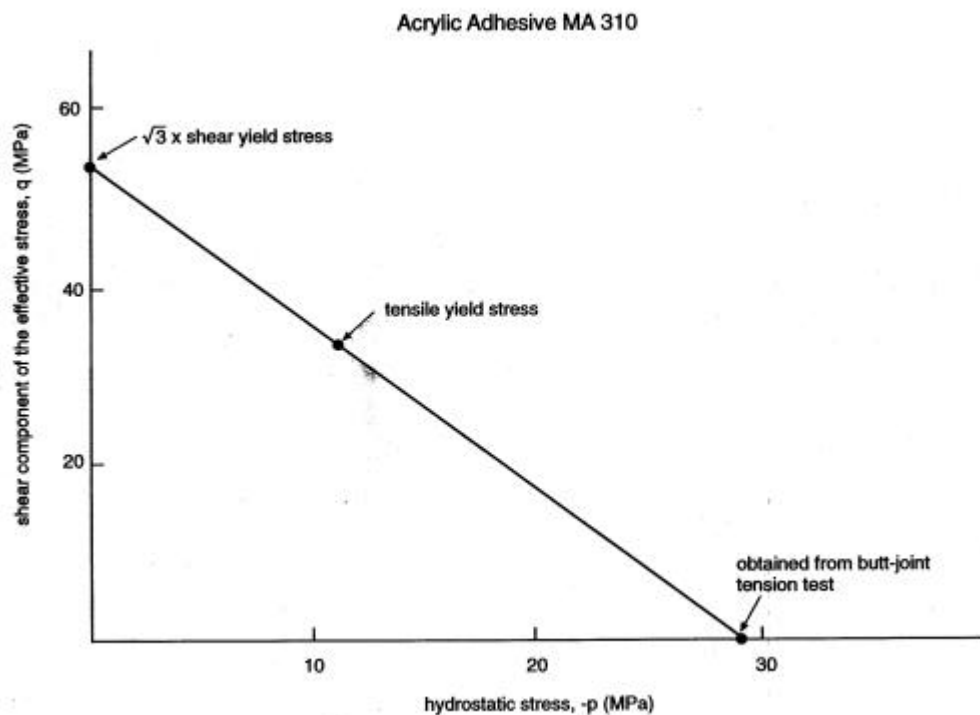


Fig 16 Experimental yield stresses for the acrylic adhesive plotted on axes of q against p (see equations 10a and 10b). The yield stress values were obtained from the flow region of the curves in figures 7 and 15.