

**A Report on Four-Wave Mixing in Optical Fibre
and its Metrological Applications**

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Summary

This report describes the non-linear optical effect known as four-wave mixing (FWM) and its implications in optical fibres. The metrological applications are discussed with particular reference to the requirements of the telecommunications industry, which represents the main driver for research in this field. This industry in particular will benefit from a detailed study of FWM in fibres because the effect can be both detrimental to systems already installed and used as the basis for new devices that will complement the ultra high speed networks of the future.

Four-wave mixing occurs when two or more frequencies (or, equivalently, wavelengths) of light propagate through an optical fibre together. Providing a condition known as *phase matching* is satisfied, light at a new frequency is generated using optical power from the original frequencies. Section 1 gives a brief overview of four-wave mixing and highlights the relevant fibre parameters. The physical mechanisms behind the effect, including the requirements for phase matching, are covered in detail in Appendix A.

Generation of light through four-wave mixing has serious implications for the rapidly expanding telecommunications field of wavelength division multiplexing (WDM). WDM systems send data through an optical fibre using a number of channels (typically 16 or 32) - each with its own designated frequency. If two or more channels interact with each other through four-wave mixing, optical power will be generated with new frequencies at the cost of a reduction of power in the original channels. This power loss makes it more difficult to correctly detect the digital data in these channels at the far end of the fibre, making errors more likely. A more damaging consequence is that the FWM between two or three channels generates light at a frequency that coincides with one of the other allocated channels. The FWM generated light then acts as noise on this channel and leads to even greater degradation of the overall system performance. It is therefore important to take steps to avoid four-wave mixing in multichannel optical communication systems. Four-wave mixing in WDM systems can be minimised by ensuring that phase matching does not occur. This can be achieved by using a number of methods including spacing channels unequally and operating at wavelengths where channels propagate at different speeds. This topic is discussed in more detail in section 2.

Since two or more frequencies of light are required for FWM, the generated signal may be switched on and off by switching one of the interacting light sources on and off. This effect forms the basis for optical frequency converters in which light at one frequency and modulated by a digital signal can be used to generate light at a different frequency but carrying the same data pattern. The data rates that can be supported by this kind of all-optical frequency conversion are significantly higher (>100 Gb/s) than those possible with optoelectronic devices, which operate below 10 Gb/s. High speed optical frequency conversion will play an important role in the optical networks of the future and is discussed along with other applications for the telecommunications industry in section 3.

How efficiently four-wave mixing occurs in an optical fibre depends on a number of fundamental fibre parameters that are of interest to an increasingly wide range of fibre

users. These parameters include the zero dispersion wavelength, second-order refractive index and the fibre birefringence. These parameters can be determined through measurements based on FWM and in some cases the FWM techniques provide more useful and accurate results than are possible with other methods. Of particular interest are distributed measurements, in which a parameter is measured at points along the length of the fibre without destroying it. Distributed measurements have applications in measuring properties of installed fibre cables and also offer a possible technique for temperature profiling in hazardous environments. The metrological applications of four-wave mixing, particularly those which will be of benefit to industry, are discussed in section 4.

It is concluded that four-wave mixing will become an increasingly significant phenomena as the number of channels and the optical powers in fibres increase. The telecommunications industry will be the main driver for research into this field since FWM must be avoided to maximise the capacity of installed fibre. The same industry also represents a large market for spin-off devices such as wavelength converters and demultiplexers based on four-wave mixing. High-speed devices such as these will form the foundation of the all-optical networks of the future. Recommendations for further work include the evaluation of FWM techniques for distributed measurements of fibre parameters for the telecommunications industry. Distributed temperature measurements using FWM in fibre should also be investigated as this could have applications in hazardous petrochemical environments.

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1. Introduction

Four-wave mixing is a non-linear process in that it is a consequence of a non-linear relationship between the polarisation induced within a dielectric medium and the amplitude of the incident electromagnetic field. The effect manifests itself in the appearance or amplification of radiation at a number of frequencies due to the incidence of radiation at two or more other frequencies onto an active material. The generation of a new frequency of radiation due to FWM has applications in the development of tunable sources and wavelength conversion¹ in all-optical routing systems whereas the amplification of an existing signal can be used for optical amplification and demultiplexing at bit rates of up to 500 Gb/s². However, frequency generation and amplification can be disadvantageous in systems where many frequencies are co-propagating but each one must remain independent. The most important example of this situation is in wavelength division multiplexed (WDM) optical communication systems.

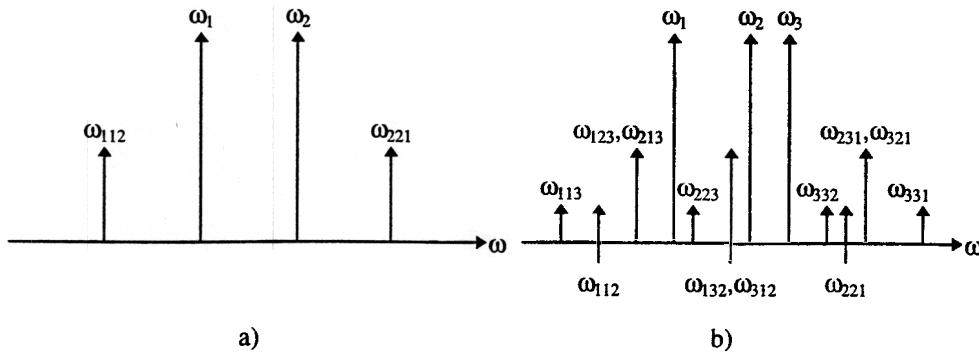


Figure 1. Additional frequencies generated through FWM in the partially degenerate (a) and non-degenerate case (b).

Figure 1 illustrates how four-wave mixing can lead to generation of a number of extra frequencies from the interaction between light at two or three incident frequencies. The relation that gives the frequency of the generated wave, ω_{ijk} is

$$\omega_{ijk} = \omega_i + \omega_j - \omega_k \quad 1-$$

If two of the three waves have the same frequency and therefore there are only two distinct frequencies initially in the fibre (figure 1a), then the effect is known as partially degenerate four-wave mixing (PDFWM). In this case, one of the incident waves assumes the role of both ω_i and ω_j in equation 1-1 and only two new frequencies are generated - one for $\omega_i = \omega_j = \omega_1$ and one for $\omega_i = \omega_j = \omega_2$. The nondegenerate case (NDFWM), in which three unique frequencies are initially present, is illustrated in figure 1b. It can be appreciated from this diagram that a large number of generated frequencies are possible through different permutations of ω_i , ω_j and ω_k .

Equation 1-1 gives the frequency of radiation that would be generated by FWM, should the process occur. It does not provide any information as to whether the process will occur or not, this is dictated by the *phase matching condition* (see section

6.2). Phase matching requires that the propagation constants of the waves satisfy a relation similar to 1-1 that ensures momentum is conserved before and after the interaction. In *dispersive* media, such as silica, light at different frequencies propagates at different speeds as opposed to a non-dispersive vacuum in which all frequencies propagate at the same speed. In order to achieve phase matching in an optical fibre, the dispersion of the fibre must be such that the frequencies involved in FWM also have the correct propagation constants to satisfy the phase matching condition.

1.1 Terminology

The study of four-wave mixing is complicated by the many conventions adopted by different authors to label the three (partially degenerate case) or four (non-degenerate case) frequencies in equation 1-1. However, terminology does not only depend on whether PDFWM or NDFWM is being considered but also on how phase matching is achieved. The terms commonly used in the literature to refer to the various waves involved in FWM are discussed below.

Partially Degenerate Four-Wave Mixing

PDFWM requires one or two incident waves and results in the generation of extra frequencies to give a total of three waves. If only a single incident wave at $\omega_1 (= \omega_2)$ is present, FWM may occur between it and noise in the fibre at a frequency ω_3 that satisfies the phase matching condition. This results in the growth of waves at frequencies ω_3 and ω_4 , where ω_4 is given by equation 1-1. Alternatively, if a wave at ω_3 is also launched into the fibre then its optical power is supplemented via the FWM process and the wave at ω_4 is also generated. The process whereby the optical power at ω_3 is increased through FWM with the wave at ω_1 is known as *parametric amplification*. Two schemes are generally used to investigate parametric amplification that differ with respect to how the phase matching condition between waves at ω_1 and ω_3 is achieved. Both of these situations are shown in figure 2 where the waves have been labelled using terms commonly applied to each situation. Note that signal and idler (or conjugate) are defined by which wave is amplified and which is generated respectively, whereas the Stokes/anti-Stokes are the lower/higher frequency sidebands respectively. The Stokes and signal waves are not necessarily the same, depending on whether the amplified wave is of a higher or lower frequency than the pump wave.

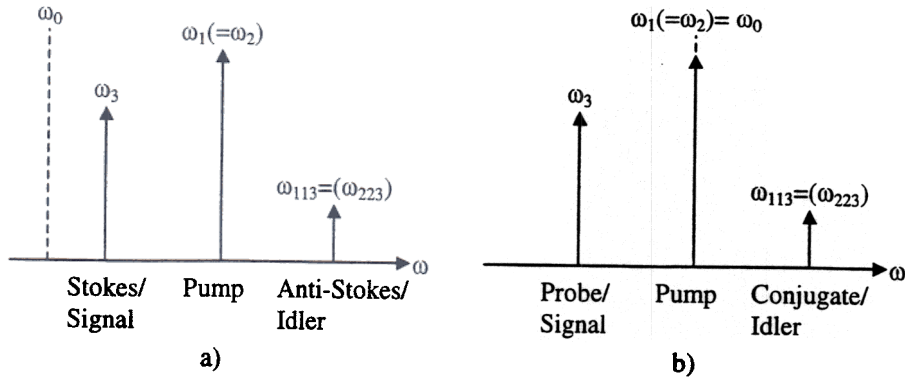


Figure 2. Terms used to describe the waves involved in PDFWM when a) the pump frequency and fibre zero dispersion frequency are different and b) the pump and zero dispersion frequencies coincide.

Non-Degenerate Four-Wave Mixing

Ironically, the increased complexity of the non-degenerate four-wave mixing case has a less confusing nomenclature. Waves are usually labelled simply by number, as in figure 1 since the multiple FWM products would otherwise be too difficult identify. One exception to this is in the case of tunable wavelength conversion near the zero dispersion wavelength of a fibre¹. In this case, two *pump* waves are used to satisfy the phase matching condition and allow an input *signal* wave to generate a fourth wave simply referred to as the *frequency converted wave*.

2. Implications of FWM for Commercial WDM Systems

Wavelength Division Multiplexed systems greatly increase the total bandwidth of each optical fibre by using a number of closely spaced channels at wavelengths within the typical 1530nm to 1565nm gain spectrum of erbium doped fibre amplifiers (EDFAs). Any interaction between these channels will lead to a degradation of the bit-error rate (BER) of the system for two reasons. Firstly, the pump channels will experience signal depletion as optical power is transferred to a different wavelength. Secondly, if the frequency of a FWM product coincides with one of the allocated system channels then this channel will suffer from noise. This is a particular problem for channels that are equally spaced in frequency such as in the ITU 100 GHz spacing recommendation for WDM³. For a WDM system with N channels, the number of four-wave mixing products, M , will be⁷

$$M = \frac{1}{2}(N^3 - N^2) \quad 2-1$$

Clearly, for a typical 8-channel WDM system, unless great care is taken to allocate the channel frequencies carefully, some crosstalk will occur due to FWM. With 32 and 40-channel WDM systems on the horizon, four-wave mixing is a serious consideration in system design.

The most detrimental cross modulation between channels in a WDM communication system occurs for closely-spaced frequencies with uniform frequency separation near the fibre dispersion zero, λ_0 . This occurs for two reasons:

- The phase matching between the signals is more easily achieved in this region.
- The reduced chromatic dispersion slows the walk-off between neighbouring signals and reduces any averaging of the cross-modulation that would otherwise occur. This will be particularly serious in NRZ systems where trains of 1's and 0's in neighbouring channels may be modulated by each other and become fragmented.

Recently, investigations have been conducted on a system with up to 27 channels separated by 100 GHz and centred on the 1550 nm zero dispersion wavelength of a dispersion shifted fibre⁴. Defining crosstalk as the ratio of FWM power to signal power in the worst affected channel, the results confirmed that it increases as the average channel wavelength approaches λ_0 . It was also demonstrated that crosstalk increased with decreasing channel separation, as expected. Crosstalk measurements were also performed by starting with a 3 channel system centred on λ_0 and adding groups of four channels. Although the actual number of four-wave mixing products increased when more channels were added, it was found that crosstalk increased to a maximum at 19 channels and did not increase any further when four more were added. This effect was attributed to the increasing separation between the newly added channels and the centre of the channel spectrum at λ_0 . This increased separation had two effects: firstly, the efficiency of the FWM was lower and secondly the products from the additional channel were not extending to the worst affected channels at the centre of the comb. This 19 channel limit would not be expected to apply to other frequency spacings between channels, such as 50 GHz, as the 19th channel in this case will be nearer to λ_0 than that in a 100 GHz spacing system. Results from these investigations also showed that the FWM power spectrum for 27 equally spaced channels centred on λ_0 was not symmetrical, as would be expected, but extended further into the longer wavelength region than into the shorter wavelengths. No explanation could be found for this effect, which results in the WDM channels above the zero dispersion wavelength receiving more power from FWM than the shorter wavelength channels.

It has also been proposed that operating WDM systems with a phase modulation method rather than amplitude modulation reduces the FWM-induced errors. Differential phase shift keying (DPSK) has been suggested for this purpose⁵, a technique which has been used to successfully transmit four channels at 10 Gb/s over a 2400 km link⁶. However, in this example, FWM was avoided by using a dispersion management scheme rather than relying on the effects of the DPSK modulation.

In summary, WDM system degradation due to four-wave mixing can be minimised by⁷:

- Using non-uniform frequency spacing between channels in dispersion shifted fibre.
- Increasing the channel separations
- Operating away from the zero dispersion wavelength of the fibre by using non-zero dispersion-shifted fibre.

- Using optical differential phase shift keying (DPSK) with direct detection rather than intensity modulation.

The disadvantage of the third technique is that a non-zero dispersion leads to optical waveform distortion at each channel frequency due to chirp in the transmitter. However, by alternating the sign of the dispersion slope along the fibre link, so-called *dispersion-managed* systems can be created that have zero net dispersion but non-zero dispersion at the transmission wavelengths at every point along the link. This can be achieved by constructing the link from alternate sections of fibre with positive and negative dispersion or by constructing a single fibre with continuously increasing or decreasing dispersion⁸. Non-destructive measurement of λ_0 along a dispersion-managed link is a possible metrological application of FWM and is discussed in section 4.1.1.

3. Applications for the Communications Industry

3.1 Optical Telecommunications Systems

3.1.1 High-Speed Optical Multiplexing/Demultiplexing

In order to make the most efficient use of the available bandwidth of optical fibres, data streams at bit rates well below the rate that can be supported by a single fibre are combined by interleaving them in time. This time division multiplexing (TDM) scheme allows a large number of low bandwidth signals to share a single channel of much greater capacity than a single signal would use alone. Optical time division multiplexed (OTDM) systems such as the Synchronous Digital Hierarchy (SDH) in Europe and the Synchronous Optical Network (SONET) in the United States have formed the backbone for global optical fibre communications for the last two decades. Increasing the capacity of OTDM systems requires that the bit rate of the multiplexed signal be increased - reducing the time interval allocated to each bit. Currently, multiplexing and demultiplexing are required at speeds in the region of 2.4 Gb/s to 10 Gb/s and may be performed electronically. However, to increase the channel capacity by using higher bit-rates of 100 Gb/s or more requires faster optical techniques for interleaving and selectively extracting optical pulses. Four-wave mixing has been successfully used to multiplex and demultiplex data at these ultrahigh-speeds and provides a 10 dB SNR improvement over the alternative Nonlinear Loop Mirror (NOLM) demultiplexer³⁴.

The four-wave mixing all-optical demultiplexer uses short, intense, co-propagating pump pulses synchronised to the time slot of the required data. Either the data or the pump pulses must be at the zero dispersion wavelength of the fibre so that the pump interacts with the data through partially degenerate FWM to generate an identical data stream at a complimentary frequency. This frequency-converted signal is then separated from the main data stream using a bandpass filter. The advantage of converting the optical frequency of the demultiplexed data before detecting it is that the bandpass filter can remove noise due to the main data stream. This gives a better SNR than other methods such as the NOLM where the demultiplexed data remains at the original optical frequency of the channel.

Pressure on the long-haul fibre links in the United States lead to the rapid development and adoption of WDM systems as a way of economically extending the bandwidth of installed fibre plant by factors of 16 or more. Previously, however, some operators chose to install dispersion-shifted fibre to maximise the transmission capabilities of their OTDM systems. As discussed in section 2, DSF is a particularly poor medium for WDM and operators must take steps to minimise the detrimental effects of FWM. However, increasing the frequency spacing between channels and using non-uniform separations become more difficult to implement when the number of WDM channels is increased. The number of WDM channels that can be used on dispersion-shifted fibre may thus be limited, in which case higher OTDM multiplexing/demultiplexing rates, such as those attainable through four-wave mixing will be required.

The pressure to adopt WDM is less in Europe than the United States but it is anticipated that these systems will become more significant in the very near future. WDM systems use of OTDM to maximise the bandwidth of each individual channel. Therefore, although four-wave mixing acts to limit the maximum number of channels, it may also be used to help maximise system bandwidth through allowing higher demultiplexing rates in each one. FWM is therefore a significant effect for maximising the bandwidth of existing OTDM and future WDM fibre systems with large markets in the US and a growing market in Europe.

3.1.2 Parametric Amplification

When a strong pump signal is incident on an optical fibre, partially degenerate four-wave mixing will lead to the generation or amplification of signals at the Stokes and anti-Stokes frequencies as power is transferred from the pump signal. These signals may grow from the noise in the fibre or, if one of them is already present, then it will be amplified by the pump signal while the complimentary signal is created. This process may be used as the basis for *parametric amplifiers* in a similar way to how Stimulated Raman Scattering (SRS) is used to produce Raman amplifiers for wavelengths not covered by the ubiquitous erbium-doped fibre amplifiers at 1550 nm. Optical amplification at wavelengths outside this region has applications in communications systems operating at 1300 nm - the transmission window for which the majority of installed fibre systems are specified. Although the gain bandwidth of parametric amplifiers (~10-100 GHz) is less than that of Raman amplifiers, (~5 THz), the spectral separation between the pump wavelength and the amplified signal wavelength is much greater (up to 100 THz compared to 13 THz for Raman gain). This increased spectral separation permits a wider choice of optical pump sources than for fibre-Raman amplifiers.

Parametric amplification has been achieved at 1292 nm using a Q-switched Nd:YAG laser at 1064 nm with phase matching achieved by varying the external stress on a birefringent fibre⁹. Amplification of a signal at 1552 nm has also been achieved in dispersion-shifted fibre using a DFB laser pump at 1549 nm - the zero dispersion wavelength of the fibre¹⁰. The main disadvantage of parametric amplification through FWM in fibre when compared to Raman amplification and semiconductor optical amplifiers is the low efficiency. Long lengths of fibre and high pump powers are required due to the low non-linear coefficient of silica and consequently steps must

also be taken to avoid stimulated Brillouin scattering, which acts to limit the maximum pump power. Conversion efficiencies of approximately -8 dB have been achieved for input pump power of 15 dBm.

3.1.3 Wavelength Conversion

Future all-optical networks based on WDM and optical routing will need wavelength converters to allow maximum flexibility when multiplexing and allocating channels. Four-wave mixing in optical fibres and DFB lasers are two possibilities for ultra-fast wavelength conversion in this environment^{1,11}. Wavelength conversion based on four-wave mixing has the advantage over other methods such as cross-gain modulation and optoelectronic methods that conversion is coherent and maintains the phase of the wave. FWM-based wavelength converters are therefore suitable for coherent systems based on phase-shift keying (PSK) as well as intensity modulation schemes¹².

Conversion efficiency in optical fibres may be limited by the effects of stimulated Brillouin scattering (SBS), which acts to limit the maximum forward propagating optical power in the fibre. Once the forward propagating optical power has exceeded a threshold value, SBS causes any increase in power to be transferred to a backscattered wave. SBS has been demonstrated to truncate the increase in FWM power for incident powers above 10 dBm in a 10 km length of DSF. However, specialised fibre manufacturing techniques have been shown to raise the SBS threshold by varying the dopant concentration along the fibre core¹³, allowing the FWM power to continue increasing for pump powers up to 15 dBm.

Tunable wavelength conversion has also been demonstrated using FWM in a 2.5 km dispersion-shifted fibre. Conversion occurred between wavelengths close to the dispersion zero of the fibre and therefore this technique is not suited to optical routing in WDM systems operating over conventional or non-zero dispersion-shifted fibre. Wavelength conversion near λ_0 may, however, be useful for integrating OTDM signals on dispersion-shifted fibre into WDM systems where the original wavelength of the signal is not acceptable on the WDM network.

3.1.4 Dispersion Compensation

Four-wave mixing has been suggested as a possible technique for compensation of chromatic dispersion in long-haul optical fibre systems¹⁴. The dispersion that occurs along an initial length of fibre is corrected by reversing the chirp on an optical pulse using FWM in an *optical phase conjugator* (OPC) to perform *mid-span spectral inversion*. The chromatic dispersion of the following fibre then brings the pulse back to its original shape. The FWM effect is known as phase conjugation and is essentially a form of wavelength conversion. A pump at the zero dispersion wavelength of the OPC fibre is used to generate a conjugate signal on the opposite side of λ_0 to the initial signal, reflecting its power spectrum into the region of opposite chromatic dispersion. Providing the resulting dispersion after the OPC is of equal magnitude to that before, zero net dispersion results.

The conversion efficiency from the signal to the conjugate wave in the OPC is maximised under the same conditions as FWM - namely phase matching, small

signal-pump wavelength separation and low dispersion. Phase matching is satisfied by using a pump signal at λ_0 but the small signal-pump separation requires that the signal should be propagating near the zero dispersion wavelength of the fibre. This means that WDM systems, which tend to use non-zero dispersion shifted fibre, are unlikely to benefit from this dispersion management technique but it can be used on conventional OTDM systems. Another important point is that mid span spectral inversion cannot correct for third-order chromatic dispersion, which can be problematic for very high bit rate systems. Polarisation sensitivity must also be considered when using this technique if the conversion efficiency is to be independent of the signal polarisation. Using two pump waves with orthogonal polarisation and different wavelengths, it has been shown that polarisation sensitivity can be reduced¹⁵.

3.2 Mobile Communications

Mobile radio communication systems, such as personal mobile telephones, operate on a cellular system in which a transmitter/receiver base station assumes responsibility for providing service to handsets within its immediate surroundings. The size of the cell is defined by the attenuation of the local environment and the maximum and minimum powers that the handsets can transmit and receive respectively. To provide continuous coverage to roaming users, neighbouring cells are packed together so that when the user is about to leave one cell, the neighbouring cell takes over handling the call without interruption.

Each cell has a set of available channels that handsets within the cell coverage area can use without conflicting with each other. To prevent crosstalk between handsets in neighbouring cells, different sets of channels are used in adjacent cells although the same channels may be used in other cells further away, from which stray signals are unlikely to have a significant effect. The simultaneous use of the same channel by multiple handsets in different cells is termed *re-use*.

The spectrum available for radio communication systems is very limited so it is essential that each operator makes the most efficient use of their allocation to maximise the number of non-conflicting channels available in a given geographical area e.g. a city centre. This can be done by using a large number of small cells, or *microcells*, rather than a single cell to cover a particular area. Using microcells permits re-use of channels that otherwise could only be used once and although each microcell cannot offer all channels there is an increase in the total system capacity over the geographical area. Another advantage of microcells is that the short distances between transmitters and receivers (~10 - 50 m) permits the use of frequencies that are highly attenuated by the atmosphere, such as 60 GHz, and which could not otherwise be used for cellular systems. The disadvantage of packing more, smaller, cells into a given coverage area is the increase in cost associated with deploying a base station in each microcell to perform the tasks of transmitting, receiving, channel allocation and handover to neighbouring cells. A cheaper alternative is to use transmitters and receivers with less functionality and operating under the control of a centralised control unit that can co-ordinate the operation of neighbouring cells.

Four-wave mixing in dispersion shifted fibre has recently been demonstrated to be an elegant method of generating a 60 GHz mm-wave suitable for broadband wireless

systems¹⁶. The CW output from a laser diode source with optical frequency f_0 was intensity modulated at frequency $\Delta f = 20$ GHz to produce two sidebands at $f_0 + \Delta f$ and $f_0 - \Delta f$. The three signals were then amplified using an EDFA and fed through a 35 km length of dispersion shifted fibre. Efficient non-degenerate FWM in the fibre generated extra sidebands at frequencies $f_0 \pm n\Delta f$ where n is an integer. All signals were optically filtered out except for the main carrier at f_0 and one of the sidebands at $f_0 + 3\Delta f$. The sideband at $f_0 + 60$ GHz was intensity modulated with a 156 MHz sinusoidal wave to simulate a data signal. The carrier and sideband were then transmitted along 108 km of standard single mode fibre and combined at a photodiode using a self-heterodyne technique. The recovered signals, consisting of a carrier wave at 60 GHz and sidebands at $60 \text{ GHz} \pm 156 \text{ MHz}$, showed excellent amplitude stability and narrow spectral widths. The amplitude and spectral stability was significantly better than that achieved when the modulated sideband was combined with a carrier signal taken directly from laser diode. This stability was attributed to the phase matching condition required for four-wave mixing.

4. Applications to the Measurement Industry

4.1 FWM-Based Measurements of Fibre Parameters

In dispersion managed transmission systems, the aggregate chromatic dispersion over the fibre link is designed to be zero to eliminate pulse spreading but the local value along the fibre is non-zero to avoid phase matching between the co-propagating channels. Non-destructive measurement of the dispersion along the fibre is an attractive tool for testing and monitoring dispersion managed cables. This is especially so since the zero dispersion wavelength of a DSF has been found to have temperature dependence of 0.03 nm/degree that could lead to significant deviations in λ_0 from the specified values of the constituent fibres depending on the ambient temperature³². The following sections describe the conditions under which four-wave mixing may be used as an analytical tool for mapping the dispersion characteristics of fibres, including the zero dispersion wavelength and the non-linear coefficient.

4.1.1 Zero Dispersion Wavelength

Destructive measurements of λ_0 in dispersion shifted fibre have shown variations of 0.5 nm over 500 m and 3.5 nm over 23 km^{17,32}. The value of the local zero dispersion wavelength of the fibre relative to those of the propagating signals determines the local FWM efficiency. It is therefore important that the λ_0 distribution in a fibre as well as the average value should be measurable non-destructively to permit system planning.

Continuous Wave Methods

Variations in the zero dispersion wavelength along an optical fibre can be qualitatively estimated by using a CW technique with pump radiation close to the zero dispersion frequency, ω_0 . In the region around λ_0 , in which the dispersion slope is approximately linear, using two pump waves either equally separated to either side of

ω_0 , or both at ω_0 gives phase matching for a probe signal at any frequency ω_3 . The FWM efficiency should be unity for all values of ω_3 but in practice the zero dispersion wavelength varies along the fibre due to fluctuations in the core composition and geometry³¹. Consequently, the ideal phase matching condition is not satisfied along the entire fibre and the FWM efficiency decreases with increasing separation of the pump and probe wavelengths. The rate at which the efficiency falls from maximum as the pump-probe frequency separation increases gives an indication of the variation in the zero dispersion wavelength over the entire fibre.

Recently, a CW technique has been proposed for measuring the λ_0 distribution along a fibre with a theoretical spatial resolution of 100 m¹⁸. The technique relies on measuring the intensity of the FWM product between a pump and probe signals as the wavelengths of the laser sources are varied, keeping their wavelength difference constant. A parameterised model of the fibre in which the zero dispersion wavelength distribution is expressed in terms of Chebychev polynomials is used to simulate the FWM intensity versus wavelength curve. The error between the experimental and simulated curves is minimised with respect to these parameters and the corresponding λ_0 distribution calculated. It is claimed that the method can resolve fluctuations in λ_0 of as little as 0.05 nm although comparisons of calculated distributions and destructively measured distributions over 7 km and 25 km of DSF indicate current uncertainties of the order of ± 1 nm.

Pulsed Methods

Using pulsed pump and probe signals, it is possible to limit the region of FWM interaction to a short section of a long fibre and thereby reduce the likely variation in λ_0 to a level where it may be assumed to be constant. The location of the interaction region is determined by the dispersion of the fibre and the frequency and temporal separation between the pump and probe. Fibre dispersion causes the pump and probe pulses to propagate at different group velocities so that they can be made to pass through each other at some region in the fibre. Scanning the position of this point along the fibre and measuring the power in the FWM product leaving the far end produces a map of the local average zero dispersion wavelength. The spatial resolution of the measurement is determined by the length of the interaction region and a compromise is required between the improved resolution and reduced FWM efficiency. Resolution may be increased by using a larger pump-probe wavelength separation to maximise the group velocity mismatch. However, the efficiency of FWM falls off rapidly with increasing pump-probe separation. Reducing the temporal pulse widths can improve spatial resolution but at the cost of reduced spectral resolution for pulses shorter than a few tens of picoseconds.

Accuracies of ~ 0.2 nm have been achieved using this method with spatial resolution of 700 m and a theoretical limit suggested of 0.1 nm accuracy with 70 m spatial resolution using 15 ps (FWHM) pulses with 60 nm separation^{17,33}. The main disadvantage of this method is that access to both ends of the fibre is required to vary the launch conditions and measure the output spectrum from the fibre.

Other pulsed techniques for measuring the zero dispersion wavelength distribution along fibres and based on four-wave mixing include OTDR-type methods in which the

dispersion is measured at a given wavelength and the zero dispersion wavelength inferred from an estimate of the dispersion slope¹⁹. Dispersion and dispersion slope measurements using OTDR techniques have the advantage of only requiring access to one end of the fibre and are discussed in section 4.1.2.

4.1.2 Dispersion and Dispersion Slope

Continuous Wave Methods

For pump wavelengths around the zero dispersion wavelength, the efficiency of partially degenerate four-wave mixing between two continuous waves displays a periodic dependence on pump wavelength. For a fibre with uniform zero dispersion wavelength, the main central peak of the resulting curve is centred on λ_0 and the period of the peaks is related to the dispersion slope of the fibre, $dD/d\lambda$. Experiments on a 1.1 km length of dispersion shifted fibre showed that the zero dispersion wavelength and dispersion slope of the fibre could be determined from a curve fitted to the periodic experimental data²⁰. The same approach, when applied to a 23 km length of fibre was unsuccessful as the efficiency curve did not exhibit any discernible periodic feature or a distinct central peak. This was attributed to variations in the zero dispersion wavelength along the fibre.

Observation of the periodic efficiency curve requires that the zero dispersion wavelength remain reasonably constant over the interaction length of the fibre. As discussed in section 4.1.1, this is not normally the case in real fibres and FWM effects in long fibres tend to have complicated characteristics that depend on the variation in phase mismatch along the full length of the fibre. Consequently, continuous wave measurements are only capable of measuring the aggregate properties of an entire fibre. However, pulsed methods may be used to restrict the interaction length so that FWM mixing occurs over restricted regions of the fibre with reduced phase mismatch variation. As with zero dispersion wavelength, distributed measurements are not possible with CW sources but may be achieved using pulsed techniques.

Pulsed Methods

Two similar methods for performing non-destructive distributed measurements of the dispersion of optical fibres have recently been reported^{19,21}. Both techniques use optical time domain reflectometry (OTDR) measurements to monitor FWM efficiency as two high power optical pulses co-propagate along the fibre. These methods are particularly useful as they only require access to one end of the fibre and are therefore ideal for monitoring installed cables.

When two pulses with closely spaced wavelengths are simultaneously launched into an optical fibre, partially degenerate FWM occurs and generates forward-propagating optical power at two new wavelengths. However, the intensity of the FWM products varies periodically along the fibre rather than simply accumulating and the period of oscillation at any point in the fibre is related to the local value of the fibre dispersion, D . For a pump wavelength of λ_p and pump-probe separation of $\delta\lambda$, the spatial frequency of the oscillation, F_s , is given by

$$F_s = cD(\lambda_p) \left(\frac{\delta\lambda}{\lambda_p} \right)^2 \quad 4.1.2-1$$

OTDR techniques measure the optical power scattered through 180° by Rayleigh scattering and joints/defects when a short pulse of light is injected into one end of a fibre. The backscattered optical power from a particular point in the fibre can be identified from the temporal delay between launching the pulse and arrival of the scattered signal. The magnitude of the detected backscattered power is proportional to the forward-propagating optical power at the point of scattering, corrected for the attenuation of the return path along the fibre. Spatial variations in the local intensity of the FWM products along the fibre can therefore be measured from the temporal variations in the backscattered power received at the OTDR. In this way, the spatial period in equation 4.1.2-1 can be measured at points along the fibre and used to calculate the dispersion at these points.

A compromise is required between increasing the spatial resolution of the measurement by using a large value of $\delta\lambda$ and maintaining a high enough level of FWM efficiency to give an acceptable SNR at the receiver. The spatial resolution of the measurement is limited by the ability to accurately determine the period of the intensity fluctuations in a given region of fibre. A spatial resolution of 500 m with a relative uncertainty of 2 % on the measured dispersion has been reported by using a pulse wavelength separation of 3.6 nm on a 25 km long dispersion shifted fibre²¹. Limits on the maximum length of fibre that can be measured are set by the sensitivity of the detector and therefore the power in the backscattered FWM products. Backscattered power may be increased by injecting Raman pump light into both ends of the fibre to induce Raman gain of the FWM product. In this way, the range of the measurement technique may be increased from 25 - 30 km to around 70 km²¹. Although this technique requires access to both ends of the fibre and negates one of the advantages of reflectometry, the measurement apparatus is still only required at one end of the fibre. A suitably stable Raman laser at the far end of the fibre need not necessarily be connected to the dispersion mapping equipment.

4.1.3 Second-Order Refractive Index

The periodic FWM efficiency vs. pump-probe wavelength separation curve discussed in section 4.1.2 within the context of CW dispersion slope measurement can also be used to measure the second-order refractive index - an increasingly important parameter in optical fibre communications. Whereas the periodicity of the curve is a function of the dispersion slope of the fibre, the ratio of the peak FWM power at maximum phase matching to the input pump power is dependent on the second-order refractive index (see equation 6.4-1 with $\eta = 1$). Calculation of n_2 requires knowledge of the effective area of the fibre and the assumption that it, and the dispersion, is uniform along the entire length. Evaluation of n_2 in a 1.1 km long DSF has been performed by this method whilst simultaneously calculating the dispersion slope³². The measured n_2 found in this case was $2.70 \times 10^{-20} \text{ m}^2/\text{W}$, which is higher than other typical values of $2.25 \times 10^{-20} \text{ m}^2/\text{W}$ to $2.55 \times 10^{-20} \text{ m}^2/\text{W}$ measured using SPM techniques²².

As with all measurements based on four-wave mixing, extracting fibre parameters from the cumulative efficiency of the process along a section of fibre will be more successful if they remain relatively uniform along the fibre. Short interaction lengths are therefore required, which implies continuous wave measurements on lengths of up to ~ 1 km and pulsed measurements on lengths longer than this. Pulsed measurements should be performed with high repetition rates to maintain the validity of the FWM power equation (6.4-1), which uses the average optical powers at all four interacting frequencies and is based on the assumption of CW operation.

4.1.4 Non-linear Coefficient

One successful technique for measuring the non-linear coefficient of optical fibre that uses two closely spaced CW sources can be regarded as an SPM method²³ or a partially degenerate FWM method²⁴. The SPM approach considers the two sources to generate a high intensity beat signal that modulates the refractive index of the fibre through the Kerr effect - leading to self-phase modulation. The phase modulation of the beat signal gives rise to extra frequencies to either side of the two main signals. From the FWM perspective, these additional frequencies are simply the mixing products that would be produced in partially degenerate FWM.

In both experiments, it is the powers at the newly generated frequencies relative to those in the original waves that are measured and used to calculate the non-linear coefficient of the fibre, n_2/A_{eff} . Analysis of the multiple peaks in the output spectrum of the fibre differs depending on which type of non-linear effect is assumed to be responsible. The SPM method uses the more direct approach - evaluating the non-linear coefficient from the measured SPM phase shift - and has achieved better accuracy than the FWM method at around 5%. The FWM method uses numerical simulation to find a value for the non-linear coefficient that gives the best fit to the experimental data. The calculation is complicated and involves parameterising two coupled nonlinear Schrödinger equations, taking into account fibre loss, chromatic dispersion and variations in the birefringence axis along the fibre. An accuracy of less than 15 %, directly related to the uncertainty on the output optical power measurements, has been quoted for this method.

It is worth clarifying at this stage that the difference between this method for measuring n_2/A_{eff} and the previous technique for n_2 described in section 4.1.3 is that in this case the two frequencies launched into the fibre are fixed and the output is a peaked spectral power distribution. The previous method used a swept pump frequency and the measurand was the peaked FWM efficiency vs. pump frequency curve.

4.2 Mechanical Sensing

High birefringence polarisation maintaining fibres (PMFs) use non-circular symmetric cores to break the degeneracy of the two polarisation states of the single mode fibre (see section 6.3.2). This ensures that they have sufficiently different propagation characteristics to eliminate coupling of optical power from one mode to another. The difference in group velocity between the two modes is dependent on the degree of

asymmetry of the core, which may be adjusted by geometrical, mechanical or electromagnetic means.

Phase mismatch in four-wave mixing may be minimised through balancing material, waveguide and non-linear dispersion (see section 6.3.3). In conventional single mode fibre, small perturbations in waveguide geometry cause power to randomly couple between two orthogonal polarisation modes as it propagates along the fibre. All power then effectively propagates in a single aggregate mode and the phase mismatch due to waveguide dispersion in these fibres is assumed to be negligible. However, in PMF two distinct propagation modes are maintained and the modal dispersion can be used to balance the material and non-linear dispersion contributions and facilitate phase matching and FWM.

The polarisation mode dispersion in high-birefringence fibre can be tuned by varying the non-circularly symmetric stress applied to the fibre core. This may be achieved by applying lateral compression, bending or varying the temperature of the fibre. The main advantage of four-wave mixing in PMF over other techniques is the broad range over which the frequency shift between the pump and signal waves can be tuned. This is because the requirements placed on the propagation constants can be more relaxed since the chromatic dispersion may be balanced by the variable polarisation mode dispersion.

By launching a fixed pump wavelength in to one polarisation mode and a probe wavelength into the other mode, parametric amplification of the probe wavelength will occur at a frequency separation determined by the birefringence of the fibre. This can be used as the basis for a measurement of the net polarisation mode dispersion in the fibre²⁵ by measuring the parametric amplification vs. pump-probe frequency separation.

Alternatively, variations in external forces on the fibre may be monitored by fixing the pump and probe frequencies and measuring the efficiency of the FWM process. In this way, high birefringence fibres could be used as real-time sensors to measure stress, bending and temperature – all of which can significantly affect the local birefringence of the fibre. For short lengths of fibre, CW methods could be used in the same way as for the dispersion measurements mentioned previously. However, distributed measurements of stress and temperature over distances of kilometres could have many applications in fields such as the petrochemical industry. Pulsed techniques using temporally delayed pulses launched into the orthogonal polarisation states of a high birefringence fibre may permit this type of measurement. The usual compromise of pump-probe frequency separation vs. spatial resolution would be more relaxed in this case than for distributed dispersion measurements. The differential group velocity between the two polarisation states would add to that due to chromatic dispersion and cause closely spaced pump and probe waves to walk through each other more quickly than with chromatic dispersion alone.

5. Conclusions

Four-wave mixing is a nonlinear process that occurs in materials such as silica with symmetric molecular structures and leads to generation of power at new optical

frequencies from two or more co-propagating waves. Efficient FWM requires high optical intensities and satisfaction of a condition known as phase matching, which is effectively conservation of energy and momentum between the incident and generated waves. Phase matching is most easily achieved between waves with very similar frequencies and in materials with low dispersion. However, in birefringent fibres or when optical power levels are high enough to cause self-phase modulation, these conditions may be relaxed through PMD and nonlinear contributions to phase matching respectively.

5.1 Industrial Drivers

The most significant negative effects of FWM are parametric gain and crosstalk in long haul, multichannel WDM systems. WDM systems increase the information carrying capacity of a single optical fibre by using it to transmit a number of channels simultaneously at different optical frequencies rather than just a single channel per fibre. Ideally, as many channels would be packed into the gain window of the erbium-doped fibre amplifier as could be separated at the fibre output and dispersion-shifted fibre would be used to minimise chromatic dispersion. However, FWM efficiency is maximised at the zero dispersion wavelength and for closely spaced input waves. Of particular concern is the case of WDM channels that are equally spaced in frequency as any FWM products will fall on frequencies used by other channels. Two general strategies are used to reduce the effects of FWM on WDM system performance:

Dispersion managed systems over non-zero dispersion-shifted fibre, which has low non-zero dispersion in the WDM operation window of 1530 nm to 1565 nm. By alternating sections of fibre with equal magnitude but opposite sign dispersion, the net chromatic dispersion of the system can be zero without having any sections with zero local dispersion where FWM would occur efficiently.

- ii. Operation over dispersion shifted fibre but with unequally spaced channels. By using unequal channel separations, the products of FWM can be made not to fall on allocated frequencies. Implementing this technique requires accurate values for the zero dispersion wavelength along the fibre – especially in the sections immediately after optical amplifiers, where optical powers will be highest.

Applications of four-wave mixing in telecommunications are mainly limited to single channel systems where there is no possibility of introducing crosstalk at other wavelengths. These systems are usually designed to operate at the zero dispersion wavelength of dispersion-shifted fibre, which is where FWM is maximised. Four-wave mixing can be used to achieve high speed all-optical multiplexing and demultiplexing as well as wavelength conversion and some dispersion management in single channel OTDM systems. Although WDM is being rapidly adopted as the technique of choice for increasing the capacity of long-haul transmission systems, OTDM is used for each channel make efficient use of the fibre bandwidth. In fact, new SDH and SONET systems are also *wavelength specified* to allow eventual integration into WDM systems, so the market for these OTDM technologies is significant and will continue to grow with the WDM market²⁶.

5.2 Meeting Industrial Needs With Metrology

The theory of four-wave mixing is well established and, providing sufficient information is available on the parameters of a fibre and the source characteristics, it should be possible to accurately predict the efficiency of the FWM. Metrological applications of four-wave mixing in optical fibre are all based on measuring the efficiency of the process and making deductions about how phase matching was achieved. In this way, most of the parameters of interest for system design have been measured using FWM-based techniques. Phase matching is a function of the input frequencies, the second-order refractive index, polarisation states of input radiation and material, waveguide and non-linear contributions to the fibre dispersion. FWM efficiency has been successfully used to measure:

- Zero dispersion wavelength, λ_0 ,
- Dispersion, D , and dispersion slope, $dD/d\lambda$,
- Second-order refractive index, n_2 ,
- Nonlinear coefficient, n_2/A_{eff}

FWM efficiency in high-birefringence fibre can be varied by altering the mechanical properties that determine the polarisation mode dispersion. This can be achieved by applying lateral stress, changing the temperature or bending the fibre. The effect could form the basis for long distance distributed sensors in hazardous environments where electrical sensors would not be suitable. The temperature dependence of the zero dispersion wavelength in ordinary dispersion-shifted fibre could also form the basis for a distributed temperature measurement.

5.3 Recommendations for Future Work

The area of distributed measurements using methods based on four-wave mixing looks particularly promising as a metrological tool. As pressure increases to squeeze more bandwidth from pre-installed fibre, wavelength division multiplexed channels will be packed closer together and the effects of FWM will need to be taken into account. Power generated as a result of FWM is dependent on the power in the pump channels, which varies along the fibre due to attenuation and amplification. Distributed, non-destructive, measurement of the factors such as dispersion and non-linear coefficient that determine FWM efficiency will therefore be of more use than measurements of the average value along the entire fibre. The possibility of distributed measurements of external mechanical quantities such as temperature and stress over lengths of up to 25 km or more should also be considered.

Based on the literature survey outlined in this report, the following areas are recommended as warranting further theoretical and experimental investigation:

- a) Distributed measurements of dispersion characteristics and the non-linear coefficient of fibres.

- b) Distributed sensors using high-birefringence fibres for temperature, bending and stress.
- c) An investigation into how variations in the zero dispersion wavelength along actual dispersion-shifted fibres affect FWM in systems with unequal channel separation.

6. Appendix A – The Theory of Four Wave Mixing

6.1 The Origins of Non-linearity

When radiation is incident upon a medium, the oscillating electromagnetic field interacts with electric dipoles in the molecules of the medium and causes them to oscillate. The result is a time-varying local electric polarisation in the medium. This oscillating electric field then re-radiates the electromagnetic field and the incident wave is considered to propagate through the medium via a series of such absorption and re-radiation processes. The polarisation vector, $\mathbf{P}$, induced by an electric field with amplitude vector $\mathbf{E}$ can be expressed as a general series expansion of the form²⁷:

$$\mathbf{P} = \epsilon_0 (\chi \cdot \mathbf{E} + \chi_2 : \mathbf{E}\mathbf{E} + \chi_3 : \mathbf{E}\mathbf{E}\mathbf{E} + \dots) \quad 6.1-1$$

where

ϵ_0 is the electric permittivity of a vacuum,

χ is the linear susceptibility tensor of the medium and

χ_2 and χ_3 are second and third order susceptibility tensors terms.

If the induced polarisation has a purely linear dependence on the applied electric field then the re-radiated electric field will be identical to the incident field. However, when second or higher-order susceptibility terms are non-zero, harmonics begin to appear in the radiated field that were not present in the incident field.

For materials that have a symmetrical molecular structure, the polarisation induced by an incident electric field is symmetrical, as illustrated in figure 3. The susceptibility of these materials contains only odd expansion terms, as opposed to anti-symmetric molecules for which even terms such as χ_2 may be non-zero.

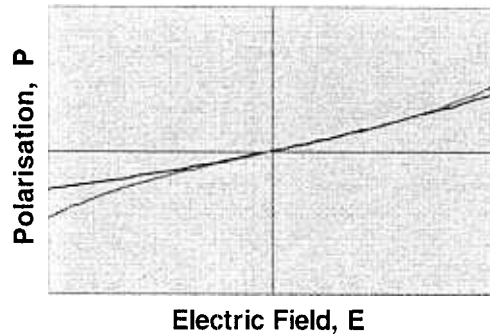


Figure 3. Electric polarisation versus electric field for materials with inversion symmetry (broken line) and with asymmetric molecular structure (solid dark line).

The dominant susceptibility term is the linear term, χ , which determines the linear refractive index of the medium, n , and the absorption attenuation coefficient, α . The

orders of the non-zero expansion terms determine the type of non-linearity to which the medium is susceptible. In materials which lack a centre of symmetry, such as quartz, KDP and ADP, the second order susceptibility is responsible for *second harmonic generation*, in which an intense wave at angular frequency ω_1 generates another wave at twice this frequency, $2\omega_1$. Another effect which is possible in these materials is *sum-frequency generation*, in which two waves at ω_1 and ω_2 interact to produce waves at $\omega_1 + \omega_2$, $2\omega_1 + \omega_2$ and $\omega_1 + 2\omega_2$.

Silica (SiO_2), the basic constituent of optical fibres is a symmetric molecule and consequently, χ_2 vanishes and second-order non-linear effects are not normally observed. Rather, it is the third-order term in equation 6.1.1 that is responsible for non-linear behaviour, which includes *self-phase modulation* (SPM) and *cross-phase modulation* (XPM) as well as four-wave mixing. SPM and XPM can be viewed as processes that result from the refractive index of the fibre material being a function of the intensity of the electromagnetic field. These effects will occur under fairly broad ranging conditions provided that the intensity of the optical field is high enough, whereas FWM requires the satisfaction of a stringent condition known as *phase matching* in order to occur efficiently. It is the degree of phase matching that is the crucial parameter when considering both the beneficial and detrimental effects of four-wave mixing in an optical fibre system.

6.2 General Requirements for Four-Wave Mixing

Four-wave mixing is a process in which optical waves at four frequencies interact via the non-linear response of a medium to the electric fields of the waves. Assuming for simplicity that four optical waves incident upon a dielectric medium are linearly polarised parallel to the x-axis, the combined electric field can be expressed as

$$\mathbf{E}_{TOT} = \hat{x} \sum_{j=1}^4 E_j \cos(\beta_j z - \omega_j t), \quad 6.2-1$$

where E_j is the amplitude of the j th electric field and β_j is the propagation constant of the wave at frequency ω_j . The propagation constant is given by the relation

$$\beta_j = \frac{n_j(\omega)\omega_j}{c} \quad 6.2-2$$

where $n_j(\omega)$ is the frequency-dependent refractive index and c is the speed of light in vacuum.

The second and third order expansion terms when equation 6.2-1 is substituted into 6.1-1 will include a number of terms involving products of the four waves. By collecting terms at each of the four incident frequencies, the non-linear polarisation in the material, $\mathbf{P}_{NL}$, can be resolved into components at each frequency, j , such that

$$\mathbf{P} = \mathbf{P}_L + \mathbf{P}_{NL} = \mathbf{P}_L + \hat{x} \sum_{j=1}^4 P_j \cos(\beta_j z - \omega_j t),$$

where
$$P_j = \frac{3\epsilon_0}{4} \chi_3 [SPM_j + XPM_j + f_j(i\theta_+) + g_j(i\theta_-) +] \quad 6.2-4$$

and f and g are functions of the electric field amplitudes and also the parameters θ_+ and θ_- discussed below.

The polarisation of the material at each of the incident frequencies can be seen from equation 6.2-4 to include terms due to self-phase modulation, cross-phase modulation and two functions of the parameters θ_+ and θ_- that represent the relative phase between the electric field and polarisation, P_j , at the j th frequency. Four-wave mixing is most efficient when θ_+ or θ_- approaches zero, where

$$\theta_+ = (\beta_1 + \beta_2 + \beta_3 - \beta_4)z - (\omega_1 + \omega_2 + \omega_3 - \omega_4)t$$

and
$$\theta_- = (\beta_1 + \beta_2 - \beta_3 - \beta_4)z - (\omega_1 + \omega_2 - \omega_3 - \omega_4)t$$

These two conditions give rise to different four-wave mixing phenomena. To minimise either θ_+ or θ_- requires that certain conditions are satisfied by both the frequencies of the signals and their propagation constants. These conditions effectively amount to conservation of energy and momentum before and after the FWM interaction. The satisfaction of the latter condition is referred to as phase matching. The frequency condition for equation 6.2-5 is that $\omega_4 = \omega_1 + \omega_2 + \omega_3$, which permits *third harmonic generation* in the fully degenerate case when $\omega_1 = \omega_2 = \omega_3$ and also generation of waves at $2\omega_1 + \omega_2$ if $\omega_1 = \omega_3 \neq \omega_2$, i.e. the partially degenerate case. Phase matching of propagation constants for this condition ($\Delta\beta = \beta_1 + \beta_2 + \beta_3 - \beta_4 = 0$) is difficult to satisfy and in practice it is the second of the two FWM mechanisms that is normally observed in fibres.

Minimisation of equation 6.2-6 occurs when the frequencies of the waves satisfy the relation $\omega_1 + \omega_2 = \omega_3 + \omega_4$ and the associated phase matching requirements are met, i.e. $\Delta\beta = \beta_1 + \beta_2 - \beta_3 - \beta_4 = 0$. This can occur for the non-degenerate case, where $\omega_1 \neq \omega_2 \neq \omega_3 \neq \omega_4$, or the partially degenerate case, in which $\omega_1 = \omega_2 \neq \omega_3 \neq \omega_4$. Both of these situations are illustrated in figure 1, in which incident frequencies ω_1 , ω_2 and ω_3 generate extra frequencies at $\omega_{ijk} = \omega_i + \omega_j - \omega_k$ through four-wave mixing.

6.3 FWM in Optical Fibre - Dispersion and Phase Matching

The small core diameters of single mode optical fibres (typically ~ 5 to 10 μm) permit high optical intensities to be maintained over long distances and therefore fibres offer excellent conditions for non-linear effects to occur. Phase matching for four-wave mixing is dependent on the relative group velocities of the interacting signals, which

is determined by the dispersion of the fibre and will be a function of the frequencies of the signals and possibly their relative polarisation.

Dispersion causes the frequency or polarisation components of an optical signal to propagate at different velocities through the fibre - leading to pulse spreading and a consequent reduction in the maximum signal bandwidth. In the interests of maximising the information-carrying capacity of an optical fibre link, it is desirable to reduce dispersion to the smallest possible value at the transmission wavelength. However, four-wave mixing becomes most efficient under these circumstances due to increased phase matching and can become a serious limitation under some circumstances. This section describes the link between fibre dispersion and four-wave mixing efficiency.

6.3.1 Chromatic Dispersion

Optical radiation propagating in bulk materials experiences a frequency-dependent refractive index and therefore a frequency-dependent velocity through the medium. For modulated optical signals and pulses with non-zero spectral width, it is the group velocity, v_g , that describes propagation of the signal through the medium. The propagation of the signal may also be described in terms of the group delay per unit length, $\tau_g = 1/v_g$, which is also a function of optical frequency, ω , or alternatively optical wavelength, λ . The derivative of the group delay with respect to wavelength in homogenous media is referred to as the *material dispersion*.

For waves propagating along the core of an optical fibre, the material dispersion is accompanied by dispersion due to the interaction of the optical field with the cladding material beyond the core-cladding interface. This *waveguide dispersion* is always negative and adds to the material dispersion to give the overall *chromatic dispersion* of the fibre, D_c . For the purposes of studying phase matching, the modal propagation constant of a single mode fibre is often expanded as a Taylor series about one of the interacting frequencies, such as ω_3 . The propagation constant difference due to chromatic dispersion $\Delta\beta = \beta(\omega_1) + \beta(\omega_2) - \beta(\omega_3) - \beta(\omega_4)$ can then be expressed as

$$\Delta\beta = \frac{\lambda_3}{\omega_3} \Delta\omega_{13} \Delta\omega_{23} \left[D_c(\omega_3) + \left(\frac{1}{2\pi\omega_3} \right) (\Delta\omega_{13} + \Delta\omega_{23}) \frac{dD_c(\omega_3)}{d\lambda} \right], \quad 6.3.1-1$$

where $\Delta\omega_{ij} = |\omega_i - \omega_j|$ and λ_3 is the vacuum wavelength of radiation with angular frequency ω_3 .

The overall chromatic dispersion of a fibre can be engineered to change sign from negative to positive near a particular wavelength by balancing chromatic and waveguide dispersion to produce *dispersion shifted fibre* (DSF). At the zero dispersion wavelength, λ_0 , the change in group delay with respect to wavelength is zero and signals at wavelengths close to λ_0 will propagate with almost identical group velocities - eliminating pulse spreading and maximising fibre bandwidth.

6.3.2 Polarisation Mode Dispersion

Single mode fibre actually supports two orthogonal polarisation states into which any input signal may be resolved. In a perfect fibre, these polarisation eigenmodes remain unchanged along the entire length and share the same phase velocity. Under these circumstances, radiation launched into the fibre at any arbitrary polarisation will emerge from the fibre with the same polarisation. However, random deviations from the perfect cylindrically symmetric core-cladding geometry found in actual fibres break the degeneracy of the two states and a phase velocity mismatch occurs between the modes.

For a given length of fibre, the random local birefringence leads to an output polarisation that varies rapidly when the frequency of polarised launched radiation is varied except for two states of launch polarisation known as the *principle states of polarisation* (PSP). Radiation launched into the fibre with either of these polarisation states remains in this single polarisation state throughout the entire fibre and consequently emerges with minimum pulse distortion. The difference in group velocity between the two PSPs is generally the difference in velocity between the fastest and slowest pulses in the fibre and the RMS delay between them is known as the *polarisation mode dispersion* (PMD) of the fibre. Optical power may be coupled between the two principle states of polarisation by fibre imperfections with spatial frequency proportional to the group velocity mismatch between them. This polarisation mode coupling causes an optical pulse launched with arbitrary polarisation to propagate with a group velocity that lies somewhere between those of the two principle states of polarisation. The random nature of the coupling causes the optical pulse to emerge temporally broadened – reducing the maximum transmission bit rate for a given BER.

By deliberately removing the cylindrical symmetry of a single mode fibre, it is possible to produce two polarisation states that are sufficiently non-degenerate that typical deviations in the fibre geometry are insufficient to cause coupling between them. Under these circumstances, light launched into one of the modes will remain in that mode along the entire length of the fibre. This is the basis of the highly birefringent *polarisation maintaining fibre* (PMF) in which the two principle polarisation modes have quite different group velocities.

The group velocity mismatch that can be achieved by launching radiation into the orthogonal modes of highly birefringent polarisation maintaining fibre may be used to achieve phase matching for four-wave mixing when chromatic and non-linear dispersion are insufficient. Since the birefringence of the fibre is dependent on the geometry, it may be tuned using a number of methods including lateral compression, bending, or varying the temperature of the fibre²⁸.

6.3.3 Phase Matching in Optical Fibres

The phase matching conditions for maximum parametric gain to occur in an optical fibre require consideration of the non-linear effects of self-phase modulation and cross-phase modulation on the intense pump signals as these effects will tend to modulate the phase of the pump. When these effects are included in the analysis, the

phase matching conditions for non-degenerate and partially degenerate four-wave mixing are given by equations 6.3.3-1 and 6.3.3-2 respectively:²⁷

$$\kappa = \Delta\beta + \gamma(p_1(0) + p_2(0)) = 0, \quad 6.3.3-1$$

$$\kappa = \Delta\beta + \gamma 2p_1(0) = 0 \quad 6.3.3-2$$

The parameter γ is the non-linearity coefficient of the fibre and $p_1(0)$ and $p_2(0)$ are the incident pump optical powers at the fibre input. It is assumed here that the optical power at the pump wavelengths is unaffected by the transfer of power to the signal wavelengths. This is known as the *undepleted pump* or *small-signal* regime. The optimum phase mismatch for the case of a depleted pump wavelength has been found to be four times smaller than in equation 6.3.3-2 for the partially degenerate case². The non-linearity coefficient is related to the third-order susceptibility through the second-order refractive index, n_2 , such that

$$\gamma = \frac{n_2 \omega}{A_{eff} c} = \frac{3}{8n} \text{Re}(\chi_3), \quad 6.3.3-3$$

where ω is the average optical frequency of the four signals, A_{eff} is the effective area of the fibre core (see Appendix B), n is the linear refractive index of the medium and $\text{Re}(\chi_3)$ is the real part of the third-order susceptibility.

The phase mismatch in equation 6.3.3-1 and 6.3.3-2 can be separated into contributions due to material dispersion, $\Delta\beta_M$, waveguide dispersion, $\Delta\beta_w$, and non-linearity, $\Delta\beta_{NL}$, such that the condition for phase matching becomes

$$\kappa = \Delta\beta_M + \Delta\beta_w + \Delta\beta_{NL} = 0 \quad 6.3.3-4$$

In standard single mode fibres, the waveguide dispersion is very small as all modes experience almost identical waveguide parameters and phase matching must therefore be achieved by using combinations of the material dispersion and non-linear effects only. However, polarisation-maintaining fibres do have non-zero waveguide dispersion due to the mismatch between the propagation constants of the two polarisation modes and therefore phase matching may be achieved by balancing all three mismatch terms. Phase matching in single mode optical fibres can therefore be achieved using any of the following conditions:

- 1) At wavelengths below the zero dispersion wavelength of the fibre, i.e. in the *normal dispersion regime*, the phase mismatch due to material dispersion is positive, so a negative value of $\Delta\beta_w$ is required. This can be achieved using the two orthogonal modes of polarisation maintaining fibre at optical powers low enough not to cause non-linear effects.
- 2) At wavelengths close to λ_0 , the material dispersion is reduced to very low levels comparable with the waveguide dispersion. In this regime, phase matching may be achieved for low optical power levels, when $\Delta\beta_{NL} \approx 0$.

- 3) Above the zero dispersion wavelength of the fibre, i.e. in the *anomalous dispersion regime*, the phase mismatch due to material dispersion is negative and may be compensated by a correspondingly positive non-linear phase mismatch component, $\Delta\beta_{NL}$, by increasing the optical powers of the pump signals²⁹.

6.4 Efficiency of Four-Wave Mixing

The time-averaged optical power generated at frequency ω_4 for co-polarised input signal at ω_1 , ω_2 and ω_3 is given by³⁰

$$p_4(L) = \eta \left(\frac{1024\pi^6}{n^4 \lambda^2 c^2} \right) \left(\frac{L_{eff}}{A_{eff}} \right)^2 (D\chi_3)^2 p_1(0)p_2(0)p_3(0) \exp(-\alpha L) \quad 6.4-$$

where η is the *four-wave mixing efficiency*, D in this case is the degeneracy factor and L_{eff} is the *effective length* of the fibre. The degeneracy factor takes a value of 1, 3 or 6 depending on whether, respectively, all, two or none of the incident frequencies are the same. Effective length is a parameter commonly used in the study of non-linear effects in fibres and is defined as the length of lossless fibre that would generate the same amount of non-linearity as the lossy fibre under consideration. The use of effective length permits simple intercomparisons of non-linearity in fibre with different attenuation characteristics. It is defined as

$$L_{eff} = \frac{1 - \exp(-\alpha L)}{\alpha}$$

where α is the attenuation of the fibre in neper/km and is related to the attenuation coefficient in dB/km by

$$\alpha(\text{neper/km}) = \frac{\ln(10)}{10} \alpha'(\text{dB/km}) \quad 6.4-3$$

The four-wave mixing efficiency parameter, η , is a function of phase mismatch, $\Delta\beta$, and is the most useful measurable quantity for describing four-wave mixing in optical fibres. The efficiency is normalised to 1 at zero phase mismatch and is given by

$$\eta = \frac{p_4(L, \kappa)}{p_4(L, \kappa = 0)} = \frac{\alpha^2}{\alpha^2 + (\kappa)^2} \left[1 + \frac{4 \exp(-\alpha L) \sin^2(\kappa L)}{\{1 - \exp(-\alpha L)\}^2} \right] \quad 6.4-4$$

where κ is defined in equation 6.4-4 as being the net phase mismatch between the four signals due to material and waveguide dispersion and non-linear effects. Four-wave mixing efficiency is usually measured as a function of the *equivalent frequency separation* between the waves involved, $\Delta\omega_{eq}$, where

$$\Delta\omega_{eq} = [\omega_{13}\omega_{23}]^{1/2}$$

6.4.1 Polarisation Dependence

Four-wave mixing is dependent on both the absolute and relative polarisations of the interacting waves, with maximum efficiency for parallel linearly polarised waves. In general, input waves are not linearly but elliptically polarised and the states of polarisation evolve along the fibre due to residual birefringence. The exception to this is standard, non-polarisation-maintaining, fibre is when the light is launched with its polarisation axis parallel to one of the principle states of polarisation (see section 6.3.2). Under these circumstances, the state of polarisation still evolves along the fibre but waves with closely spaced frequencies evolve together and maintain parallel, if not linear, polarisation. Experimental investigations into FWM in fibres are usually performed using polarisation controllers with each optical source to allow adjustment of the relative and absolute states of input polarisation. Typically, the polarisation controllers are simply adjusted until FWM is maximised, without attempting to locate the PSPs of the fibre or ensure linear polarisation^{31, 30, 2}.

6.5 CW vs. Pulsed / Modulated Signals

When the waves interacting via four-wave mixing are continuous, the efficiency of the process depends on the aggregate phase matching between the signals over the entire length of the fibre. The dependence of the efficiency on the distribution of phase matching along the fibre is non-trivial, having a number of peaks that become increasingly complex for longer lengths of fibre³¹. Fibre parameters such as the dispersion slope and second-order refractive index have been successfully derived from the efficiency curve of a 1.1 km fibre along which phase matching remained effectively constant. For longer fibres, the FWM can be predicted fairly successfully if the fibre parameter distribution is known³² and although the reverse is also possible, it requires fitting the results of a model to the measured efficiency and becomes more difficult with increasing fibre length.

In the case of pulsed signals, dispersion causes the pump and signal pulses to travel at different velocities so that they may not overlap along the full length of the fibre. Four-wave mixing efficiency between pulsed signals is then governed by the length of the overlap region in the fibre but is modified by the phase matching within this window. The length and position of the overlap region in the fibre depends on the lengths of the pump and signal pulses, their relative delay at the fibre input, their wavelengths and the dispersion of the fibre.

Four-wave mixing of pulses in optical fibres have implications in WDM communication systems, where interaction between neighbouring channels may be dependent on the type of modulation scheme employed (i.e. RZ or NRZ) and the actual data being transmitted in each channel. Applications of FWM between pulses include distributed fibre measurements³³ and high speed all-optical demultiplexing of optical time division multiplexed (OTDM) signals^{2,34}.

7. Appendix B - The Effective Area Parameter

Silica, the basic constituent of optical fibres has one of the lowest second-order refractive indices of any optical material. Nevertheless, non-linear effects occur in optical fibres because the high optical intensity required can be achieved due to tight confinement of the optical field over long distances. The average intensity is given by the total optical power divided by the area over which it is distributed. The field distribution of the fundamental mode in a single mode fibre is non-uniform inside the fibre core and also extends into the cladding, so a weighted *effective area* is used rather than the geometrical area of the actual fibre core when calculating the non-linear characteristics of the fibre. The effective area is calculated from the modal field distribution, $F(x, y)$, using

$$A_{eff} = \frac{\left(\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy \right)^2}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |F(x, y)|^4 dx dy} \quad 7-1$$

The fundamental mode of a conventional single mode fibres is often approximated by a Gaussian field distribution such that

$$F(x, y) = \exp\left\{-\frac{(x^2 + y^2)}{w^2}\right\}, \quad 7-2$$

where $2w$ is the *mode field diameter* (MFD). Under this Gaussian field assumption, the effective area can be calculated to be simply

$$A_{eff} = \pi w^2$$

Dispersion shifted fibres use more complicated refractive index profiles than conventional single mode fibre so that the waveguide dispersion adds to the chromatic dispersion and shifts the zero dispersion wavelength to a new value. The mode fields in these fibres cannot be approximated by a Gaussian distribution and the effective area must be calculated from the mode field diameter using a fibre-dependent correction factor, K , such that³⁵

$$A_{eff} = K\pi w^2 \quad 7-3$$

The modal field distribution is strongly dependent on the wavelength of the guided radiation and consequently the effective area for a typical fibre can vary from 10-20 μm^2 at visible wavelengths to 50-80 μm^2 at communications wavelengths²⁷. The nonlinearity coefficient of the fibre is therefore also a function of the wavelength.

8. Appendix C - Glossary of Symbols

α	Attenuation Coefficient (neper/m)
α'	Attenuation Coefficient (dB/m)
β	Propagation Constant (radians/m)
$\Delta\beta$	Phase Mismatch (radians/m)
χ , χ_2 and χ_3	Linear, Second, and Third-Order Susceptibility Tensors
ϵ_0	Electric Permittivity of Free Space (F/m)
γ	Non-Linearity Coefficient (radians/m ³)
η	Four-Wave Mixing Efficiency (normalised to 1)
κ	Phase Mismatch (radians/m)
λ	Wavelength (m)
λ_0	Zero Dispersion Wavelength (m)
τ_g	Group Delay per Unit Length (s/m)
ω	Angular Frequency (radians/s)
A_{eff}	Effective Area (m ²)
c	Velocity of Light in Vacuum
D	Fibre Dispersion (ps/(nm km))
$D_c(\lambda)$	Chromatic Dispersion at Vacuum Wavelength λ
$\mathbf{E}$	Electric Field Vector
L , L_{eff}	Length and Effective Length Respectively (m)
p	Optical Power (Watts)
$\mathbf{P}$	Polarisation vector
v_g	Group Velocity (m/s)
w	Mode Field Radius

9. Appendix D - Glossary of terms

ADP	Ammonium Dihydrogen Phosphate
BER	Bit-Error Ratio
CW	Continuous Wave
NDFWM	Non-Degenerate Four-Wave Mixing
DFB	Distributed Feedback
DPSK	Differential Phase Shift Keying
DSF	Dispersion-Shifted Fibre
EDFA	Erbium-Doped Fibre Amplifier
FWM	Four-Wave Mixing
ITU	International Telecommunication Union
KDP	Potassium Dihydrogen Phosphate
MFD	Mode Field Diameter
NOLM	Nonlinear Loop Mirror
NRZ	Non-Return-to-Zero
NZDSF	Non-Zero Dispersion-Shifted Fibre
OPC	Optical Phase Conjugator
OTDM	Optical Time Division Multiplexing
OTDR	Optical Time Domain Reflectometry
PDFWM	Partially Degenerate Four-Wave Mixing
PMD	Polarisation Mode Dispersion
PMF	Polarisation Maintaining Fibre
PSK	Phase-Shift Keying
PSP	Principle State of Polarisation
RZ	Return-to-Zero
SDH	Synchronous Digital Hierarchy
SNR	Signal to Noise Ratio
SONET	Synchronous Optical Network
SPM	Self-Phase Modulation
SRS	Stimulated Raman Scattering
TDM	Time-Division Multiplexing
WDM	Wavelength Division Multiplexing
XPM	Cross-Phase Modulation

10. Appendix E - References

- ¹ Inoue K., "Tunable and Selective Wavelength Conversion Using Fibre Four-Wave Mixing with Two Pump Lights", *IEEE Photonics Technology Letters*, **6**, pp. 1451-1453, 1994.
- ² Hedekvist P. O. *et al.*, "Fibre Four-Wave Mixing Demultiplexing with Inherent Parametric Amplification", *IEEE Journal of Lightwave Technology*, **15**, pp. 2051-2058, 1997.
- ITU Recommendation G.692 "Optical Interfaces for Multichannel Systems With Optical Amplifiers"
- ⁴ Saint Georges E. and Watson T. J., "Experimental Study of Four-wave Mixing on Dispersion-Shifted Fiber and Low Dispersion Fibers With a Dense Multiwavelength Source", *Technical Digest, Optical Fibre Communication Conference*, pp. 67-69, 1997.
- ⁵ Yonenaga K. *et al.*, in *Optical Fiber Communication Conference*, Vol. 2 of 1996 OSA Technical Digest Series (Optical Society of America, Washington, DC., 1996), PD1-4.
- ⁶ Yonenaga K. and Hagimoto K., "10-Gbit/s Four-Channel WDM Transmission Experiment Over 2400-km DSF Using Optical DPSK Direct Detection Scheme", *Technical Digest, Optical Fibre Communication Conference*, pp. 331-332, 1997.
- ⁷ Tkach R. W. *et al.*, "Four-Photon Mixing and High-Speed WDM Systems", *Journal of Lightwave Technology*, **13**, pp. 841-849, 1995.
- ⁸ Nakajima K. *et al.*, "Four-Wave-Mixing Suppression Effect of Dispersion Varying Fiber", *IEEE Photonics Technology Letters*, **10**, pp. 537-539, (1998).
- ⁹ Ohashi M., *et al.*, "Phase-Matched Light Amplification by Three-Wave Mixing Process in a Birefringent Fiber due to Externally Applied Stress", *Applied Physics Letters*, **41**, pp. 1111-1113, 1982.
- ¹⁰ Watanabe S. and Chikama T., "Highly Efficient Conversion and Parametric Gain of Nondegenerate Forward Four-Wave Mixing in a Singlemode Fibre", *Electronics Letters*, **30**, pp. 163-164, 1994.
- ¹¹ Chi W. D. J., *et al.*, "Analysis of Wavelength Conversion Using Highly Nondegenerate Four-Wave Mixing in $\lambda/4$ -Shifted DFB Lasers", *IEEE Journal of Quantum Electronics*, **34**, pp. 1861-1866, 1998.
- ¹² Weisenfeld J. M., *et al.*, "Wavelength Conversion Techniques for Optical Networks", pp. 655-656 *Proc. European Conference on Optical Communications*, Madrid, 1998.
- ¹³ Tsujikawa K. *et al.*, "Enhancement of Four-Wave Mixing in a Tapered Core Fiber With Uniform Chromatic Dispersion", *Technical Digest, Optical Fibre Communication Conference*, pp. 70-71, 1997.
- ¹⁴ Watanabe S., *et al.*, "Compensation of Chromatic Dispersion in a Single-Mode Fiber by Optical Phase Conjugation", *IEEE Photonics Technology Letters*, **5**, pp. 92-95, 1993.
- ¹⁵ Hedekvist P. O., *et al.*, "Polarisation Dependence and Efficiency in a Fiber Four-Wave Mixing Phase Conjugator with Orthogonal Pump Waves", *IEEE Photonics Technology Letters*, **8**, pp. 776-778, 1996.
- ¹⁶ Kitayama K., "Highly Stabilised Millimetre-Waves Generation and Transport by Using Fiber-Optic Four-Wave Mixings", *Technical Digest, Optical Fibre Communication Conference*, pp. 338-339, 1997.
- ¹⁷ Eisert M. and Jopson M., "Nondestructive Position-Resolved Measurement of the Zero-Dispersion Wavelength in an Optical Fibre", *IEEE Journal of Lightwave Technology*, **15**, pp. 135-143, 1997.

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- ¹⁸ Brener I. *et al.*, "A New Technique for Zero-Dispersion Wavelength Mapping in Single Mode Fiber with High Spatial Resolution", Proceedings of The Symposium on Optical Fiber Measurements, *pp.* 163-164, Boulder, Colorado, 1998.
- ¹⁹ Artiglia M. *et al.*, "Measurement of Local Zero-Dispersion Wavelengths on Optical Links for WDM Optically Amplified Transmission Systems", *pp.* 68-71, Proc. 4th Optical Fibre Measurement Conference, Teddington, UK, 1997.
- ²⁰ Onaka H., *et al.*, "Measuring the Longitudinal Distribution of Four-wave Mixing Efficiency in Dispersion-Shifted Fibers", IEEE Photonics Technology Letters, **6**, *pp.* 1454-1456, 1994.
- ²¹ Gripp J. and Mollenauer L. F., "OTDR-like Dispersion-map Measurements", *pp.* 153-158, Technical Digest: Symposium on Optical Fiber Measurements - SOFM '98, Boulder, Colorado, 1998.
- ²² Stolen R.H. *et al.*, "Measurement of the Nonlinear Refractive Index of Long Dispersion-Shifted Fibers by Self-Phase Modulation at 1.55 μm ", IEEE Journal of Lightwave Technology, **16**, *pp.* 1006-1012, 1998.
- ²³ Boskovic A., *et al.*, "Direct Continuous-wave Measurement of n_2 in Various Types of Telecommunication Fiber at 1.55 μm ", Optics Letters, **21**, *pp.* 1966-1968, 1996.
- ²⁴ Prigent L. and Hamaide J.-P., "Measurement of Fiber Nonlinear Kerr Coefficient by Four-Wave Mixing", IEEE Photonics Technology Letters, **5**, *pp.* 1092-1095, 1993.
- ²⁵ Shibata N. *et al.*, Optics Letters, **10**, *pp.* 154 - , 1985.
- ²⁶ Ewart Lowe, "Current European WDM Deployment Trends", IEEE Communication Magazine, **36**, No. 2, *pp.* 46-50, 1998.
- ²⁷ Agrawal G. P., "Nonlinear Fiber Optics - 2nd edition", Academic Press, London, 1995.
- ²⁸ Rashleigh S.C., "Origins and Control of Polarization Effects in Single-Mode Fibers", IEEE Journal of Lightwave Technology, **LT-1**, *pp.* 312-331, 1983.
- ²⁹ Yamamoto T. and Nakazawa M., "Highly Efficient Four-Wave Mixing in an Optical Fiber with Intensity Dependent Phase Matching", IEEE Photonics Technology Letters, **9**, *pp.* 327-329, 1997.
- ³⁰ Shibata N., *et al.*, "Phase-Mismatch Dependence of Efficiency of Wave Generation through Four-Wave Mixing in a Single-Mode Optical Fiber", IEEE Journal of Quantum Electronics, **QE-23**, *pp.* 1205-1210, 1987.
- ³¹ Inoue K., "Four-Wave Mixing in an Optical Fiber in the Zero-Dispersion Wavelength Region". IEEE Journal of Lightwave Technology, **10**, *pp.* 1553-1561, 1992.
- ³² Onaka H., *et al.*, "Measuring the Longitudinal Distribution of Four-wave Mixing Efficiency in Dispersion-Shifted Fibers", IEEE Photonics Technology Letters, **6**, *pp.* 1454-1456, 1994.
- ³³ Jopson R.M., "Non-Destructive Dispersion-Zero Measurements Along an Optical Fibre", Electronics Letters, **31**, *pp.* 2115-2117, 1995.
- ³⁴ Kawanishi S., "Ultrahigh-Speed Optical Time-Division-Multiplexed Transmission Technology Based on Optical Signal Processing", IEEE Journal of Quantum Electronics, **34**, *pp.* 2064-2078, 1998.
- ³⁵ Namihira Y., "Relationship Between Nonlinear Effective Area and Mode Field Diameter for Dispersion Shifted Fibres", Electronics Letters, **30**, *pp.* 262-263, 1994.